Computer algebra independent integration tests

1_Algebraic_functions/1.1_Binomial_products/1.1.2Quadratic/1.1.2.8P(x)(cx)^m(a+bx^2)

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

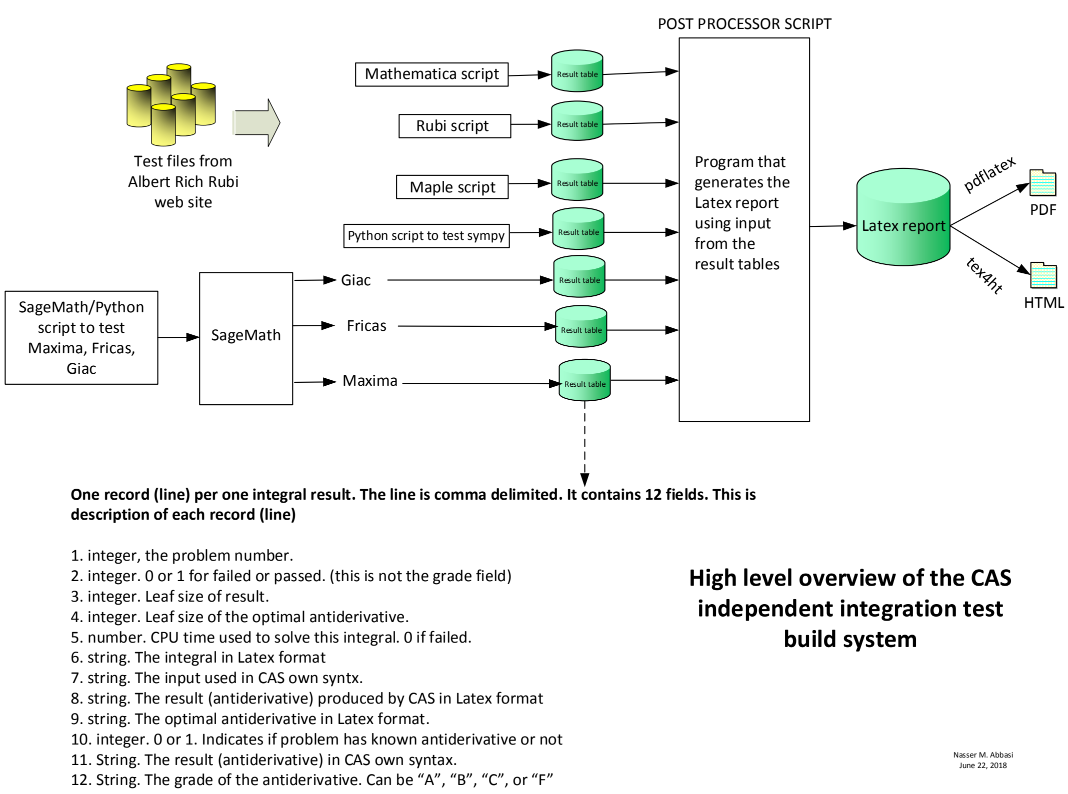
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expressi

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (174)	% 0. (0)
Rubi in Sympy	% 70.69 (123)	% 29.31 (51)
Mathematica	% 100. (174)	% 0. (0)
Maple	% 97.7 (170)	% 2.3 (4)
Maxima	% 24.71 (43)	% 75.29 (131)
Fricas	% 97.7 (170)	% 2.3 (4)
Sympy	% 83.91 (146)	% 16.09 (28)
Giac	% 97.7 (170)	% 2.3 (4)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

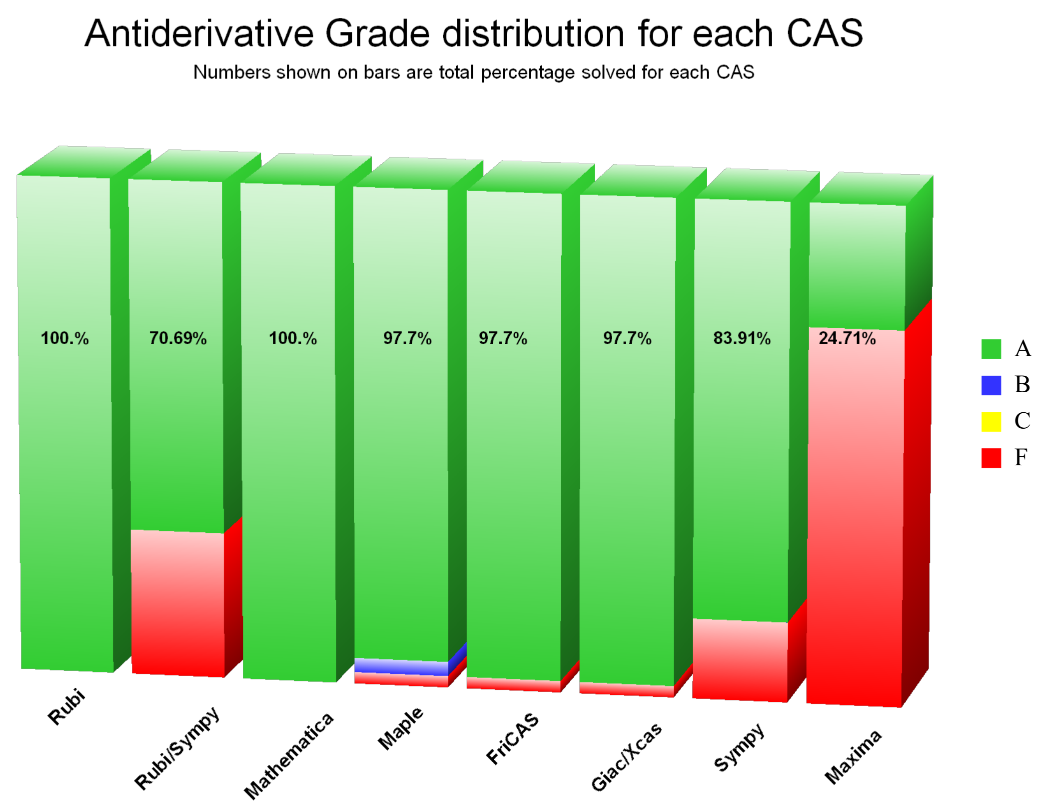
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

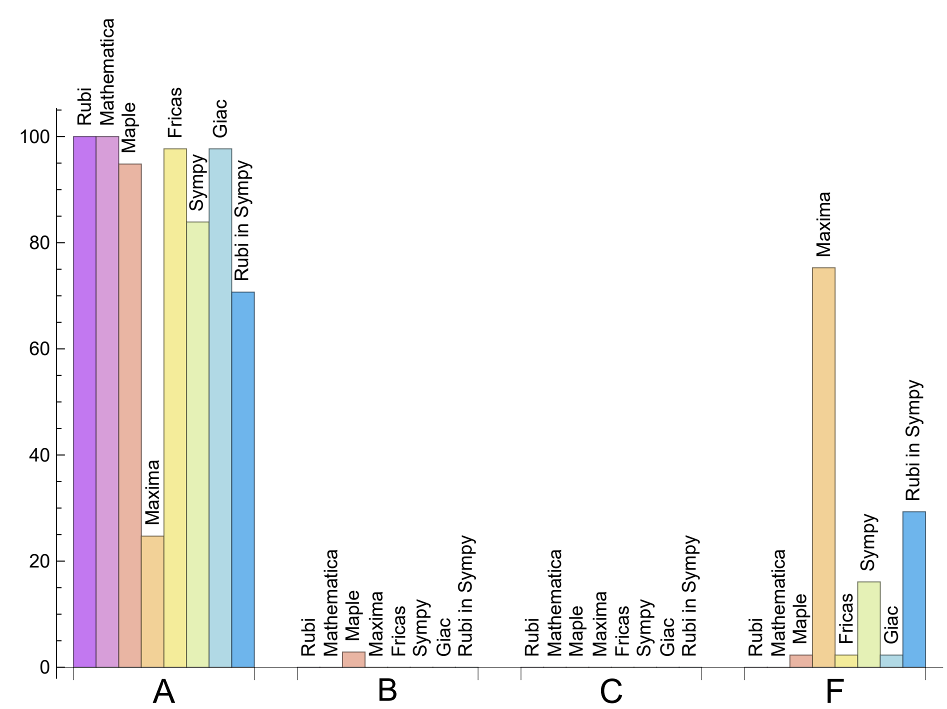
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	70.69	0.	0.	29.31
Mathematica	100.	0.	0.	0.
Maple	94.83	2.87	0.	2.3
Maxima	24.71	0.	0.	75.29
Fricas	97.7	0.	0.	2.3
Sympy	83.91	0.	0.	16.09
Giac	97.7	0.	0.	2.3

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.36	130.41	1.	121.	1.
Rubi in Sympy	52.13	111.67	0.92	105.	0.9
Mathematica	0.17	113.76	0.91	103.	0.91
Maple	0.01	154.91	1.13	134.	1.1
Maxima	1.38	131.16	1.45	130.	1.34
Fricas	0.31	42.54	0.41	1.	0.01
Sympy	16.6	299.03	2.59	208.5	1.79
Giac	0.22	189.06	1.44	166.5	1.33

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 102, 113, 114, 115, 116, 117, 118, 119, 124, 125, 133, 134, 135, 136, 139, 140, 141, 142, 159, 160, 161, 162, 168, 172, 173, 174}

Not solved by Mathematica {}

Not solved by Maple {58, 59, 60, 61}

Not solved by Maxima {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 40, 41, 42, 47, 48, 55, 56, 57, 58, 59, 60, 61, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174}

Not solved by Fricas {58, 59, 60, 61}

Not solved by Sympy {47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 132, 140, 141, 142, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174}

Not solved by Giac {58, 59, 60, 61}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	100	115	0	1	192	126	114
normalized size	1	1.	0.79	0.91	0.	0.01	1.51	0.99	0.9
time (sec)	N/A	0.312	0.119	0.011	0.	0.266	10.285	0.215	27.581

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	94	0	1	165	109	94
normalized size	1	1.	0.85	0.9	0.	0.01	1.59	1.05	0.9
time (sec)	N/A	0.192	0.092	0.009	0.	0.268	6.905	0.218	15.769

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	75	0	1	124	92	70
normalized size	1	1.	0.99	0.94	0.	0.01	1.55	1.15	0.88
time (sec)	N/A	0.094	0.07	0.007	0.	0.259	6.511	0.223	8.708

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	53	0	1	70	74	58
normalized size	1	1.	1.	0.79	0.	0.01	1.04	1.1	0.87
time (sec)	N/A	0.064	0.058	0.007	0.	0.263	3.742	0.231	7.015

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	91	78	0	1	107	105	70
normalized size	1	1.	1.15	0.99	0.	0.01	1.35	1.33	0.89
time (sec)	N/A	0.213	0.099	0.008	0.	0.285	5.413	0.23	20.151

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	87	97	0	1	124	138	65
normalized size	1	1.	1.16	1.29	0.	0.01	1.65	1.84	0.87
time (sec)	N/A	0.205	0.084	0.011	0.	0.273	4.835	0.233	18.678

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	96	121	0	1	107	220	73
normalized size	1	1.	1.2	1.51	0.	0.01	1.34	2.75	0.91
time (sec)	N/A	0.21	0.17	0.013	0.	0.275	5.66	0.231	21.717

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	119	134	0	1	318	155	139
normalized size	1	1.	0.79	0.89	0.	0.01	2.12	1.03	0.93
time (sec)	N/A	0.327	0.143	0.012	0.	0.28	24.549	0.229	30.291

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	108	113	0	1	287	139	114
normalized size	1	1.	0.85	0.89	0.	0.01	2.26	1.09	0.9
time (sec)	N/A	0.209	0.129	0.01	0.	0.261	16.405	0.222	17.954

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	94	0	1	223	120	90
normalized size	1	1.	0.97	0.91	0.	0.01	2.17	1.17	0.87
time (sec)	N/A	0.113	0.114	0.007	0.	0.277	15.408	0.224	10.622

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	69	0	1	219	103	80
normalized size	1	1.	1.01	0.79	0.	0.01	2.52	1.18	0.92
time (sec)	N/A	0.076	0.082	0.006	0.	0.264	9.32	0.223	8.661

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	112	107	0	1	218	135	97
normalized size	1	1.	1.06	1.01	0.	0.01	2.06	1.27	0.92
time (sec)	N/A	0.311	0.166	0.008	0.	0.275	11.205	0.225	35.657

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	113	126	0	1	184	167	97
normalized size	1	1.	1.05	1.17	0.	0.01	1.7	1.55	0.9
time (sec)	N/A	0.306	0.155	0.011	0.	0.283	7.99	0.229	30.454

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	150	0	1	182	258	109
normalized size	1	1.	1.02	1.35	0.	0.01	1.64	2.32	0.98
time (sec)	N/A	0.292	0.181	0.01	0.	0.274	10.089	0.23	28.355

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	138	153	0	1	469	189	162
normalized size	1	1.	0.8	0.88	0.	0.01	2.71	1.09	0.94
time (sec)	N/A	0.356	0.176	0.012	0.	0.282	46.672	0.229	34.775

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	126	132	0	1	442	173	139
normalized size	1	1.	0.84	0.88	0.	0.01	2.95	1.15	0.93
time (sec)	N/A	0.23	0.154	0.008	0.	0.271	32.261	0.225	20.682

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	113	0	1	354	154	116
normalized size	1	1.	0.94	0.9	0.	0.01	2.81	1.22	0.92
time (sec)	N/A	0.132	0.146	0.007	0.	0.268	30.001	0.231	13.41

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	108	85	0	1	348	136	100
normalized size	1	1.	1.01	0.79	0.	0.01	3.25	1.27	0.93
time (sec)	N/A	0.098	0.137	0.006	0.	0.266	18.919	0.217	10.549

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	138	0	1	323	169	121
normalized size	1	1.	1.	1.05	0.	0.01	2.45	1.28	0.92
time (sec)	N/A	0.414	0.251	0.009	0.	0.283	21.391	0.22	50.353

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	135	158	0	1	318	203	124
normalized size	1	1.	0.99	1.16	0.	0.01	2.34	1.49	0.91
time (sec)	N/A	0.405	0.224	0.013	0.	0.286	15.052	0.225	45.502

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	181	0	1	279	296	136
normalized size	1	1.	0.96	1.28	0.	0.01	1.98	2.1	0.96
time (sec)	N/A	0.398	0.25	0.013	0.	0.275	16.783	0.236	42.957

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	96	0	1	150	100	94
normalized size	1	1.	0.76	0.92	0.	0.01	1.44	0.96	0.9
time (sec)	N/A	0.278	0.104	0.011	0.	0.261	8.067	0.227	24.688

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	67	75	0	1	94	82	73
normalized size	1	1.	0.83	0.93	0.	0.01	1.16	1.01	0.9
time (sec)	N/A	0.17	0.068	0.009	0.	0.267	4.991	0.227	14.725

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	55	0	1	70	68	48
normalized size	1	1.	1.09	0.98	0.	0.02	1.25	1.21	0.86
time (sec)	N/A	0.08	0.054	0.006	0.	0.262	4.575	0.219	7.393

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	0	1	102	53	37
normalized size	1	1.	1.07	0.86	0.	0.02	2.37	1.23	0.86
time (sec)	N/A	0.048	0.032	0.007	0.	0.269	1.351	0.228	6.197

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	52	0	1	99	78	48
normalized size	1	1.	1.26	0.98	0.	0.02	1.87	1.47	0.91
time (sec)	N/A	0.133	0.057	0.01	0.	0.294	2.548	0.223	11.299

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	49	0	1	41	88	39
normalized size	1	1.	1.23	1.04	0.	0.02	0.87	1.87	0.83
time (sec)	N/A	0.122	0.068	0.01	0.	0.255	3.017	0.218	9.292

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	68	0	1	66	197	60
normalized size	1	1.	1.	0.94	0.	0.01	0.92	2.74	0.83
time (sec)	N/A	0.193	0.08	0.012	0.	0.273	5.068	0.225	15.156

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	93	0	1	117	95	78
normalized size	1	1.	0.93	1.15	0.	0.01	1.44	1.17	0.96
time (sec)	N/A	0.175	0.139	0.01	0.	0.272	9.077	0.222	16.402

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	72	0	1	83	78	65
normalized size	1	1.	0.91	1.09	0.	0.02	1.26	1.18	0.98
time (sec)	N/A	0.137	0.105	0.009	0.	0.261	6.064	0.22	14.708

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	54	0	1	66	65	41
normalized size	1	1.	1.1	1.12	0.	0.02	1.38	1.35	0.85
time (sec)	N/A	0.074	0.06	0.006	0.	0.26	5.964	0.219	7.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	42	47	46	31	22
normalized size	1	1.	0.96	0.93	1.5	1.68	1.64	1.11	0.79
time (sec)	N/A	0.033	0.028	0.005	1.343	0.265	4.233	0.215	4.261

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	60	0	1	206	80	39
normalized size	1	1.	1.23	1.28	0.	0.02	4.38	1.7	0.83
time (sec)	N/A	0.135	0.102	0.01	0.	0.26	6.89	0.217	13.437

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	80	0	1	235	130	60
normalized size	1	1.	1.04	1.14	0.	0.01	3.36	1.86	0.86
time (sec)	N/A	0.213	0.161	0.013	0.	0.264	9.374	0.223	20.015

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	101	0	1	124	231	88
normalized size	1	1.	0.87	1.06	0.	0.01	1.31	2.43	0.93
time (sec)	N/A	0.304	0.266	0.011	0.	0.269	13.087	0.22	31.506

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	91	0	1	400	95	73
normalized size	1	1.	0.92	1.15	0.	0.01	5.06	1.2	0.92
time (sec)	N/A	0.163	0.153	0.012	0.	0.27	22.728	0.229	16.12

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	41	95	85	141	49	46
normalized size	1	1.	0.83	0.77	1.79	1.6	2.66	0.92	0.87
time (sec)	N/A	0.095	0.052	0.009	1.359	0.256	19.12	0.219	6.834

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	32	29	69	66	95	35	36
normalized size	1	1.	0.68	0.62	1.47	1.4	2.02	0.74	0.77
time (sec)	N/A	0.072	0.04	0.004	1.331	0.266	18.796	0.217	5.806

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	65	84	146	50	42
normalized size	1	1.	0.84	0.78	1.27	1.65	2.86	0.98	0.82
time (sec)	N/A	0.043	0.039	0.004	1.334	0.248	18.697	0.216	5.173

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	79	92	0	1	840	111	65
normalized size	1	1.	1.04	1.21	0.	0.01	11.05	1.46	0.86
time (sec)	N/A	0.219	0.296	0.008	0.	0.257	29.072	0.221	26.638

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	112	0	1	910	161	88
normalized size	1	1.	0.91	1.08	0.	0.01	8.75	1.55	0.85
time (sec)	N/A	0.304	0.219	0.014	0.	0.281	41.41	0.219	35.014

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	103	134	0	1	1034	266	119
normalized size	1	1.	0.8	1.04	0.	0.01	8.02	2.06	0.92
time (sec)	N/A	0.405	0.355	0.012	0.	0.267	55.756	0.222	48.73

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	38	123	24	26	29
normalized size	1	1.	0.89	1.07	1.41	4.56	0.89	0.96	1.07
time (sec)	N/A	0.046	0.034	0.01	1.504	0.252	0.274	0.213	5.943

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	38	123	24	26	29
normalized size	1	1.	0.89	1.07	1.41	4.56	0.89	0.96	1.07
time (sec)	N/A	0.05	0.013	0.007	1.494	0.231	0.268	0.219	7.12

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	30	35	27	41	17
normalized size	1	1.	1.94	0.88	1.76	2.06	1.59	2.41	1.
time (sec)	N/A	0.022	0.017	0.003	1.483	0.227	0.077	0.213	4.406

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	5	8	5
normalized size	1	1.	1.	1.17	1.33	1.33	0.83	1.33	0.83
time (sec)	N/A	0.015	0.007	0.003	1.502	0.229	0.073	0.209	4.391

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	157	265	0	1	0	275	192
normalized size	1	1.	0.74	1.24	0.	0.	0.	1.29	0.9
time (sec)	N/A	0.818	0.299	0.049	0.	0.293	0.	0.228	59.165

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	127	277	0	1	0	186	133
normalized size	1	1.	0.85	1.85	0.	0.01	0.	1.24	0.89
time (sec)	N/A	0.431	0.311	0.018	0.	0.283	0.	0.222	42.434

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	89	95	324	185	0	151	117
normalized size	1	1.	0.67	0.72	2.45	1.4	0.	1.14	0.89
time (sec)	N/A	0.383	0.13	0.009	1.361	0.263	0.	0.231	27.021

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	78	76	342	165	0	109	128
normalized size	1	1.	0.52	0.51	2.3	1.11	0.	0.73	0.86
time (sec)	N/A	0.446	0.121	0.009	1.372	0.278	0.	0.217	31.039

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	84	85	242	177	0	128	124
normalized size	1	1.	0.6	0.61	1.74	1.27	0.	0.92	0.89
time (sec)	N/A	0.359	0.096	0.011	1.385	0.286	0.	0.223	29.082

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	87	88	266	181	0	127	126
normalized size	1	1.	0.63	0.63	1.91	1.3	0.	0.91	0.91
time (sec)	N/A	0.297	0.108	0.009	1.362	0.312	0.	0.219	23.569

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	73	166	161	0	111	107
normalized size	1	1.	0.63	0.61	1.39	1.35	0.	0.93	0.9
time (sec)	N/A	0.182	0.078	0.008	1.35	0.361	0.	0.22	21.33

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	207	185	0	151	110
normalized size	1	1.	0.72	0.76	1.63	1.46	0.	1.19	0.87
time (sec)	N/A	0.153	0.092	0.007	1.484	0.343	0.	0.221	13.633

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	131	169	0	1	0	205	121
normalized size	1	1.	0.95	1.22	0.	0.01	0.	1.49	0.88
time (sec)	N/A	0.418	0.791	0.011	0.	0.328	0.	0.224	61.402

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	158	240	0	1	0	323	163
normalized size	1	1.	0.84	1.28	0.	0.01	0.	1.72	0.87
time (sec)	N/A	0.698	0.348	0.014	0.	0.334	0.	0.224	72.815

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	288	0	1	0	439	207
normalized size	1	1.	0.84	1.32	0.	0.	0.	2.	0.95
time (sec)	N/A	0.867	0.492	0.016	0.	0.379	0.	0.231	85.828

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0	34
normalized size	1	1.	0.96	0.	0.	0.	2.16	0.	0.76
time (sec)	N/A	0.055	0.026	0.039	0.	0.	2.261	0.	7.129

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	192	0	68
normalized size	1	1.	0.9	0.	0.	0.	2.11	0.	0.75
time (sec)	N/A	0.136	0.082	0.045	0.	0.	6.034	0.	15.158

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	0	0	0	204	0	56
normalized size	1	1.	0.76	0.	0.	0.	2.68	0.	0.74
time (sec)	N/A	0.124	0.068	0.047	0.	0.	7.888	0.	14.362

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	0	0	0	298	0	90
normalized size	1	1.	0.83	0.	0.	0.	2.46	0.	0.74
time (sec)	N/A	0.279	0.104	0.047	0.	0.	9.679	0.	39.393

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	1	60	77	60
normalized size	1	1.	1.	0.83	1.11	0.02	0.92	1.18	0.92
time (sec)	N/A	0.147	0.029	0.004	1.353	0.251	0.053	0.211	22.387

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	1	60	77	60
normalized size	1	1.	1.	0.83	1.11	0.02	0.92	1.18	0.92
time (sec)	N/A	0.148	0.03	0.002	1.347	0.258	0.053	0.213	25.708

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	1	60	77	0
normalized size	1	1.	1.	0.83	1.11	0.02	0.92	1.18	0.
time (sec)	N/A	0.123	0.019	0.002	1.351	0.228	0.052	0.212	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	1	56	73	0
normalized size	1	1.	1.	0.85	1.13	0.02	0.93	1.22	0.
time (sec)	N/A	0.091	0.017	0.003	1.345	0.228	0.05	0.218	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	65	65	54	72	0
normalized size	1	1.	1.	0.95	1.16	1.16	0.96	1.29	0.
time (sec)	N/A	0.081	0.03	0.006	1.345	0.247	0.597	0.226	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	65	74	49	68	0
normalized size	1	1.	1.	0.93	1.2	1.37	0.91	1.26	0.
time (sec)	N/A	0.099	0.059	0.01	1.347	0.257	0.623	0.215	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	65	74	49	65	0
normalized size	1	1.	0.94	0.89	1.2	1.37	0.91	1.2	0.
time (sec)	N/A	0.099	0.052	0.009	1.346	0.264	0.851	0.215	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	51	66	74	53	68	0
normalized size	1	1.	1.02	0.94	1.22	1.37	0.98	1.26	0.
time (sec)	N/A	0.1	0.033	0.01	1.348	0.223	1.687	0.212	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	136	1	110	142	100
normalized size	1	1.	0.9	0.94	1.25	0.01	1.01	1.3	0.92
time (sec)	N/A	0.259	0.085	0.003	1.347	0.206	0.074	0.208	35.627

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	136	1	110	142	100
normalized size	1	1.	0.84	0.94	1.25	0.01	1.01	1.3	0.92
time (sec)	N/A	0.258	0.097	0.002	1.334	0.203	0.074	0.239	39.589

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	136	1	110	142	94
normalized size	1	1.	0.88	0.98	1.31	0.01	1.06	1.37	0.9
time (sec)	N/A	0.317	0.096	0.003	1.346	0.205	0.073	0.221	40.116

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	132	1	107	138	0
normalized size	1	1.	0.89	1.	1.33	0.01	1.08	1.39	0.
time (sec)	N/A	0.183	0.084	0.001	1.336	0.2	0.071	0.222	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	100	130	130	104	135	0
normalized size	1	1.	0.96	1.09	1.41	1.41	1.13	1.47	0.
time (sec)	N/A	0.152	0.096	0.005	1.349	0.221	0.722	0.23	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	130	139	99	132	0
normalized size	1	1.	0.98	1.09	1.44	1.54	1.1	1.47	0.
time (sec)	N/A	0.198	0.139	0.01	1.356	0.223	0.758	0.224	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	97	130	139	99	131	0
normalized size	1	1.	0.89	0.99	1.33	1.42	1.01	1.34	0.
time (sec)	N/A	0.186	0.085	0.01	1.351	0.219	1.029	0.223	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	97	131	139	99	131	0
normalized size	1	1.	0.85	0.99	1.34	1.42	1.01	1.34	0.
time (sec)	N/A	0.188	0.103	0.011	1.355	0.234	1.986	0.226	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	150	196	1	163	207	139
normalized size	1	1.	1.	1.01	1.32	0.01	1.09	1.39	0.93
time (sec)	N/A	0.373	0.058	0.002	1.347	0.202	0.086	0.222	46.281

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	150	196	1	165	207	141
normalized size	1	1.	0.84	1.01	1.32	0.01	1.11	1.39	0.95
time (sec)	N/A	0.368	0.155	0.002	1.353	0.208	0.087	0.223	50.038

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	150	196	1	163	207	129
normalized size	1	1.	0.9	1.09	1.42	0.01	1.18	1.5	0.93
time (sec)	N/A	0.405	0.146	0.003	1.386	0.199	0.086	0.239	46.683

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	121	147	192	1	158	201	0
normalized size	1	1.	0.91	1.11	1.44	0.01	1.19	1.51	0.
time (sec)	N/A	0.25	0.138	0.003	1.427	0.197	0.085	0.222	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	148	189	189	158	200	0
normalized size	1	1.	0.94	1.15	1.47	1.47	1.22	1.55	0.
time (sec)	N/A	0.198	0.205	0.006	1.332	0.229	0.872	0.221	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	188	198	150	196	0
normalized size	1	1.	0.99	1.17	1.52	1.6	1.21	1.58	0.
time (sec)	N/A	0.266	0.272	0.01	1.34	0.227	0.903	0.222	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	144	188	198	150	194	0
normalized size	1	1.	0.92	1.07	1.39	1.47	1.11	1.44	0.
time (sec)	N/A	0.284	0.143	0.011	1.345	0.224	1.199	0.221	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	124	146	192	198	153	197	0
normalized size	1	1.	0.89	1.05	1.38	1.42	1.1	1.42	0.
time (sec)	N/A	0.284	0.138	0.011	1.363	0.229	2.157	0.223	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	130	176	0	1	308	217	0
normalized size	1	1.	0.86	1.17	0.	0.01	2.04	1.44	0.
time (sec)	N/A	0.325	0.156	0.008	0.	0.263	2.079	0.222	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	152	0	1	269	185	0
normalized size	1	1.	0.88	1.17	0.	0.01	2.07	1.42	0.
time (sec)	N/A	0.278	0.21	0.008	0.	0.258	2.047	0.223	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	128	0	1	243	151	0
normalized size	1	1.	0.86	1.15	0.	0.01	2.19	1.36	0.
time (sec)	N/A	0.249	0.102	0.007	0.	0.272	1.907	0.22	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	106	0	1	211	119	0
normalized size	1	1.	0.88	1.15	0.	0.01	2.29	1.29	0.
time (sec)	N/A	0.21	0.124	0.006	0.	0.239	1.803	0.238	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	83	0	1	219	89	0
normalized size	1	1.	0.93	1.14	0.	0.01	3.	1.22	0.
time (sec)	N/A	0.161	0.075	0.005	0.	0.233	1.791	0.237	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	80	0	1	1268	89	0
normalized size	1	1.	1.01	1.11	0.	0.01	17.61	1.24	0.
time (sec)	N/A	0.21	0.099	0.009	0.	0.271	39.118	0.225	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	83	0	1	1258	92	61
normalized size	1	1.	0.99	1.09	0.	0.01	16.55	1.21	0.8
time (sec)	N/A	0.206	0.096	0.012	0.	0.265	36.148	0.242	36.99

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	102	0	1	1686	108	76
normalized size	1	1.	0.91	1.11	0.	0.01	18.33	1.17	0.83
time (sec)	N/A	0.238	0.145	0.011	0.	0.282	39.141	0.227	43.292

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	139	201	0	1	333	215	0
normalized size	1	1.	0.79	1.14	0.	0.01	1.89	1.22	0.
time (sec)	N/A	0.553	0.24	0.013	0.	0.234	7.293	0.225	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	128	177	0	1	287	177	0
normalized size	1	1.	0.83	1.15	0.	0.01	1.86	1.15	0.
time (sec)	N/A	0.501	0.144	0.013	0.	0.239	6.904	0.223	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	100	154	0	1	284	150	0
normalized size	1	1.	0.75	1.15	0.	0.01	2.12	1.12	0.
time (sec)	N/A	0.478	0.148	0.013	0.	0.24	6.562	0.224	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	127	0	1	212	109	83
normalized size	1	1.	0.91	1.26	0.	0.01	2.1	1.08	0.82
time (sec)	N/A	0.255	0.1	0.013	0.	0.241	5.397	0.221	32.492

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	100	0	1	233	119	75
normalized size	1	1.	0.89	1.08	0.	0.01	2.51	1.28	0.81
time (sec)	N/A	0.146	0.165	0.017	0.	0.235	4.377	0.24	32.482

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	127	0	1	797	126	85
normalized size	1	1.	0.89	1.34	0.	0.01	8.39	1.33	0.89
time (sec)	N/A	0.269	0.144	0.019	0.	0.263	14.503	0.236	43.913

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	136	0	1	782	139	92
normalized size	1	1.	1.	1.24	0.	0.01	7.11	1.26	0.84
time (sec)	N/A	0.283	0.134	0.018	0.	0.265	15.886	0.241	44.248

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	112	169	0	1	1807	170	105
normalized size	1	1.	0.83	1.25	0.	0.01	13.39	1.26	0.78
time (sec)	N/A	0.413	0.216	0.02	0.	0.287	60.91	0.227	51.695

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	139	235	0	1	357	212	0
normalized size	1	1.	0.75	1.27	0.	0.01	1.93	1.15	0.
time (sec)	N/A	0.713	0.235	0.017	0.	0.236	41.516	0.247	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	126	206	0	1	282	165	131
normalized size	1	1.	0.81	1.33	0.	0.01	1.82	1.06	0.85
time (sec)	N/A	0.494	0.151	0.015	0.	0.242	38.268	0.228	133.624

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	138	0	1	303	173	114
normalized size	1	1.	0.9	1.01	0.	0.01	2.23	1.27	0.84
time (sec)	N/A	0.357	0.205	0.019	0.	0.238	33.89	0.227	100.162

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	110	0	1	177	131	88
normalized size	1	1.	0.83	0.92	0.	0.01	1.49	1.1	0.74
time (sec)	N/A	0.25	0.204	0.013	0.	0.239	25.825	0.223	31.224

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	111	0	1	184	143	100
normalized size	1	1.	0.9	0.96	0.	0.01	1.59	1.23	0.86
time (sec)	N/A	0.173	0.2	0.012	0.	0.238	16.448	0.22	34.503

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	117	186	0	1	872	173	121
normalized size	1	1.	0.9	1.43	0.	0.01	6.71	1.33	0.93
time (sec)	N/A	0.341	0.212	0.019	0.	0.267	30.494	0.228	63.886

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	141	195	0	1	860	190	124
normalized size	1	1.	0.98	1.35	0.	0.01	5.97	1.32	0.86
time (sec)	N/A	0.446	0.183	0.023	0.	0.251	37.928	0.226	71.766

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	147	250	0	1	1904	219	136
normalized size	1	1.	0.84	1.44	0.	0.01	10.94	1.26	0.78
time (sec)	N/A	0.628	0.322	0.024	0.	0.3	96.732	0.225	74.116

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	32	15	34	15
normalized size	1	1.	1.	0.95	1.2	1.6	0.75	1.7	0.75
time (sec)	N/A	0.05	0.013	0.014	1.347	0.223	0.097	0.22	9.174

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	30	93	22	23	17
normalized size	1	1.	0.78	0.65	1.3	4.04	0.96	1.	0.74
time (sec)	N/A	0.053	0.012	0.008	1.329	0.237	0.48	0.215	7.203

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	27	34	20	27	20
normalized size	1	1.	1.	0.84	1.08	1.36	0.8	1.08	0.8
time (sec)	N/A	0.059	0.019	0.005	1.504	0.22	0.086	0.221	11.698

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	32	22	32	0
normalized size	1	1.	1.	0.83	1.07	1.07	0.73	1.07	0.
time (sec)	N/A	0.06	0.009	0.004	1.496	0.225	0.091	0.222	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	278	0	1	366	338	0
normalized size	1	1.	1.	1.32	0.	0.	1.74	1.61	0.
time (sec)	N/A	0.347	0.292	0.006	0.	0.231	1.776	0.224	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	162	230	0	1	325	270	0
normalized size	1	1.	0.94	1.34	0.	0.01	1.89	1.57	0.
time (sec)	N/A	0.262	0.258	0.004	0.	0.238	2.254	0.222	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	182	0	1	180	205	0
normalized size	1	1.	0.94	1.34	0.	0.01	1.32	1.51	0.
time (sec)	N/A	0.216	0.192	0.004	0.	0.231	1.575	0.22	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	135	0	1	158	143	0
normalized size	1	1.	0.98	1.35	0.	0.01	1.58	1.43	0.
time (sec)	N/A	0.14	0.161	0.005	0.	0.233	1.608	0.218	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	114	0	1	150	116	0
normalized size	1	1.	0.99	1.36	0.	0.01	1.79	1.38	0.
time (sec)	N/A	0.187	0.129	0.008	0.	0.233	2.311	0.217	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	115	0	1	151	109	0
normalized size	1	1.	1.01	1.4	0.	0.01	1.84	1.33	0.
time (sec)	N/A	0.175	0.177	0.01	0.	0.234	4.407	0.216	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	103	142	0	1	167	142	90
normalized size	1	1.	0.99	1.37	0.	0.01	1.61	1.37	0.87
time (sec)	N/A	0.208	0.182	0.01	0.	0.234	10.545	0.219	41.92

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	139	190	0	1	301	204	124
normalized size	1	1.	1.01	1.39	0.	0.01	2.2	1.49	0.91
time (sec)	N/A	0.277	0.235	0.012	0.	0.24	25.544	0.223	51.974

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	174	238	0	1	354	271	156
normalized size	1	1.	0.99	1.36	0.	0.01	2.02	1.55	0.89
time (sec)	N/A	0.33	0.348	0.014	0.	0.236	52.536	0.22	60.851

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	286	0	1	398	336	192
normalized size	1	1.	1.	1.36	0.	0.	1.89	1.59	0.91
time (sec)	N/A	0.391	0.363	0.013	0.	0.238	90.198	0.217	71.655

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	227	309	0	1	430	340	0
normalized size	1	1.	0.95	1.29	0.	0.	1.79	1.42	0.
time (sec)	N/A	0.649	0.238	0.017	0.	0.237	4.617	0.216	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	187	258	0	1	250	271	0
normalized size	1	1.	0.93	1.28	0.	0.	1.24	1.34	0.
time (sec)	N/A	0.545	0.187	0.017	0.	0.234	4.441	0.216	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	212	0	1	216	205	141
normalized size	1	1.	0.91	1.3	0.	0.01	1.33	1.26	0.87
time (sec)	N/A	0.51	0.15	0.018	0.	0.238	4.233	0.216	138.425

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	177	0	1	199	170	114
normalized size	1	1.	1.03	1.5	0.	0.01	1.69	1.44	0.97
time (sec)	N/A	0.277	0.166	0.014	0.	0.24	3.69	0.216	81.022

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	165	0	1	197	165	122
normalized size	1	1.	1.03	1.47	0.	0.01	1.76	1.47	1.09
time (sec)	N/A	0.322	0.115	0.017	0.	0.235	9.256	0.216	140.885

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	182	0	1	212	166	146
normalized size	1	1.	1.03	1.5	0.	0.01	1.75	1.37	1.21
time (sec)	N/A	0.365	0.137	0.019	0.	0.241	23.851	0.216	143.952

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	151	219	0	1	226	204	175
normalized size	1	1.	0.99	1.44	0.	0.01	1.49	1.34	1.15
time (sec)	N/A	0.417	0.169	0.021	0.	0.248	56.109	0.217	149.568

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	190	268	0	1	394	271	202
normalized size	1	1.	1.01	1.42	0.	0.01	2.08	1.43	1.07
time (sec)	N/A	0.577	0.194	0.023	0.	0.239	127.943	0.217	160.894

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	230	318	0	1	0	340	231
normalized size	1	1.	1.	1.38	0.	0.	0.	1.48	1.
time (sec)	N/A	0.739	0.221	0.024	0.	0.241	0.	0.214	168.751

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	272	394	0	1	491	406	0
normalized size	1	1.	0.87	1.27	0.	0.	1.58	1.31	0.
time (sec)	N/A	1.082	0.312	0.02	0.	0.242	25.673	0.22	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	232	343	0	1	311	338	0
normalized size	1	1.	0.85	1.26	0.	0.	1.14	1.24	0.
time (sec)	N/A	0.967	0.292	0.019	0.	0.235	25.271	0.219	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	176	294	0	1	279	270	0
normalized size	1	1.	0.75	1.25	0.	0.	1.19	1.15	0.
time (sec)	N/A	0.868	0.307	0.018	0.	0.241	24.52	0.216	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	156	259	0	1	258	234	0
normalized size	1	1.	0.81	1.34	0.	0.01	1.34	1.21	0.
time (sec)	N/A	0.74	0.255	0.016	0.	0.236	21.23	0.217	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	234	0	1	243	201	146
normalized size	1	1.	0.96	1.59	0.	0.01	1.65	1.37	0.99
time (sec)	N/A	0.355	0.224	0.018	0.	0.24	15.37	0.219	84.462

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	155	237	0	1	250	207	160
normalized size	1	1.	1.03	1.58	0.	0.01	1.67	1.38	1.07
time (sec)	N/A	0.464	0.237	0.02	0.	0.239	40.362	0.218	151.552

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	169	264	0	1	270	230	0
normalized size	1	1.	1.01	1.57	0.	0.01	1.61	1.37	0.
time (sec)	N/A	0.557	0.294	0.022	0.	0.241	99.166	0.221	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	300	0	1	0	267	0
normalized size	1	1.	1.	1.53	0.	0.01	0.	1.36	0.
time (sec)	N/A	0.667	0.229	0.024	0.	0.237	0.	0.217	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	234	351	0	1	0	338	0
normalized size	1	1.	1.	1.5	0.	0.	0.	1.44	0.
time (sec)	N/A	0.912	0.253	0.026	0.	0.247	0.	0.219	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	276	401	0	1	0	406	0
normalized size	1	1.	1.	1.45	0.	0.	0.	1.47	0.
time (sec)	N/A	1.161	0.284	0.028	0.	0.236	0.	0.216	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	0	239	442	387	206
normalized size	1	1.	0.74	0.9	0.	1.12	2.07	1.81	0.96
time (sec)	N/A	0.465	0.22	0.01	0.	0.257	6.805	0.22	85.851

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	0	181	340	296	156
normalized size	1	1.	0.73	0.87	0.	1.08	2.04	1.77	0.93
time (sec)	N/A	0.355	0.152	0.01	0.	0.249	4.047	0.224	62.436

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	0	127	238	207	110
normalized size	1	1.	0.74	0.82	0.	1.05	1.97	1.71	0.91
time (sec)	N/A	0.271	0.121	0.007	0.	0.239	2.549	0.219	45.798

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	97	134	0	1	192	171	92
normalized size	1	1.	0.94	1.3	0.	0.01	1.86	1.66	0.89
time (sec)	N/A	0.263	0.259	0.013	0.	0.249	23.53	0.221	49.328

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	127	0	1	138	154	87
normalized size	1	1.	1.01	1.27	0.	0.01	1.38	1.54	0.87
time (sec)	N/A	0.408	0.303	0.013	0.	0.26	37.233	0.221	156.743

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	125	162	0	1	194	190	104
normalized size	1	1.	1.1	1.42	0.	0.01	1.7	1.67	0.91
time (sec)	N/A	0.534	0.328	0.015	0.	0.275	62.403	0.224	147.611

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	166	238	0	1	303	313	141
normalized size	1	1.	1.14	1.63	0.	0.01	2.08	2.14	0.97
time (sec)	N/A	0.6	0.273	0.016	0.	0.315	86.713	0.229	152.04

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	199	320	0	1	444	487	190
normalized size	1	1.	1.02	1.64	0.	0.01	2.28	2.5	0.97
time (sec)	N/A	0.818	0.376	0.021	0.	0.527	145.495	0.229	162.471

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	368	0	1	586	302	238
normalized size	1	1.	0.77	1.5	0.	0.	2.39	1.23	0.97
time (sec)	N/A	0.65	0.243	0.032	0.	0.557	52.53	0.228	70.83

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	152	284	0	1	444	236	187
normalized size	1	1.	0.78	1.46	0.	0.01	2.29	1.22	0.96
time (sec)	N/A	0.532	0.22	0.014	0.	0.374	34.252	0.226	70.05

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	203	0	1	362	174	138
normalized size	1	1.	0.83	1.4	0.	0.01	2.5	1.2	0.95
time (sec)	N/A	0.271	0.148	0.01	0.	0.281	19.264	0.221	47.392

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	100	140	0	1	250	163	104
normalized size	1	1.	0.85	1.2	0.	0.01	2.14	1.39	0.89
time (sec)	N/A	0.29	0.238	0.014	0.	0.261	10.799	0.227	52.785

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	117	0	1	197	238	95
normalized size	1	1.	0.85	1.06	0.	0.01	1.79	2.16	0.86
time (sec)	N/A	0.325	0.184	0.015	0.	0.268	6.069	0.231	82.593

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	136	0	1	456	437	117
normalized size	1	1.	0.83	1.15	0.	0.01	3.86	3.7	0.99
time (sec)	N/A	0.36	0.215	0.015	0.	0.278	4.536	0.229	79.01

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	111	0	135	891	748	163
normalized size	1	1.	0.74	0.79	0.	0.96	6.36	5.34	1.16
time (sec)	N/A	0.335	0.14	0.009	0.	0.416	7.218	0.23	94.433

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	157	0	190	1642	900	216
normalized size	1	1.	0.71	0.83	0.	1.01	8.69	4.76	1.14
time (sec)	N/A	0.495	0.151	0.01	0.	0.717	11.069	0.229	101.347

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	252	517	0	1	0	462	0
normalized size	1	1.	0.66	1.36	0.	0.	0.	1.21	0.
time (sec)	N/A	1.62	0.525	0.402	0.	1.113	0.	0.231	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	208	460	0	1	0	358	0
normalized size	1	1.	0.75	1.65	0.	0.	0.	1.28	0.
time (sec)	N/A	0.989	0.384	0.016	0.	0.722	0.	0.229	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	174	405	0	1	0	274	0
normalized size	1	1.	0.83	1.93	0.	0.	0.	1.3	0.
time (sec)	N/A	0.98	0.367	0.02	0.	0.518	0.	0.23	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	192	147	363	0	1	0	216	0
normalized size	1	1.07	0.82	2.03	0.	0.01	0.	1.21	0.
time (sec)	N/A	0.761	0.383	0.016	0.	0.462	0.	0.229	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	98	109	452	190	0	177	206
normalized size	1	1.	0.73	0.81	3.37	1.42	0.	1.32	1.54
time (sec)	N/A	0.404	0.133	0.008	1.37	0.359	0.	0.224	82.863

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	179	133	157	0	246	0	285	230
normalized size	1	0.97	0.72	0.85	0.	1.33	0.	1.54	1.24
time (sec)	N/A	0.499	0.181	0.01	0.	0.418	0.	0.225	155.612

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	165	205	0	304	0	471	270
normalized size	1	1.	0.68	0.85	0.	1.26	0.	1.95	1.12
time (sec)	N/A	0.67	0.212	0.012	0.	0.68	0.	0.227	163.22

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	275	202	253	0	365	0	799	309
normalized size	1	0.98	0.72	0.9	0.	1.3	0.	2.84	1.1
time (sec)	N/A	0.843	0.294	0.012	0.	1.125	0.	0.233	171.121

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	234	301	0	420	0	1	354
normalized size	1	1.	0.7	0.9	0.	1.26	0.	0.	1.06
time (sec)	N/A	0.993	0.428	0.012	0.	1.97	0.	0.255	179.705

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	380	270	349	0	478	0	1	0
normalized size	1	0.97	0.69	0.89	0.	1.22	0.	0.	0.
time (sec)	N/A	1.135	0.475	0.013	0.	3.116	0.	0.259	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	0	239	442	387	206
normalized size	1	1.	0.74	0.9	0.	1.12	2.07	1.81	0.96
time (sec)	N/A	0.463	0.222	0.01	0.	0.263	6.768	0.231	86.485

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	0	181	340	296	156
normalized size	1	1.	0.73	0.87	0.	1.08	2.04	1.77	0.93
time (sec)	N/A	0.366	0.154	0.008	0.	0.263	4.011	0.224	73.834

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	0	127	238	207	110
normalized size	1	1.	0.74	0.82	0.	1.05	1.97	1.71	0.91
time (sec)	N/A	0.285	0.122	0.006	0.	0.275	2.493	0.22	58.647

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	257	201	478	0	1	0	302	0
normalized size	1	0.98	0.77	1.83	0.	0.	0.	1.16	0.
time (sec)	N/A	1.543	0.438	0.016	0.	0.604	0.	0.229	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	252	176	427	0	1	0	275	0
normalized size	1	1.18	0.82	2.	0.	0.	0.	1.29	0.
time (sec)	N/A	0.947	0.424	0.01	0.	0.605	0.	0.228	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	138	166	0	252	0	297	0
normalized size	1	1.	0.72	0.86	0.	1.31	0.	1.54	0.
time (sec)	N/A	0.656	0.227	0.011	0.	0.513	0.	0.229	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [13] had the largest ratio of [0.4]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	20	0.25
2	A	5	5	1.	20	0.25
3	A	4	4	1.	18	0.222
4	A	4	4	1.	17	0.235
5	A	7	7	1.	20	0.35
6	A	7	7	1.	20	0.35
7	A	7	7	1.	20	0.35
8	A	7	5	1.	20	0.25
9	A	6	5	1.	20	0.25
10	A	5	4	1.	18	0.222
11	A	5	4	1.	17	0.235
12	A	8	7	1.	20	0.35
13	A	8	8	1.	20	0.4
14	A	8	7	1.	20	0.35
15	A	8	5	1.	20	0.25
16	A	7	5	1.	20	0.25
17	A	6	4	1.	18	0.222
18	A	6	4	1.	17	0.235
19	A	9	7	1.	20	0.35
20	A	9	8	1.	20	0.4
21	A	9	8	1.	20	0.4
22	A	5	4	1.	20	0.2
23	A	4	4	1.	20	0.2
24	A	3	3	1.	18	0.167
25	A	3	3	1.	17	0.176
26	A	6	6	1.	20	0.3
27	A	4	4	1.	20	0.2
28	A	5	5	1.	20	0.25
29	A	4	4	1.	20	0.2
30	A	4	4	1.	20	0.2
31	A	3	3	1.	18	0.167
32	A	1	1	1.	17	0.059
33	A	5	5	1.	20	0.25
34	A	5	5	1.	20	0.25
35	A	6	6	1.	20	0.3
36	A	4	4	1.	20	0.2
37	A	2	2	1.	20	0.1
38	A	2	2	1.	18	0.111
39	A	2	2	1.	17	0.118
40	A	6	5	1.	20	0.25
41	A	6	5	1.	20	0.25
42	A	7	6	1.	20	0.3
43	A	2	2	1.	18	0.111
44	A	3	3	1.	19	0.158
45	A	2	2	1.	13	0.154

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
46	A	2	2	1.	13	0.154
47	A	7	5	1.	25	0.2
48	A	6	5	1.	25	0.2
49	A	5	4	1.	25	0.16
50	A	4	4	1.	25	0.16
51	A	4	4	1.	25	0.16
52	A	4	4	1.	25	0.16
53	A	4	4	1.	23	0.174
54	A	5	4	1.	22	0.182
55	A	8	6	1.	25	0.24
56	A	8	5	1.	25	0.2
57	A	9	6	1.	25	0.24
58	A	2	2	1.	16	0.125
59	A	3	2	1.	20	0.1
60	A	2	2	1.	22	0.091
61	A	5	3	1.	25	0.12
62	A	2	1	1.	26	0.038
63	A	2	1	1.	26	0.038
64	A	2	1	1.	24	0.042
65	A	2	1	1.	23	0.043
66	A	2	1	1.	26	0.038
67	A	2	1	1.	26	0.038
68	A	2	1	1.	26	0.038
69	A	2	1	1.	26	0.038
70	A	2	1	1.	28	0.036
71	A	2	1	1.	28	0.036
72	A	4	3	1.	26	0.115
73	A	3	2	1.	25	0.08
74	A	3	2	1.	28	0.071
75	A	3	2	1.	28	0.071
76	A	2	1	1.	28	0.036
77	A	2	1	1.	28	0.036
78	A	2	1	1.	28	0.036
79	A	2	1	1.	28	0.036
80	A	4	3	1.	26	0.115
81	A	3	2	1.	25	0.08
82	A	3	2	1.	28	0.071
83	A	3	2	1.	28	0.071
84	A	2	1	1.	28	0.036
85	A	2	1	1.	28	0.036
86	A	5	4	1.	28	0.143
87	A	5	4	1.	28	0.143
88	A	5	4	1.	28	0.143
89	A	5	4	1.	26	0.154
90	A	5	4	1.	25	0.16
91	A	5	4	1.	28	0.143
92	A	5	4	1.	28	0.143
93	A	5	4	1.	28	0.143
94	A	6	5	1.	28	0.179

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	5	1.	28	0.179
96	A	6	5	1.	28	0.179
97	A	6	5	1.	26	0.192
98	A	4	4	1.	25	0.16
99	A	6	5	1.	28	0.179
100	A	6	5	1.	28	0.179
101	A	6	5	1.	28	0.179
102	A	7	5	1.	28	0.179
103	A	6	5	1.	28	0.179
104	A	5	4	1.	28	0.143
105	A	4	4	1.	26	0.154
106	A	3	3	1.	25	0.12
107	A	7	6	1.	28	0.214
108	A	7	5	1.	28	0.179
109	A	7	5	1.	28	0.179
110	A	4	3	1.	17	0.176
111	A	4	3	1.	17	0.176
112	A	4	4	1.	21	0.19
113	A	6	5	1.	15	0.333
114	A	3	2	1.	30	0.067
115	A	3	2	1.	30	0.067
116	A	3	2	1.	30	0.067
117	A	3	2	1.	27	0.074
118	A	3	2	1.	30	0.067
119	A	3	2	1.	30	0.067
120	A	3	2	1.	30	0.067
121	A	3	2	1.	30	0.067
122	A	3	2	1.	30	0.067
123	A	3	2	1.	30	0.067
124	A	5	4	1.	30	0.133
125	A	5	4	1.	30	0.133
126	A	5	4	1.	30	0.133
127	A	4	3	1.	27	0.111
128	A	4	3	1.	30	0.1
129	A	4	3	1.	30	0.1
130	A	4	3	1.	30	0.1
131	A	4	3	1.	30	0.1
132	A	4	3	1.	30	0.1
133	A	8	6	1.	30	0.2
134	A	8	6	1.	30	0.2
135	A	8	6	1.	30	0.2
136	A	7	6	1.	30	0.2
137	A	4	3	1.	27	0.111
138	A	4	3	1.	30	0.1
139	A	5	3	1.	30	0.1
140	A	5	3	1.	30	0.1
141	A	5	3	1.	30	0.1
142	A	5	3	1.	30	0.1
143	A	3	2	1.	32	0.062

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	3	2	1.	32	0.062
145	A	3	2	1.	30	0.067
146	A	5	4	1.	32	0.125
147	A	6	5	1.	32	0.156
148	A	6	6	1.	32	0.188
149	A	6	6	1.	32	0.188
150	A	7	7	1.	32	0.219
151	A	7	5	1.	32	0.156
152	A	6	5	1.	32	0.156
153	A	5	4	1.	29	0.138
154	A	6	6	1.	32	0.188
155	A	7	6	1.	32	0.188
156	A	7	6	1.	32	0.188
157	A	5	3	1.	32	0.094
158	A	6	4	1.	32	0.125
159	A	11	8	1.	32	0.25
160	A	10	8	1.	32	0.25
161	A	9	8	1.	32	0.25
162	A	8	7	1.07	32	0.219
163	A	5	4	1.	29	0.138
164	A	6	5	0.97	32	0.156
165	A	7	5	1.	32	0.156
166	A	8	4	0.98	32	0.125
167	A	9	5	1.	32	0.156
168	A	10	5	0.97	32	0.156
169	A	4	3	1.	33	0.091
170	A	4	3	1.	33	0.091
171	A	4	3	1.	31	0.097
172	A	10	8	0.98	37	0.216
173	A	6	4	1.18	34	0.118
174	A	6	4	1.	37	0.108

3 Listing of integrals

3.1 $\int x^3(A + Bx)\sqrt{a + bx^2}\, dx$

Optimal. Leaf size=127

$$\frac{a^3 B \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2 Bx\sqrt{a+bx^2}}{16b^2}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5
*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b
*x^2)^(3/2))/(120*b^2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(16*b^(5/2))
```

Rubi [A] time = 0.311658, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{a^3 B \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2 Bx\sqrt{a+bx^2}}{16b^2}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5
*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b
*x^2)^(3/2))/(120*b^2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(16*b^(5/2))
```

Rubi in Sympy [A] time = 27.5814, size = 114, normalized size = 0.9

$$\frac{Ax^2\left(a+bx^2\right)^{\frac{3}{2}}}{5b}+\frac{Ba^3 \operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{5}{2}}}+\frac{Ba^2x\sqrt{a+bx^2}}{16b^2}+\frac{Bx^3\left(a+bx^2\right)^{\frac{3}{2}}}{6b}-\frac{a(48A+45Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{360b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*x**2*(a + b*x**2)**(3/2)/(5*b) + B*a**3*atanh(sqrt(b)*x/sqrt(a
+ b*x**2))/(16*b**(5/2)) + B*a**2*x*sqrt(a + b*x**2)/(16*b**2) +
B*x**3*(a + b*x**2)**(3/2)/(6*b) - a*(48*A + 45*B*x)*(a + b*x**2)
**(3/2)/(360*b**2)
```

Mathematica [A] time = 0.11948, size = 100, normalized size = 0.79

$$\frac{15a^3 B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}\sqrt{a+bx^2}\left(-a^2(32A+15Bx)+2abx^2(8A+5Bx)+8b^2x^4(6A+5Bx)\right)}{240b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(8*
A + 5*B*x) - a^2*(32*A + 15*B*x)) + 15*a^3*B*Log[b*x + Sqrt[b]*Sq
```

```
rt[a + b*x^2]])/(240*b^(5/2))
```

Maple [A] time = 0.011, size = 115, normalized size = 0.9

$$\frac{Ax^2}{5\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{2\,Aa}{15\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Bx^3}{6\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bxa}{8\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Bxa^2}{16\,b^2}\sqrt{bx^2+a}+\frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*A*x^2*(b*x^2+a)^(3/2)/b-2/15*A*a/b^2*(b*x^2+a)^(3/2)+1/6*B*x^
3*(b*x^2+a)^(3/2)/b-1/8*B*a/b^2*x*(b*x^2+a)^(3/2)+1/16*a^2*B*x*(b
*x^2+a)^(1/2)/b^2+1/16*B*a^3/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266099, size = 1, normalized size = 0.01

$$\left[\frac{15\,Ba^3\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(40\,Bb^2x^5+48\,Ab^2x^4+10\,Babx^3+16\,Aabx^2-15\,Ba^2x-32\,Aa^2\right)\sqrt{bx^2}}{480\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A)*x^3,x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(
b)) + 2*(40*B*b^2*x^5 + 48*A*b^2*x^4 + 10*B*a*b*x^3 + 16*A*a*b*x^
2 - 15*B*a^2*x - 32*A*a^2)*sqrt(b*x^2 + a)*sqrt(b))/b^(5/2), 1/24
0*(15*B*a^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^2*x^5 +
48*A*b^2*x^4 + 10*B*a*b*x^3 + 16*A*a*b*x^2 - 15*B*a^2*x - 32*A*a^
2)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 10.2851, size = 192, normalized size = 1.51

$$A\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)-\frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a +
 b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x*
*4/4, True)) - B*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(
3/2)*x**3/(48*b*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1
 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) + B
*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.214942, size = 126, normalized size = 0.99

$$-\frac{Ba^3 \ln\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}} + \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A)*x^3,x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240
*sqrt(b*x^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*
x - 15*B*a^2/b^2)*x - 32*A*a^2/b^2)
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b)
- ((8*a*B - 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.192447, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b)
- ((8*a*B - 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi in Sympy [A] time = 15.7692, size = 94, normalized size = 0.9

$$-\frac{Aa^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}}-\frac{Aax\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{\frac{3}{2}}}{5b}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}(-15Abx+8Ba)}{60b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*b**(3/2)) - A*a*x*sq
rt(a + b*x**2)/(8*b) + B*x**2*(a + b*x**2)**(3/2)/(5*b) - (a + b*
x**2)**(3/2)*(-15*A*b*x + 8*B*a)/(60*b**2)
```

Mathematica [A] time = 0.0924764, size = 88, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(-16a^2B+abx(15A+8Bx)+6b^2x^3(5A+4Bx)\right)-15a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{120b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15
*A + 8*B*x)) - 15*a^2*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
])/(120*b^2)
```

Maple [A] time = 0.009, size = 94, normalized size = 0.9

$$\frac{Ax}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{aAx}{8\,b}\sqrt{bx^2+a}-\frac{Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Bx^2}{5\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{2\,Ba}{15\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)/b-1/8*a*A*x*(b*x^2+a)^(1/2)/b-1/8*A*a^2/b
^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/5*B*x^2*(b*x^2+a)^(3/2)/b-
2/15*B*a/b^2*(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268216, size = 1, normalized size = 0.01

$$\left[\frac{15\,Aa^2b\log\left(2\,\sqrt{bx^2+a}\,abx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}\sqrt{b}}{240\,b^{\frac{5}{2}}},\right.$$
$$\left.-\frac{15\,Aa^2b\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}\sqrt{-b}}{120\,\sqrt{-b}b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*b*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt
(b)) + 2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x
- 16*B*a^2)*sqrt(b*x^2 + a)*sqrt(b))/b^(5/2), -1/120*(15*A*a^2*b*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (24*B*b^2*x^4 + 30*A*b^2*x^3
 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(b*x^2 + a)*sqrt(-b))
/(sqrt(-b)*b^2)]
```

Sympy [A] time = 6.90488, size = 165, normalized size = 1.59

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$
$$+B\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(
1 + b*x**2/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A
*b*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*Piecewise((-2*a**2*sqr
t(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*s
qrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

GIAC/XCAS [A] time = 0.217751, size = 109, normalized size = 1.05

$$\frac{Aa^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*s
qrt(b*x^2 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x
- 16*B*a^2/b^2)
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2)
)/(12*b) - (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2
))
```

Rubi [A] time = 0.0942558, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2)
)/(12*b) - (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2
))
```

Rubi in Sympy [A] time = 8.70822, size = 70, normalized size = 0.88

$$-\frac{Ba^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}}-\frac{Bax\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -B*a**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*b**(3/2)) - B*a*x*sq
rt(a + b*x**2)/(8*b) + (4*A + 3*B*x)*(a + b*x**2)**(3/2)/(12*b)
```

Mathematica [A] time = 0.0698619, size = 79, normalized size = 0.99

$$\frac{\sqrt{b}\sqrt{a+bx^2}\left(8aA+3aBx+8Abx^2+6bBx^3\right)-3a^2B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3
) - 3*a^2*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(24*b^(3/2))
```

Maple [A] time = 0.007, size = 75, normalized size = 0.9

$$\frac{A}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Bx}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bxa}{8\,b}\sqrt{bx^2+a}-\frac{a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*A/b*(b*x^2+a)^(3/2)+1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*B*a/b*x*(b*
x^2+a)^(1/2)-1/8*B*a^2/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258562, size = 1, normalized size = 0.01

$$\left[\frac{3\,Ba^2\log\left(2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(6\,Bbx^3+8\,Abx^2+3\,Bax+8\,Aa\right)\sqrt{bx^2+a}\sqrt{b}}{48\,b^{\frac{3}{2}}},\right.$$
$$\left.-\frac{3\,Ba^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(6\,Bbx^3+8\,Abx^2+3\,Bax+8\,Aa\right)\sqrt{bx^2+a}\sqrt{-b}}{24\,\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x,x, algorithm="fricas")
```

```
[Out] [1/48*(3*B*a^2*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))
 + 2*(6*B*b*x^3 + 8*A*b*x^2 + 3*B*a*x + 8*A*a)*sqrt(b*x^2 + a)*sq
rt(b))/b^(3/2), -1/24*(3*B*a^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (6*B*b*x^3 + 8*A*b*x^2 + 3*B*a*x + 8*A*a)*sqrt(b*x^2 + a)*sqrt
(-b))/(sqrt(-b)*b)]
```

Sympy [A] time = 6.5106, size = 124, normalized size = 1.55

$$A\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b
), True)) + B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x
**3/(8*sqrt(1 + b*x**2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b
**(3/2)) + B*b*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.22314, size = 92, normalized size = 1.15

$$\frac{Ba^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(3\,Bx+4\,A\right)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)*x,x, algorithm="giac")
```

```
[Out] 1/8*B*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sq
rt(b*x^2 + a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] time = 0.0639736, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi in Sympy [A] time = 7.01506, size = 58, normalized size = 0.87

$$\frac{Aa\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{Ax\sqrt{a+bx^2}}{2}+\frac{B\left(a+bx^2\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*sqrt(b)) + A*x*sqrt(a +
b*x**2)/2 + B*(a + b*x**2)**(3/2)/(3*b)
```

Mathematica [A] time = 0.0584011, size = 67, normalized size = 1.

$$\frac{\sqrt{a+bx^2}(2aB+bx(3A+2Bx))+3aA\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{6b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*a*B + b*x*(3*A + 2*B*x)) + 3*a*A*Sqrt[b]*Log[
b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(6*b)
```

Maple [A] time = 0.007, size = 53, normalized size = 0.8

$$\frac{Ax}{2}\sqrt{bx^2+a}+\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{B}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/3*B*(b*x^2+a)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262778, size = 1, normalized size = 0.01

$$\left[\frac{3\,Aab\log\left(-2\,\sqrt{bx^2+a}\,bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(2\,Bbx^2+3\,Abx+2\,Ba\right)\sqrt{bx^2+a}\sqrt{b}}{12\,b^{\frac{3}{2}}},\ \frac{3\,Aab\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(2\,Bbx^2+\right.}{6\,\sqrt{-bb}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*b*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)
) + 2*(2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a)*sqrt(b))/b^(3
/2), 1/6*(3*A*a*b*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2
 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b)]
```

Sympy [A] time = 3.74226, size = 70, normalized size = 1.04

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+B\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(
2*sqrt(b)) + B*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2
)**(3/2)/(3*b), True))
```

GIAC/XCAS [A] time = 0.230956, size = 74, normalized size = 1.1

$$-\frac{Aa\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{6}\,\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A),x, algorithm="giac")
```

```
[Out] -1/2*A*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt
(b*x^2 + a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]]
```

Rubi [A] time = 0.213448, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]]
```

Rubi in Sympy [A] time = 20.1511, size = 70, normalized size = 0.89

$$-A\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{Ba\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{(2A+Bx)\sqrt{a+bx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*atanh(sqrt(a + b*x**2)/sqrt(a)) + B*a*atanh(sqrt(b)*x/
sqrt(a + b*x**2))/(2*sqrt(b)) + (2*A + B*x)*sqrt(a + b*x**2)/2
```

Mathematica [A] time = 0.0994853, size = 91, normalized size = 1.15

$$\sqrt{a+bx^2}\left(A+\frac{Bx}{2}\right)-\sqrt{a}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a}A\log(x)+\frac{aB\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] (A + (B*x)/2)*Sqrt[a + b*x^2] + Sqrt[a]*A*Log[x] - Sqrt[a]*A*Log[
a + Sqrt[a]*Sqrt[a + b*x^2]] + (a*B*Log[b*x + Sqrt[b]*Sqrt[a + b*
x^2]])/(2*Sqrt[b])
```

Maple [A] time = 0.008, size = 78, normalized size = 1.

$$\frac{Bx}{2}\sqrt{bx^2+a}+\frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-A\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)+A\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*x*B*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))-A*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(b*x^2+a)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285082, size = 1, normalized size = 0.01

$$\left[\frac{Ba\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+2A\sqrt{a}\sqrt{b}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}(Bx+2A)\sqrt{b}}{4\sqrt{b}}, \frac{Ba\arctan\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{}\right.$$

$$-\frac{4A\sqrt{-a}\sqrt{b}\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-Ba\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)-2\sqrt{bx^2+a}(Bx+2A)\sqrt{b}}{4\sqrt{b}}, \frac{Ba\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)) + 2
*A*sqrt(a)*sqrt(b)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*sqrt(b*x^2 + a)*(B*x + 2*A)*sqrt(b))/sqrt(b), 1/2*(B*a*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + A*sqrt(a)*sqrt(-b)*log(-(b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*x^2 + a)*(B*x
 + 2*A)*sqrt(-b))/sqrt(-b), -1/4*(4*A*sqrt(-a)*sqrt(b)*arctan(a/(
sqrt(b*x^2 + a)*sqrt(-a))) - B*a*log(-2*sqrt(b*x^2 + a)*b*x - (2*
b*x^2 + a)*sqrt(b)) - 2*sqrt(b*x^2 + a)*(B*x + 2*A)*sqrt(b))/sqrt
(b), 1/2*(B*a*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*A*sqrt(-a)*s
qrt(-b)*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) + sqrt(b*x^2 + a)*(B
*x + 2*A)*sqrt(-b))/sqrt(-b)]
```

Sympy [A] time = 5.41282, size = 107, normalized size = 1.35

$$-A\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*
x**2) + 1)) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*sqrt(a)*x*sqrt
(1 + b*x**2/a)/2 + B*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

GIAC/XCAS [A] time = 0.229876, size = 105, normalized size = 1.33

$$\frac{2\,Aa\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{Ba\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{2}\,\sqrt{bx^2+a}(Bx+2\,A)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(b*x^2 + a)*(B*x + A)/x,x, algorithm="giac")
```

```
[Out] 2*A*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) -
1/2*B*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/2*sqrt(
b*x^2 + a)*(B*x + 2*A)
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a+bx^2}(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.205008, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\sqrt{a+bx^2}(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 18.6781, size = 65, normalized size = 0.87

$$A\sqrt{b}\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-B\sqrt{a}\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(A-Bx)\sqrt{a+bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] A*sqrt(b)*atanh(sqrt(b)*x/sqrt(a + b*x**2)) - B*sqrt(a)*atanh(sqr
t(a + b*x**2)/sqrt(a)) - (A - B*x)*sqrt(a + b*x**2)/x
```

Mathematica [A] time = 0.0835402, size = 87, normalized size = 1.16

$$\sqrt{a+bx^2}\left(B-\frac{A}{x}\right)+A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)-\sqrt{a}B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a}B\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] (B - A/x)*Sqrt[a + b*x^2] + Sqrt[a]*B*Log[x] - Sqrt[a]*B*Log[a +
Sqrt[a]*Sqrt[a + b*x^2]] + A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b
*x^2]]
```

Maple [A] time = 0.011, size = 97, normalized size = 1.3

$$-\frac{A}{ax}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Axb}{a}\sqrt{bx^2+a}+A\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$
$$-B\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+B\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(3/2)+A*b/a*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))-B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2)
)/x)+B*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272651, size = 1, normalized size = 0.01

$$\left[\frac{A\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\ \frac{2\,A\sqrt{-b}x\arctan\left(\frac{\ldots}{\sqrt{bx\ldots}}\right)\ldots}{\ldots}\right.$$

$$-\frac{2\,B\sqrt{-a}x\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-A\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\ \frac{A\sqrt{-b}x\arctan\left(\frac{bx}{\sqrt{bx^2+a\ldots}}\right)\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*sqrt(b*x^2 + a)*(B*x - A))/x, 1/2*(2*A*sqrt(-b)*x*arctan(b*
x/(sqrt(b*x^2 + a)*sqrt(-b))) + B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a)*(B*x - A))/x,
-1/2*(2*B*sqrt(-a)*x*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - A*sqr
t(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b
*x^2 + a)*(B*x - A))/x, (A*sqrt(-b)*x*arctan(b*x/(sqrt(b*x^2 + a)
*sqrt(-b))) - B*sqrt(-a)*x*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) +
 sqrt(b*x^2 + a)*(B*x - A))/x]
```

Sympy [A] time = 4.8349, size = 124, normalized size = 1.65

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqr
t(a)) - A*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt
(a)/(sqrt(b)*x)) + B*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(
b)*x/sqrt(a/(b*x**2) + 1)
```

GIAC/XCAS [A] time = 0.23325, size = 138, normalized size = 1.84

$$\frac{2\,Ba\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - A\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \sqrt{bx^2+a}B + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) -
A*sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)
*B + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3} dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*Sqrt[a])
```

Rubi [A] time = 0.20957, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]   Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*Sqrt[a])
```

Rubi in Sympy [A] time = 21.7174, size = 73, normalized size = 0.91

$$-\frac{Ab\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+B\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*b*atanh(sqrt(a + b*x**2)/sqrt(a))/(2*sqrt(a)) + B*sqrt(b)*atan
h(sqrt(b)*x/sqrt(a + b*x**2)) - (A + 2*B*x)*sqrt(a + b*x**2)/(2*x
**2)
```

Mathematica [A] time = 0.169981, size = 96, normalized size = 1.2

$$\frac{1}{2}\left(-\frac{\sqrt{a+bx^2}(A+2Bx)}{x^2}-\frac{Ab\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{Ab\log(x)}{\sqrt{a}}+2\sqrt{b}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] (-(((A + 2*B*x)*Sqrt[a + b*x^2])/x^2) + (A*b*Log[x])/Sqrt[a] - (A
*b*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/Sqrt[a] + 2*Sqrt[b]*B*Log[b*
x + Sqrt[b]*Sqrt[a + b*x^2]])/2
```

Maple [A] time = 0.013, size = 121, normalized size = 1.5

$$-\frac{A}{2\,a{x}^{2}} \left(b{x}^{2}+a \right) ^{\frac{3}{2}}-\frac{Ab}{2}\ln \left(\frac{1}{x} \left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a} \right) \right) \frac{1}{\sqrt{a}}+\frac{Ab}{2\,a}\sqrt{b{x}^{2}+a}-\frac{B}{ax} \left(b{x}^{2}+a \right) ^{\frac{3}{2}}+\frac{bBx}{a}\sqrt{b{x}^{2}+a}+B\sqrt{b}\ln \left(x\sqrt{b}+\sqrt{b{x}^{2}+a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(3/2)-1/2*A*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)+1/2*A*b/a*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(3/2)
+B*b/a*x*(b*x^2+a)^(1/2)+B*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275049, size = 1, normalized size = 0.01

$$\left[\frac{2\,B\sqrt{a}\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+Abx^2\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)-2\,\sqrt{bx^2+a}(2\,Bx+A)\sqrt{a}}{4\,\sqrt{a}x^2},\ \frac{4\,B\sqrt{a}\sqrt{-\cdots}}{\cdots}\right.$$

$$-\frac{Abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{bx^2+a}(2\,Bx+A)\sqrt{-a}}{2\,\sqrt{-a}x^2},\ \frac{2\,B\sqrt{-a}\sqrt{-b}x^2\arctan\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*sqrt(a)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + A*b*x^2*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2
 + a)*a)/x^2) - 2*sqrt(b*x^2 + a)*(2*B*x + A)*sqrt(a))/(sqrt(a)*x
^2), 1/4*(4*B*sqrt(a)*sqrt(-b)*x^2*arctan(b*x/(sqrt(b*x^2 + a)*sq
rt(-b))) + A*b*x^2*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a
)*a)/x^2) - 2*sqrt(b*x^2 + a)*(2*B*x + A)*sqrt(a))/(sqrt(a)*x^2),
 -1/2*(A*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - B*sqrt(-a)*sqrt
(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt(b*
x^2 + a)*(2*B*x + A)*sqrt(-a))/(sqrt(-a)*x^2), 1/2*(2*B*sqrt(-a)*
sqrt(-b)*x^2*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - A*b*x^2*arc
tan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(2*B*x + A)*sqrt(
-a))/(sqrt(-a)*x^2)]
```

Sympy [A] time = 5.65985, size = 107, normalized size = 1.34

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b
)*x))/(2*sqrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*
asinh(sqrt(b)*x/sqrt(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.230635, size = 220, normalized size = 2.75

$$\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - B\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(B*x + A)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*
sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqr
t(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/((sqr
t(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2
))/(64*b^2) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2
)^(5/2))/(8*b) - (a*(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2)
+ (3*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.327142, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2
))/(64*b^2) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2
)^(5/2))/(8*b) - (a*(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2)
+ (3*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi in Sympy [A] time = 30.291, size = 139, normalized size = 0.93

$$\frac{Ax^2\left(a+bx^2\right)^{\frac{5}{2}}}{7b}+\frac{3Ba^4\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{5}{2}}}+\frac{3Ba^3x\sqrt{a+bx^2}}{128b^2}+\frac{Ba^2x\left(a+bx^2\right)^{\frac{3}{2}}}{64b^2}+\frac{Bx^3\left(a+bx^2\right)^{\frac{5}{2}}}{8b}-\frac{a(96A+105Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{1680b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*x**2*(a + b*x**2)**(5/2)/(7*b) + 3*B*a**4*atanh(sqrt(b)*x/sqrt(
a + b*x**2))/(128*b**(5/2)) + 3*B*a**3*x*sqrt(a + b*x**2)/(128*b*
*2) + B*a**2*x*(a + b*x**2)**(3/2)/(64*b**2) + B*x**3*(a + b*x**2
)**(5/2)/(8*b) - a*(96*A + 105*B*x)*(a + b*x**2)**(5/2)/(1680*b**
2)
```

Mathematica [A] time = 0.142776, size = 119, normalized size = 0.79

$$\frac{105a^4B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}\sqrt{a+bx^2}\left(-a^3(256A+105Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)+80b^3\right.}{4480b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*
(64*A + 35*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 10
5*B*x)) + 105*a^4*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(4480*b^(
5/2))
```

Maple [A] time = 0.012, size = 134, normalized size = 0.9

$$\frac{Ax^2}{7b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{2Aa}{35b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bx^3}{8b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bxa}{16b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bxa^2}{64b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3a^3Bx}{128b^2}\sqrt{bx^2+a}+\frac{3Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*A*x^2*(b*x^2+a)^(5/2)/b-2/35*A*a/b^2*(b*x^2+a)^(5/2)+1/8*B*x^
3*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x*(b*x^2+a)^(5/2)+1/64*B*a^2/b^2
*x*(b*x^2+a)^(3/2)+3/128*B*a^3/b^2*x*(b*x^2+a)^(1/2)+3/128*B*a^4/
b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280218, size = 1, normalized size = 0.01

$$\left[\frac{105\,Ba^4\log\left(-2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(560\,Bb^3x^7+640\,Ab^3x^6+840\,Bab^2x^5+1024\,Aab^2x^4+70\,Ba^2bx^3+128\right.}{8960\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^3,x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqr
t(b)) + 2*(560*B*b^3*x^7 + 640*A*b^3*x^6 + 840*B*a*b^2*x^5 + 1024
*A*a*b^2*x^4 + 70*B*a^2*b*x^3 + 128*A*a^2*b*x^2 - 105*B*a^3*x - 2
56*A*a^3)*sqrt(b*x^2 + a)*sqrt(b))/b^(5/2), 1/4480*(105*B*a^4*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (560*B*b^3*x^7 + 640*A*b^3*x^6
+ 840*B*a*b^2*x^5 + 1024*A*a*b^2*x^4 + 70*B*a^2*b*x^3 + 128*A*a^2
*b*x^2 - 105*B*a^3*x - 256*A*a^3)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt
(-b)*b^2)]
```

Sympy [A] time = 24.5491, size = 318, normalized size = 2.12

$$Aa\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}$$
$$-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Ba^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a
 + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*
x**4/4, True)) + A*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3
) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x
**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6
, True)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - B*a**(5
/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64*sqrt
(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*
B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + B*b**2*x**9/(8*s
qrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.228703, size = 155, normalized size = 1.03

$$-\frac{3\,Ba^4\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}-\frac{1}{4480}\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\left(128\,Aa+5\left(21\,Ba+2\left(7\,Bbx+8\,Ab\right)x\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^3,x, algorithm="giac")
```

```
[Out] -3/128*B*a^4*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/44
80*sqrt(b*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/
b + (35*B*a^2/b + 4*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)
*x)*x)*x)*x)*x)
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24
*b) + (B*x^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a +
 b*x^2)^(5/2))/(210*b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.208948, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24
*b) + (B*x^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a +
 b*x^2)^(5/2))/(210*b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(16*b^(3/2))
```

Rubi in Sympy [A] time = 17.9537, size = 114, normalized size = 0.9

$$-\frac{Aa^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{3}{2}}}-\frac{Aa^2x\sqrt{a+bx^2}}{16b}-\frac{Aax\left(a+bx^2\right)^{\frac{3}{2}}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{\frac{5}{2}}}{7b}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}(-35Abx+12Ba)}{210b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] -A*a**3*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*b**(3/2)) - A*a**2*
x*sqrt(a + b*x**2)/(16*b) - A*a*x*(a + b*x**2)**(3/2)/(24*b) + B*
x**2*(a + b*x**2)**(5/2)/(7*b) - (a + b*x**2)**(5/2)*(-35*A*b*x +
 12*B*a)/(210*b**2)
```

Mathematica [A] time = 0.128862, size = 108, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(-96a^3B+3a^2bx(35A+16Bx)+2ab^2x^3(245A+192Bx)+40b^3x^5(7A+6Bx)\right)-105a^3A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right.}{1680b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*
x*(35*A + 16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x)) - 105*a^3*A*Sq
rt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(1680*b^2)
```

Maple [A] time = 0.01, size = 113, normalized size = 0.9

$$\frac{Ax}{6b}\left(bx^{2}+a\right)^{\frac{5}{2}}-\frac{aAx}{24b}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{a^{2}Ax}{16b}\sqrt{bx^{2}+a}-\frac{Aa^{3}}{16}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{3}{2}}+\frac{Bx^{2}}{7b}\left(bx^{2}+a\right)^{\frac{5}{2}}-\frac{2Ba}{35b^{2}}\left(bx^{2}+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*A*x*(b*x^2+a)^(5/2)/b-1/24*A*a/b*x*(b*x^2+a)^(3/2)-1/16*A*a^2
/b*x*(b*x^2+a)^(1/2)-1/16*A*a^3/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/7*B*x^2*(b*x^2+a)^(5/2)/b-2/35*B*a/b^2*(b*x^2+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261218, size = 1, normalized size = 0.01

$$\left[\frac{105\,Aa^{3}b\log\left(2\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)+2\left(240\,Bb^{3}x^{6}+280\,Ab^{3}x^{5}+384\,Bab^{2}x^{4}+490\,Aab^{2}x^{3}+48\,Ba^{2}bx^{2}+105\,A\right.}{3360\,b^{\frac{5}{2}}}\right.$$

$$-\frac{105\,Aa^{3}b\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-\left(240\,Bb^{3}x^{6}+280\,Ab^{3}x^{5}+384\,Bab^{2}x^{4}+490\,Aab^{2}x^{3}+48\,Ba^{2}bx^{2}+105\,Aa^{2}bx-96\,Ba^{3}\right)\sqrt{bx}}{1680\sqrt{-b}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^2,x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*b*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sq
rt(b)) + 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490
*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*
x^2 + a)*sqrt(b))/b^(5/2), -1/1680*(105*A*a^3*b*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) - (240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*
x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3
)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 16.405, size = 287, normalized size = 2.26

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+Bb\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*s
qrt(1 + b*x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a))
- A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*
sqrt(a)*sqrt(1 + b*x**2/a)) + B*a*Piecewise((-2*a**2*sqrt(a + b*x
**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b
*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b*Piecewise((8*a
**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(1
05*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)
/7, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

GIAC/XCAS [A] time = 0.222428, size = 139, normalized size = 1.09

$$\frac{Aa^3\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}-\frac{1}{1680}\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2}-\left(\frac{105\,Aa^2}{b}+2\left(\frac{24\,Ba^2}{b}+(245\,Aa+4\,(48\,Ba+5\,(6\,Bbx+7\,Ab)x)x)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x^2,x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680
*sqrt(b*x^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (
245*A*a + 4*(48*B*a + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24
*b) + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.113047, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```


```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24
*b) + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi in Sympy [A] time = 10.6217, size = 90, normalized size = 0.87

$$-\frac{Ba^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{3}{2}}}-\frac{Ba^2x\sqrt{a+bx^2}}{16b}-\frac{Bax\left(a+bx^2\right)^{\frac{3}{2}}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{30b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] -B*a**3*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*b**(3/2)) - B*a**2*
x*sqrt(a + b*x**2)/(16*b) - B*a*x*(a + b*x**2)**(3/2)/(24*b) + (6
*A + 5*B*x)*(a + b*x**2)**(5/2)/(30*b)
```

Mathematica [A] time = 0.113902, size = 100, normalized size = 0.97

$$\frac{\sqrt{b}\sqrt{a+bx^2}\left(3a^2(16A+5Bx)+2abx^2(48A+35Bx)+8b^2x^4(6A+5Bx)\right)-15a^3B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{240b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 3*a^2*(16*A +
 5*B*x) + 2*a*b*x^2*(48*A + 35*B*x)) - 15*a^3*B*Log[b*x + Sqrt[b]
*Sqrt[a + b*x^2]])/(240*b^(3/2))
```

Maple [A] time = 0.007, size = 94, normalized size = 0.9

$$\frac{A}{5\,b}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bx}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bxa}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bxa^2}{16\,b}\sqrt{bx^2+a}-\frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*A/b*(b*x^2+a)^(5/2)+1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*a*B*x*(b*x
^2+a)^(3/2)/b-1/16*a^2*B*x*(b*x^2+a)^(1/2)/b-1/16*B*a^3/b^(3/2)*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277005, size = 1, normalized size = 0.01

$$\left[\frac{15\,Ba^3\log\left(2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\,\left(40\,Bb^2x^5+48\,Ab^2x^4+70\,Babx^3+96\,Aabx^2+15\,Ba^2x+48\,Aa^2\right)\sqrt{bx^2+a}\sqrt{b}}{480\,b^{\frac{3}{2}}},\right.$$
$$\left.-\frac{15\,Ba^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(40\,Bb^2x^5+48\,Ab^2x^4+70\,Babx^3+96\,Aabx^2+15\,Ba^2x+48\,Aa^2\right)\sqrt{bx^2+a}\sqrt{-b}}{240\,\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x,x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b
)) + 2*(40*B*b^2*x^5 + 48*A*b^2*x^4 + 70*B*a*b*x^3 + 96*A*a*b*x^2
 + 15*B*a^2*x + 48*A*a^2)*sqrt(b*x^2 + a)*sqrt(b))/b^(3/2), -1/24
0*(15*B*a^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^2*x^5 +
48*A*b^2*x^4 + 70*B*a*b*x^3 + 96*A*a*b*x^2 + 15*B*a^2*x + 48*A*a^
2)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b)]
```

Sympy [A] time = 15.4076, size = 223, normalized size = 2.17

$$Aa\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Bb^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3
*b), True)) + A*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) +
 a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b,
0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2
/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b*x**2/a)) + 11*B*sqrt(a)*
```

```
b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.224078, size = 120, normalized size = 1.17

$$\frac{Ba^3\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)*x,x, algorithm="giac")
```

```
[Out] 1/16*B*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*
sqrt(b*x^2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a +
 4*(5*B*b*x + 6*A*b)*x)*x)*x)*x)
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a +
 b*x^2)^(5/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0763275, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a +
 b*x^2)^(5/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(8*Sqrt[b])
```

Rubi in Sympy [A] time = 8.6607, size = 80, normalized size = 0.92

$$\frac{3Aa^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3Aax\sqrt{a+bx^2}}{8}+\frac{Ax\left(a+bx^2\right)^{\frac{3}{2}}}{4}+\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] 3*A*a**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*sqrt(b)) + 3*A*a*x*
sqrt(a + b*x**2)/8 + A*x*(a + b*x**2)**(3/2)/4 + B*(a + b*x**2)**
(5/2)/(5*b)
```

Mathematica [A] time = 0.0820824, size = 88, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\left(8a^2B+abx(25A+16Bx)+2b^2x^3(5A+4Bx)\right)+15a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{40b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A
 + 16*B*x)) + 15*a^2*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]
)/(40*b)
```

Maple [A] time = 0.006, size = 69, normalized size = 0.8

$$\frac{Ax}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,aAx}{8}\sqrt{bx^{2}+a}+\frac{3\,Aa^{2}}{8}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)\frac{1}{\sqrt{b}}+\frac{B}{5\,b}\left(bx^{2}+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)+3/8*a*A*x*(b*x^2+a)^(1/2)+3/8*A*a^2/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/5*B*(b*x^2+a)^(5/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263626, size = 1, normalized size = 0.01

$$\left[\frac{15\,Aa^{2}b\log\left(-2\,\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)+2\left(8\,Bb^{2}x^{4}+10\,Ab^{2}x^{3}+16\,Babx^{2}+25\,Aabx+8\,Ba^{2}\right)\sqrt{bx^{2}+a}\sqrt{b}}{80\,b^{\frac{3}{2}}},\ \frac{15\,Aa}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*b*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt
(b)) + 2*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x
+ 8*B*a^2)*sqrt(b*x^2 + a)*sqrt(b))/b^(3/2), 1/40*(15*A*a^2*b*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^2*x^4 + 10*A*b^2*x^3 + 1
6*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x^2 + a)*sqrt(-b))/(sq
rt(-b)*b)]
```

Sympy [A] time = 9.32045, size = 219, normalized size = 2.52

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3A\sqrt{a}bx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Aa^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^{2}x^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$
$$+Ba\left(\begin{cases}\frac{\sqrt{a}x^{2}}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{4}}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x*
*2/a)) + 3*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asi
nh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + B*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b
```

```
*x**2)**(3/2)/(3*b), True)) + B*b*Piecewise((-2*a**2*sqrt(a + b*x
**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b
*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

GIAC/XCAS [A] time = 0.22258, size = 103, normalized size = 1.18

$$-\frac{3\,Aa^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*s
qrt(b*x^2 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b
)*x)*x)*x)
```

3.12 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x}\,dx$

Optimal. Leaf size=106

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^
(3/2))/12 + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqr
t[b]) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.3105, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^
(3/2))/12 + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqr
t[b]) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 35.6575, size = 97, normalized size = 0.92

$$-Aa^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3Ba^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{a(8A+3Bx)\sqrt{a+bx^2}}{8}+\frac{(4A+3Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*atanh(sqrt(a + b*x**2)/sqrt(a)) + 3*B*a**2*atanh(sqrt
(b)*x/sqrt(a + b*x**2))/(8*sqrt(b)) + a*(8*A + 3*B*x)*sqrt(a + b*
x**2)/8 + (4*A + 3*B*x)*(a + b*x**2)**(3/2)/12
```

Mathematica [A] time = 0.166392, size = 112, normalized size = 1.06

$$-a^{3/2}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+a^{3/2}A\log(x)+\frac{3a^2B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8\sqrt{b}}+\frac{1}{24}\sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/24
+ a^(3/2)*A*Log[x] - a^(3/2)*A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] +
 (3*a^2*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Maple [A] time = 0.008, size = 107, normalized size = 1.

$$\frac{Bx}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,Bxa}{8}\sqrt{bx^2+a}+\frac{3\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}} +\frac{A}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Aa^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+A\sqrt{bx^2+a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/4*x*B*(b*x^2+a)^(3/2)+3/8*B*a*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(b*x^2+a)^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2748, size = 1, normalized size = 0.01

$$\left[\frac{9\,Ba^2\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+24\,Aa^{\frac{3}{2}}\sqrt{b}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bbx^3+8\,Abx^2+15\,Bax+32\,Aa\right.}{48\,\sqrt{b}}\right.$$

$$-\frac{48\,A\sqrt{-a}a\sqrt{b}\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-9\,Ba^2\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)-2\left(6\,Bbx^3+8\,Abx^2+15\,Bax+32\,Aa\right)}{48\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)
) + 24*A*a^(3/2)*sqrt(b)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a)
+ 2*a)/x^2) + 2*(6*B*b*x^3 + 8*A*b*x^2 + 15*B*a*x + 32*A*a)*sqrt(
b*x^2 + a)*sqrt(b))/sqrt(b), 1/24*(9*B*a^2*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) + 12*A*a^(3/2)*sqrt(-b)*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + (6*B*b*x^3 + 8*A*b*x^2 + 15*B*a*x + 32*
A*a)*sqrt(b*x^2 + a)*sqrt(-b))/sqrt(-b), -1/48*(48*A*sqrt(-a)*a*s
qrt(b)*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - 9*B*a^2*log(-2*sqrt
(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)) - 2*(6*B*b*x^3 + 8*A*b*x
^2 + 15*B*a*x + 32*A*a)*sqrt(b*x^2 + a)*sqrt(b))/sqrt(b), 1/24*(9
*B*a^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24*A*sqrt(-a)*a*sqrt(
-b)*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) + (6*B*b*x^3 + 8*A*b*x^2
 + 15*B*a*x + 32*A*a)*sqrt(b*x^2 + a)*sqrt(-b))/sqrt(-b)]
```

Sympy [A] time = 11.2046, size = 218, normalized size = 2.06

$$-Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$
$$+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Bb^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a
/(b*x**2) + 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecew
ise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + B*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 +
b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*B*a**2
*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*x**5/(4*sqrt(a)*sq
rt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.224871, size = 135, normalized size = 1.27

$$\frac{2\,Aa^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,Ba^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$
$$+\frac{1}{24}\,\sqrt{bx^2+a}(32\,Aa+(15\,Ba+2\,(3\,Bbx+4\,Ab)x)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
- 3/8*B*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24*
sqrt(b*x^2 + a)*(32*A*a + (15*B*a + 2*(3*B*b*x + 4*A*b)*x)*x)
```

3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=108

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^
(3/2))/(3*x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/2 - a^(3/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.305782, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^
(3/2))/(3*x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/2 - a^(3/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 30.4541, size = 97, normalized size = 0.9

$$\frac{3Aa\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2}-Ba^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{\sqrt{a+bx^2}\,(6Abx+4Ba)}{4}-\frac{(3A-Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] 3*A*a*sqrt(b)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/2 - B*a**(3/2)*at
anh(sqrt(a + b*x**2)/sqrt(a)) + sqrt(a + b*x**2)*(6*A*b*x + 4*B*a
)/4 - (3*A - B*x)*(a + b*x**2)**(3/2)/(3*x)
```

Mathematica [A] time = 0.154832, size = 113, normalized size = 1.05

$$-a^{3/2}B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+a^{3/2}B\log(x)+\sqrt{a+bx^2}\left(-\frac{aA}{x}+\frac{4aB}{3}+\frac{Abx}{2}+\frac{1}{3}bBx^2\right)+\frac{3}{2}aA\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] Sqrt[a + b*x^2]*((4*a*B)/3 - (a*A)/x + (A*b*x)/2 + (b*B*x^2)/3) +
 a^(3/2)*B*Log[x] - a^(3/2)*B*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] +
(3*a*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/2
```

Maple [A] time = 0.011, size = 126, normalized size = 1.2

$$-\frac{A}{ax}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{Axb}{a}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,Axb}{2}\sqrt{bx^{2}+a}+\frac{3\,Aa}{2}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)$$
$$+\frac{B}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}-Ba^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)+B\sqrt{bx^{2}+a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(5/2)+A*b/a*x*(b*x^2+a)^(3/2)+3/2*A*b*x*(b*x^2+a
)^(1/2)+3/2*A*b^(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*B*(b*x^
2+a)^(3/2)-B*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x
^2+a)^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283154, size = 1, normalized size = 0.01

$$\left[\frac{9\,Aa\sqrt{b}x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(2\,Bbx^{3}+3\,Abx^{2}+8\,Bax-6\,Aa\right)\sqrt{b}}{12\,x}\right.$$
$$-\frac{12\,B\sqrt{-a}ax\arctan\left(\frac{a}{\sqrt{bx^{2}+a}\sqrt{-a}}\right)-9\,Aa\sqrt{b}x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\left(2\,Bbx^{3}+3\,Abx^{2}+8\,Bax-6\,Aa\right)\sqrt{b}}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) + 6*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2
 + a))/x, 1/6*(9*A*a*sqrt(-b)*x*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(
-b))) + 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
 a))/x, -1/12*(12*B*sqrt(-a)*a*x*arctan(a/(sqrt(b*x^2 + a)*sqrt(-
a))) - 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
a))/x, 1/6*(9*A*a*sqrt(-b)*x*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b)
)) - 6*B*sqrt(-a)*a*x*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) + (2*B
*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x]
```

Sympy [A] time = 7.99014, size = 184, normalized size = 1.7

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}$$
$$-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**
2/a)/2 - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(s
qrt(b)*x/sqrt(a))/2 - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a
**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x*
*2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2
)**(3/2)/(3*b), True))
```

GIAC/XCAS [A] time = 0.229309, size = 167, normalized size = 1.55

$$\frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Aa\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{6}\sqrt{bx^2+a}(8\,Ba+(2\,Bbx+3\,Ab)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
- 3/2*A*a*sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2
*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 +
 a)*(8*B*a + (2*B*b*x + 3*A*b)*x)
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)
^(3/2))/(2*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/2 - (3*Sqrt[a]*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.291912, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)
^(3/2))/(2*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/2 - (3*Sqrt[a]*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi in Sympy [A] time = 28.3551, size = 109, normalized size = 0.98

$$-\frac{3A\sqrt{a}b\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2}+\frac{3Ba\sqrt{b}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2}-\frac{3\sqrt{a+bx^2}\,(-4Abx+4Ba)}{8x}-\frac{(2A-2Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*atanh(sqrt(a + b*x**2)/sqrt(a))/2 + 3*B*a*sqrt(b)*
atanh(sqrt(b)*x/sqrt(a + b*x**2))/2 - 3*sqrt(a + b*x**2)*(-4*A*b*
x + 4*B*a)/(8*x) - (2*A - 2*B*x)*(a + b*x**2)**(3/2)/(4*x**2)
```

Mathematica [A] time = 0.180684, size = 113, normalized size = 1.02

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\left(bx^2(2A+Bx)-a(A+2Bx)\right)}{x^2}-3\sqrt{a}Ab\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)\right.$$
$$\left.+3\sqrt{a}Ab\log(x)+3a\sqrt{b}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(b*x^2*(2*A + B*x) - a*(A + 2*B*x)))/x^2 + 3*Sq
rt[a]*A*b*Log[x] - 3*Sqrt[a]*A*b*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]
 + 3*a*Sqrt[b]*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/2
```

Maple [A] time = 0.01, size = 150, normalized size = 1.4

$$-\frac{A}{2\,ax^{2}}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{Ab}{2\,a}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{3\,Ab}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)+\frac{3\,Ab}{2}\sqrt{bx^{2}+a}$$
$$-\frac{B}{ax}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{bBx}{a}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,bBx}{2}\sqrt{bx^{2}+a}+\frac{3\,Ba}{2}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(5/2)+1/2*A*b/a*(b*x^2+a)^(3/2)-3/2*A*b*a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/
2)-B/a/x*(b*x^2+a)^(5/2)+B*b/a*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2
+a)^(1/2)+3/2*B*b^(1/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27355, size = 1, normalized size = 0.01

$$\left[\frac{3\,Ba\sqrt{b}x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+3\,A\sqrt{a}bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{b}}{4\,x^{2}}\right.$$
$$-\frac{6\,A\sqrt{-a}bx^{2}\arctan\left(\frac{a}{\sqrt{bx^{2}+a}\sqrt{-a}}\right)-3\,Ba\sqrt{b}x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\,\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{bx^{2}}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) + 3*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^
2 + a))/x^2, 1/4*(6*B*a*sqrt(-b)*x^2*arctan(b*x/(sqrt(b*x^2 + a)*
sqrt(-b))) + 3*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(
b*x^2 + a))/x^2, -1/4*(6*A*sqrt(-a)*b*x^2*arctan(a/(sqrt(b*x^2 +
a)*sqrt(-a))) - 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b
*x^2 + a))/x^2, 1/2*(3*B*a*sqrt(-b)*x^2*arctan(b*x/(sqrt(b*x^2 +
a)*sqrt(-b))) - 3*A*sqrt(-a)*b*x^2*arctan(a/(sqrt(b*x^2 + a)*sqrt
(-a))) + (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x
^2]
```

Sympy [A] time = 10.0889, size = 182, normalized size = 1.64

$$-\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/
(b*x**2) + 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b*
*(3/2)*x/sqrt(a/(b*x**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a))
 + B*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*
x**2/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
```

GIAC/XCAS [A] time = 0.230129, size = 258, normalized size = 2.32

$$\frac{3\,Aab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Ba\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^2b-2\,Ba^3\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(B*x + A)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
- 3/2*B*a*sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*
b*x + 2*A*b)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A
*a*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)
*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3*sqrt(b))/((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^2
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=173

$$\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2
))/(128*b^2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a
+ b*x^2)^(7/2))/(9*b) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(16
0*A + 189*B*x)*(a + b*x^2)^(7/2))/(5040*b^2) + (3*a^5*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi [A] time = 0.356346, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2
))/(128*b^2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a
+ b*x^2)^(7/2))/(9*b) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(16
0*A + 189*B*x)*(a + b*x^2)^(7/2))/(5040*b^2) + (3*a^5*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi in Sympy [A] time = 34.7748, size = 162, normalized size = 0.94

$$\frac{Ax^2\left(a+bx^2\right)^{\frac{7}{2}}}{9b}+\frac{3Ba^5\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{5}{2}}}+\frac{3Ba^4x\sqrt{a+bx^2}}{256b^2}+\frac{Ba^3x\left(a+bx^2\right)^{\frac{3}{2}}}{128b^2}+\frac{Ba^2x\left(a+bx^2\right)^{\frac{5}{2}}}{160b^2}+\frac{Bx^3\left(a+bx^2\right)^{\frac{7}{2}}}{10b}-\frac{a(160A+189Bx)\left(a+bx^2\right)^{\frac{7}{2}}}{5040b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**2*(a + b*x**2)**(7/2)/(9*b) + 3*B*a**5*atanh(sqrt(b)*x/sqrt(
a + b*x**2))/(256*b**(5/2)) + 3*B*a**4*x*sqrt(a + b*x**2)/(256*b*
*2) + B*a**3*x*(a + b*x**2)**(3/2)/(128*b**2) + B*a**2*x*(a + b*x
**2)**(5/2)/(160*b**2) + B*x**3*(a + b*x**2)**(7/2)/(10*b) - a*(1
60*A + 189*B*x)*(a + b*x**2)**(7/2)/(5040*b**2)
```

Mathematica [A] time = 0.176267, size = 138, normalized size = 0.8

$$\frac{945a^5B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}\sqrt{a+bx^2}\left(-5a^4(512A+189Bx)+10a^3bx^2(128A+63Bx)+24a^2b^2x^4(800A+651Bx)+\right.}{80640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x
^2*(128*A + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(8
00*A + 651*B*x) + 16*a*b^3*x^6*(1520*A + 1323*B*x)) + 945*a^5*B*L
og[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(80640*b^(5/2))
```

Maple [A] time = 0.012, size = 153, normalized size = 0.9

$$\frac{Ax^2}{9b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{2Aa}{63b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bx^3}{10b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3Bxa}{80b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bxa^2}{160b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^3Bx}{128b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3a^4Bx}{256b^2}\sqrt{bx^2+a}+\frac{3Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/9*A*x^2*(b*x^2+a)^(7/2)/b-2/63*A*a/b^2*(b*x^2+a)^(7/2)+1/10*B*x
^3*(b*x^2+a)^(7/2)/b-3/80*B*a/b^2*x*(b*x^2+a)^(7/2)+1/160*a^2*B*x
*(b*x^2+a)^(5/2)/b^2+1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+3/256*a^4*
B*x*(b*x^2+a)^(1/2)/b^2+3/256*B*a^5/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282127, size = 1, normalized size = 0.01

$$\left[\frac{945\,Ba^5\log\left(-2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(8064\,Bb^4x^9+8960\,Ab^4x^8+21168\,Bab^3x^7+24320\,Aab^3x^6+15624\,Ba^2\right.}{161280\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^3,x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*s
qrt(b)) + 2*(8064*B*b^4*x^9 + 8960*A*b^4*x^8 + 21168*B*a*b^3*x^7
+ 24320*A*a*b^3*x^6 + 15624*B*a^2*b^2*x^5 + 19200*A*a^2*b^2*x^4 +
 630*B*a^3*b*x^3 + 1280*A*a^3*b*x^2 - 945*B*a^4*x - 2560*A*a^4)*s
qrt(b*x^2 + a)*sqrt(b))/b^(5/2), 1/80640*(945*B*a^5*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) + (8064*B*b^4*x^9 + 8960*A*b^4*x^8 + 21168*
B*a*b^3*x^7 + 24320*A*a*b^3*x^6 + 15624*B*a^2*b^2*x^5 + 19200*A*a
^2*b^2*x^4 + 630*B*a^3*b*x^3 + 1280*A*a^3*b*x^2 - 945*B*a^4*x - 2
560*A*a^4)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 46.6717, size = 469, normalized size = 2.71

$$Aa^2 \left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases} \right) + 2Aab \left(\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^6}{6} & \text{otherwise} \end{cases} \right) + Ab^2 \left(\begin{cases} -\frac{16a^4\sqrt{a+bx^2}}{315b^4} + \frac{8a^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2a^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{ax^6\sqrt{a+bx^2}}{63b} + \frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^8}{8} & \text{otherwise} \end{cases} \right) - \frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{29B\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} + \frac{Bb^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqr
t(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(
a)*x**4/4, True)) + 2*A*a*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(1
05*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(
a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)
*x**6/6, True)) + A*b**2*Piecewise((-16*a**4*sqrt(a + b*x**2)/(31
5*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*s
qrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**
8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)) - 3*B*a*
*(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*s
qrt(1 + b*x**2/a)) + 129*B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a))
 + 73*B*a**(3/2)*b*x**7/(160*sqrt(1 + b*x**2/a)) + 29*B*sqrt(a)*b
**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*a**5*asinh(sqrt(b)*x/sqrt(
a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.228995, size = 189, normalized size = 1.09

$$-\frac{3\,Ba^5\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}} - \frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2} + \left(\frac{945\,Ba^4}{b^2} - 2\left(\frac{640\,Aa^3}{b} + \left(\frac{315\,Ba^3}{b} + 4\left(2400\,Aa^2 + \left(1953\,Ba^2 + 2\left(1520\,Aab + 7\left(189\,Bab + 8\left(9\,Bb^2\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^3,x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80
640*(2560*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^
3/b + 4*(2400*A*a^2 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b
+ 8*(9*B*b^2*x + 10*A*b^2)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a
)
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/
2))/(192*b) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^
2)^(7/2))/(9*b) - ((16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^
2) - (5*a^4*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.22998, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/
2))/(192*b) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^
2)^(7/2))/(9*b) - ((16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^
2) - (5*a^4*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi in Sympy [A] time = 20.6821, size = 139, normalized size = 0.93

$$-\frac{5Aa^4\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{3}{2}}}-\frac{5Aa^3x\sqrt{a+bx^2}}{128b}-\frac{5Aa^2x\left(a+bx^2\right)^{\frac{3}{2}}}{192b}-\frac{Aax\left(a+bx^2\right)^{\frac{5}{2}}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{\frac{7}{2}}}{9b}-\frac{\left(a+bx^2\right)^{\frac{7}{2}}(-63Abx+16Ba)}{504b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] -5*A*a**4*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(128*b**(3/2)) - 5*A*
a**3*x*sqrt(a + b*x**2)/(128*b) - 5*A*a**2*x*(a + b*x**2)**(3/2)/
(192*b) - A*a*x*(a + b*x**2)**(5/2)/(48*b) + B*x**2*(a + b*x**2)*
*(7/2)/(9*b) - (a + b*x**2)**(7/2)*(-63*A*b*x + 16*B*a)/(504*b**2
)
```

Mathematica [A] time = 0.153675, size = 126, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-256a^4B+a^3bx(315A+128Bx)+6a^2b^2x^3(413A+320Bx)+8ab^3x^5(357A+304Bx)+112b^4x^7(9A+8Bx)\right)-315}{8064b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*
x*(315*A + 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x
^3*(413*A + 320*B*x)) - 315*a^4*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[
a + b*x^2]])/(8064*b^2)
```

Maple [A] time = 0.008, size = 132, normalized size = 0.9

$$\frac{Ax}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{aAx}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,a^2Ax}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,a^3Ax}{128\,b}\sqrt{bx^2+a}$$
$$-\frac{5\,Aa^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Bx^2}{9\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{2\,Ba}{63\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*A*x*(b*x^2+a)^(7/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b-5/192*a^2*
A*x*(b*x^2+a)^(3/2)/b-5/128*a^3*A*x*(b*x^2+a)^(1/2)/b-5/128*A*a^4
/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/9*B*x^2*(b*x^2+a)^(7/2)/
b-2/63*B*a/b^2*(b*x^2+a)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270992, size = 1, normalized size = 0.01

$$\left[\frac{315\,Aa^4b\log\left(2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4\right.}{16128\,b^{\frac{5}{2}}}\right.$$
$$-\frac{315\,Aa^4b\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4+2478\,Aa^2b^2x^3+12\right.}{8064\,\sqrt{-b}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^2,x, algorithm="fricas")
```

```
[Out] [1/16128*(315*A*a^4*b*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*s
qrt(b)) + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 +
2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*
B*a^3*b*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a)*sqrt(b))
/b^(5/2), -1/8064*(315*A*a^4*b*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*
b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x
^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-
b)*b^2)]
```

Sympy [A] time = 32.2609, size = 442, normalized size = 2.95

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}$$
$$+\frac{Ab^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$+2Bab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$
$$+Bb^2\left(\begin{cases}-\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(
384*sqrt(1 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x
**2/a)) + 23*A*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a*
*4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + A*b**3*x**9/(8*sqrt(
a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewise((-2*a**2*sqrt(a + b*x**
2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x
**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + 2*B*a*b*Piecewise((8
*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/
(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**
2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + B*b**2*Piecewise((-16*
a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(
315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt
(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a
)*x**8/8, True))
```

GIAC/XCAS [A] time = 0.224881, size = 173, normalized size = 1.15

$$\frac{5\,Aa^4\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,Aab+2\left(152\,Bab+7\left(8\,Bb^2x+9\,Ab^2\right)x\right)x\right)x\right)x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x^2,x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/806
4*(256*B*a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4
*(240*B*a^2 + (357*A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/
2))/(192*b) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(
a + b*x^2)^(7/2))/(56*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.131859, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/
2))/(192*b) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(
a + b*x^2)^(7/2))/(56*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(128*b^(3/2))
```

Rubi in Sympy [A] time = 13.4098, size = 116, normalized size = 0.92

$$-\frac{5Ba^4\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{3}{2}}}-\frac{5Ba^3x\sqrt{a+bx^2}}{128b}-\frac{5Ba^2x\left(a+bx^2\right)^{\frac{3}{2}}}{192b}-\frac{Bax\left(a+bx^2\right)^{\frac{5}{2}}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{\frac{7}{2}}}{56b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] -5*B*a**4*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(128*b**(3/2)) - 5*B*
a**3*x*sqrt(a + b*x**2)/(128*b) - 5*B*a**2*x*(a + b*x**2)**(3/2)/
(192*b) - B*a*x*(a + b*x**2)**(5/2)/(48*b) + (8*A + 7*B*x)*(a + b
*x**2)**(7/2)/(56*b)
```

Mathematica [A] time = 0.146143, size = 119, normalized size = 0.94

$$\frac{\sqrt{b}\sqrt{a+bx^2}\left(3a^3(128A+35Bx)+2a^2bx^2(576A+413Bx)+8ab^2x^4(144A+119Bx)+48b^3x^6(8A+7Bx)\right)-105a^4B\log\left(\sqrt{b}\right.}{2688b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(48*b^3*x^6*(8*A + 7*B*x) + 3*a^3*(128*A
 + 35*B*x) + 8*a*b^2*x^4*(144*A + 119*B*x) + 2*a^2*b*x^2*(576*A +
 413*B*x)) - 105*a^4*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2688*
b^(3/2))
```

Maple [A] time = 0.007, size = 113, normalized size = 0.9

$$\frac{A}{7b}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bx}{8b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Bxa}{48b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5Bxa^2}{192b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5a^3Bx}{128b}\sqrt{bx^2+a}-\frac{5Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*A/b*(b*x^2+a)^(7/2)+1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*a*B*x*(b*x
^2+a)^(5/2)/b-5/192*a^2*B*x*(b*x^2+a)^(3/2)/b-5/128*a^3*B*x*(b*x^
2+a)^(1/2)/b-5/128*B*a^4/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267743, size = 1, normalized size = 0.01

$$\left[\frac{105\,Ba^4\log\left(2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+2\left(336\,Bb^3x^7+384\,Ab^3x^6+952\,Bab^2x^5+1152\,Aab^2x^4+826\,Ba^2bx^3+115\right.}{5376\,b^{\frac{3}{2}}}\right.$$

$$-\frac{105\,Ba^4\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(336\,Bb^3x^7+384\,Ab^3x^6+952\,Bab^2x^5+1152\,Aab^2x^4+826\,Ba^2bx^3+1152\,Aa^2bx^2+105\,Ba^3x\right.}{2688\sqrt{-b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)*x,x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt
(b)) + 2*(336*B*b^3*x^7 + 384*A*b^3*x^6 + 952*B*a*b^2*x^5 + 1152*
A*a*b^2*x^4 + 826*B*a^2*b*x^3 + 1152*A*a^2*b*x^2 + 105*B*a^3*x +
384*A*a^3)*sqrt(b*x^2 + a)*sqrt(b))/b^(3/2), -1/2688*(105*B*a^4*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (336*B*b^3*x^7 + 384*A*b^3*x^
6 + 952*B*a*b^2*x^5 + 1152*A*a*b^2*x^4 + 826*B*a^2*b*x^3 + 1152*A
*a^2*b*x^2 + 105*B*a^3*x + 384*A*a^3)*sqrt(b*x^2 + a)*sqrt(-b))/(
sqrt(-b)*b)]
```

Sympy [A] time = 30.001, size = 354, normalized size = 2.81

$$Aa^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$+Ab^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{133Ba^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23B\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)
/(3*b), True)) + 2*A*a*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*
b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5,
 Ne(b, 0)), (sqrt(a)*x**4/4, True)) + A*b**2*Piecewise((8*a**3*sq
rt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**
2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne
(b, 0)), (sqrt(a)*x**6/6, True)) + 5*B*a**(7/2)*x/(128*b*sqrt(1 +
 b*x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*
B*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x*
*7/(48*sqrt(1 + b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + B*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.230988, size = 154, normalized size = 1.22

$$\frac{5\,Ba^4\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^3}{b}+\left(\frac{105\,Ba^3}{b}+2\left(576\,Aa^2+\left(413\,Ba^2+4\left(144\,Aab+\left(119\,Bab+6\left(7\,Bb^2x+8\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)*x,x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/268
8*(384*A*a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(14
4*A*a*b + (119*B*a*b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*
sqrt(b*x^2 + a)
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 +
 (A*x*(a + b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3
*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] time = 0.0975907, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 +
 (A*x*(a + b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3
*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi in Sympy [A] time = 10.5491, size = 100, normalized size = 0.93

$$\frac{5Aa^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5Aa^2x\sqrt{a+bx^2}}{16}+\frac{5Aax\left(a+bx^2\right)^{\frac{3}{2}}}{24}+\frac{Ax\left(a+bx^2\right)^{\frac{5}{2}}}{6}+\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**3*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*sqrt(b)) + 5*A*a**
2*x*sqrt(a + b*x**2)/16 + 5*A*a*x*(a + b*x**2)**(3/2)/24 + A*x*(a
 + b*x**2)**(5/2)/6 + B*(a + b*x**2)**(7/2)/(7*b)
```

Mathematica [A] time = 0.136852, size = 108, normalized size = 1.01

$$\frac{105a^3A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{a+bx^2}\left(48a^3B+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx)+8b^3x^5(7A+6Bx)\right)}{336b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*
(77*A + 48*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) + 105*a^3*A*Sqrt[b
]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(336*b)
```

Maple [A] time = 0.006, size = 85, normalized size = 0.8

$$\frac{Ax}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{5\,aAx}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{5\,a^{2}Ax}{16}\sqrt{bx^{2}+a}+\frac{5\,Aa^{3}}{16}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)\frac{1}{\sqrt{b}}+\frac{B}{7\,b}\left(bx^{2}+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*A*x*(b*x^2+a)^(5/2)+5/24*a*A*x*(b*x^2+a)^(3/2)+5/16*a^2*A*x*(
b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1
/7*B*(b*x^2+a)^(7/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265796, size = 1, normalized size = 0.01

$$\left[\frac{105\,Aa^{3}b\log\left(-2\,\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)+2\left(48\,Bb^{3}x^{6}+56\,Ab^{3}x^{5}+144\,Bab^{2}x^{4}+182\,Aab^{2}x^{3}+144\,Ba^{2}bx^{2}+231\right.}{672\,b^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*b*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sq
rt(b)) + 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A
*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x
^2 + a)*sqrt(b))/b^(3/2), 1/336*(105*A*a^3*b*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + (48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 +
 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sq
rt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b)]
```

Sympy [A] time = 18.9186, size = 348, normalized size = 3.25

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17A\sqrt{a}b^{2}x^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5Aa^{3}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^{3}x^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$
$$+Ba^{2}\left(\begin{cases}\frac{\sqrt{a}x^{2}}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Bab\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{4}}{4} & \text{otherwise}\end{cases}\right)$$
$$+Bb^{2}\left(\begin{cases}\frac{8a^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4a^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{x^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{6}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b
*x**2/a)) + 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*s
qrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)
*x/sqrt(a))/(16*sqrt(b)) + A*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)
**(3/2)/(3*b), True)) + 2*B*a*b*Piecewise((-2*a**2*sqrt(a + b*x**
2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x
**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b**2*Piecewise((8*
a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(
105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2
)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

GIAC/XCAS [A] time = 0.21724, size = 136, normalized size = 1.27

$$-\frac{5\,Aa^3\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336
*(48*B*a^3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b
 + (6*B*b^2*x + 7*A*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.19 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{16}a^2\sqrt{a+bx^2}(16A+5Bx)$$
$$+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b
*x^2)^(3/2))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.414349, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{16}a^2\sqrt{a+bx^2}(16A+5Bx)$$
$$+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b
*x^2)^(3/2))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 50.3528, size = 121, normalized size = 0.92

$$-Aa^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5Ba^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{a^2(48A+15Bx)\sqrt{a+bx^2}}{48}$$
$$+\frac{a(24A+15Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{72}+\frac{(6A+5Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*atanh(sqrt(a + b*x**2)/sqrt(a)) + 5*B*a**3*atanh(sqrt
(b)*x/sqrt(a + b*x**2))/(16*sqrt(b)) + a**2*(48*A + 15*B*x)*sqrt(
a + b*x**2)/48 + a*(24*A + 15*B*x)*(a + b*x**2)**(3/2)/72 + (6*A
+ 5*B*x)*(a + b*x**2)**(5/2)/30
```

Mathematica [A] time = 0.250968, size = 132, normalized size = 1.

$$-a^{5/2}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+a^{5/2}A\log(x)+\frac{5a^3B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16\sqrt{b}}$$
$$+\frac{1}{240}\sqrt{a+bx^2}\left(a^2(368A+165Bx)+2abx^2(88A+65Bx)+8b^2x^4(6A+5Bx)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*
B*x) + a^2*(368*A + 165*B*x)))/240 + a^(5/2)*A*Log[x] - a^(5/2)*A
*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] + (5*a^3*B*Log[b*x + Sqrt[b]*Sq
rt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A]   time = 0.009, size = 138, normalized size = 1.1

$$\frac{Bx}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Bxa}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,Bxa^2}{16}\sqrt{bx^2+a}+\frac{5\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{A}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Aa}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Aa^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+A\sqrt{bx^2+a}aa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x+A)*(b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/6*x*B*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(
b*x^2+a)^(1/2)+5/16*B*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1
/5*A*(b*x^2+a)^(5/2)+1/3*A*a*(b*x^2+a)^(3/2)-A*a^(5/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(b*x^2+a)^(1/2)*a^2
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.282997, size = 1, normalized size = 0.01

$$\left[\frac{75\,Ba^3\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+240\,Aa^{\frac{5}{2}}\sqrt{b}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(40\,Bb^2x^5+48\,Ab^2x^4+130\,Bab\right.}{480\,\sqrt{b}}\right.$$

$$-\frac{480\,A\sqrt{-a}a^2\sqrt{b}\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-75\,Ba^3\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)-2\left(40\,Bb^2x^5+48\,Ab^2x^4+130\,Babx\right.}{480\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(
b)) + 240*A*a^(5/2)*sqrt(b)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(40*B*b^2*x^5 + 48*A*b^2*x^4 + 130*B*a*b*x^3 +
 176*A*a*b*x^2 + 165*B*a^2*x + 368*A*a^2)*sqrt(b*x^2 + a)*sqrt(b)
)/sqrt(b), 1/240*(75*B*a^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 1
20*A*a^(5/2)*sqrt(-b)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + (40*B*b^2*x^5 + 48*A*b^2*x^4 + 130*B*a*b*x^3 + 176*A*a
*b*x^2 + 165*B*a^2*x + 368*A*a^2)*sqrt(b*x^2 + a)*sqrt(-b))/sqrt(
-b), -1/480*(480*A*sqrt(-a)*a^2*sqrt(b)*arctan(a/(sqrt(b*x^2 + a)
```

```
*sqrt(-a))) - 75*B*a^3*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)
*sqrt(b)) - 2*(40*B*b^2*x^5 + 48*A*b^2*x^4 + 130*B*a*b*x^3 + 176*
A*a*b*x^2 + 165*B*a^2*x + 368*A*a^2)*sqrt(b*x^2 + a)*sqrt(b))/sqr
t(b), 1/240*(75*B*a^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 240*A*
sqrt(-a)*a^2*sqrt(-b)*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) + (40*
B*b^2*x^5 + 48*A*b^2*x^4 + 130*B*a*b*x^3 + 176*A*a*b*x^2 + 165*B*
a^2*x + 368*A*a^2)*sqrt(b*x^2 + a)*sqrt(-b))/sqrt(-b)]
```

Sympy [A] time = 21.3915, size = 323, normalized size = 2.45

$$-Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17B\sqrt{a}b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Bb^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a
/(b*x**2) + 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b
*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b)
, True)) + A*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) +
 a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b,
0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x*sqrt(1 + b*x**2/a)/2
+ 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x**3/(
48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x*
*2/a)) + 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + B*b**3*
x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.21954, size = 169, normalized size = 1.28

$$\frac{2\,Aa^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,Ba^3\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{240}\left(368\,Aa^2+\left(165\,Ba^2+2\left(88\,Aab+\left(65\,Bab+4\left(5\,Bb^2x+6\,Ab^2\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
- 5/16*B*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24
0*(368*A*a^2 + (165*B*a^2 + 2*(88*A*a*b + (65*B*a*b + 4*(5*B*b^2*
x + 6*A*b^2)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.20 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=136

$$a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}(8aB+15Abx)$$
$$-\frac{(a+bx^2)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}(a+bx^2)^{3/2}(4aB+15Abx)$$

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a
 + b*x^2)^(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15
*a^2*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*
B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.404596, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}(8aB+15Abx)$$
$$-\frac{(a+bx^2)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}(a+bx^2)^{3/2}(4aB+15Abx)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a
 + b*x^2)^(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15
*a^2*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*
B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 45.5015, size = 124, normalized size = 0.91

$$\frac{15Aa^2\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8}-Ba^{\frac{5}{2}}\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{a\sqrt{a+bx^2}\,(30Abx+16Ba)}{16}$$
$$+\frac{(a+bx^2)^{\frac{3}{2}}(30Abx+8Ba)}{24}-\frac{(5A-Bx)(a+bx^2)^{\frac{5}{2}}}{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] 15*A*a**2*sqrt(b)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/8 - B*a**(5/2
)*atanh(sqrt(a + b*x**2)/sqrt(a)) + a*sqrt(a + b*x**2)*(30*A*b*x
+ 16*B*a)/16 + (a + b*x**2)**(3/2)*(30*A*b*x + 8*B*a)/24 - (5*A -
 B*x)*(a + b*x**2)**(5/2)/(5*x)
```

Mathematica [A] time = 0.22353, size = 135, normalized size = 0.99

$$-a^{5/2}B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)$$
$$+a^{5/2}B\log(x)+\frac{\sqrt{a+bx^2}\left(-8a^2(15A-23Bx)+abx^2(135A+88Bx)+6b^2x^4(5A+4Bx)\right)}{120x}$$
$$+\frac{15}{8}a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*(15*A - 23*B*x) + 6*b^2*x^4*(5*A + 4*B*x
) + a*b*x^2*(135*A + 88*B*x)))/(120*x) + a^(5/2)*B*Log[x] - a^(5/
2)*B*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] + (15*a^2*A*Sqrt[b]*Log[b*x
 + Sqrt[b]*Sqrt[a + b*x^2]])/8
```

Maple [A] time = 0.013, size = 158, normalized size = 1.2

$$-\frac{A}{ax}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Axb}{a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Axb}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{15\,Axab}{8}\sqrt{bx^2+a}+\frac{15\,Aa^2}{8}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)+\frac{B}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ba}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Ba^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+B\sqrt{bx^2+a}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(7/2)+A*b/a*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a
)^(3/2)+15/8*A*b*a*x*(b*x^2+a)^(1/2)+15/8*A*b^(1/2)*a^2*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))+1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2
)-B*a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/
2)*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285574, size = 1, normalized size = 0.01

$$\left[\frac{225\,Aa^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx\right.}{240\,x}\right.$$

$$-\frac{240\,B\sqrt{-a}a^2x\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-225\,Aa^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx\right.}{240\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 +
 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/1
20*(225*A*a^2*sqrt(-b)*x*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) +
 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
```

```
2) + (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2
+ 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/240*(240*B*sqrt
(-a)*a^2*x*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - 225*A*a^2*sqrt(
b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(24*B*b^
2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x
 - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/120*(225*A*a^2*sqrt(-b)*x*arc
tan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - 120*B*sqrt(-a)*a^2*x*arctan
(a/(sqrt(b*x^2 + a)*sqrt(-a))) + (24*B*b^2*x^5 + 30*A*b^2*x^4 + 8
8*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2
 + a))/x]
```

Sympy [A] time = 15.0522, size = 318, normalized size = 2.34

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Bab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x*
*2/a) - 7*A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**
2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + A*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(
5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3/(sqrt(b)*x*sqrt(a/(b*x**
2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*B*a*b*Piecew
ise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2
*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (s
qrt(a)*x**4/4, True))
```

GIAC/XCAS [A] time = 0.224961, size = 203, normalized size = 1.49

$$\frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Aa^2\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{120}\left(184\,Ba^2+\left(135\,Aab+2\left(44\,Bab+3\left(4\,Bb^2x+5\,Ab^2\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
- 15/8*A*a^2*sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*
a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*
a^2 + (135*A*a*b + 2*(44*B*a*b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)
*sqrt(b*x^2 + a)
```

3.21 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+\frac{5}{8}ab\sqrt{a+bx^2}(4A+3Bx)$$

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a
 + b*x^2)^(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2)
 + (15*a^2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5
*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.398463, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+\frac{5}{8}ab\sqrt{a+bx^2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a
 + b*x^2)^(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2)
 + (15*a^2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5
*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi in Sympy [A] time = 42.9574, size = 136, normalized size = 0.96

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2}+\frac{15Ba^2\sqrt{b}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8}+\frac{5ab(32A+24Bx)\sqrt{a+bx^2}}{64}-\frac{5\left(a+bx^2\right)^{\frac{3}{2}}(-8Abx+12Ba)}{48x}-\frac{(4A-2Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*atanh(sqrt(a + b*x**2)/sqrt(a))/2 + 15*B*a**2*sqr
t(b)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/8 + 5*a*b*(32*A + 24*B*x)*
sqrt(a + b*x**2)/64 - 5*(a + b*x**2)**(3/2)*(-8*A*b*x + 12*B*a)/(
48*x) - (4*A - 2*B*x)*(a + b*x**2)**(5/2)/(8*x**2)
```

Mathematica [A] time = 0.249846, size = 135, normalized size = 0.96

$$\frac{1}{24}\left(-60a^{3/2}Ab\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+60a^{3/2}Ab\log(x)+\frac{\sqrt{a+bx^2}\left(-12a^2(A+2Bx)+abx^2(56A+27Bx)+2b^2x^4(4A+3Bx)\right)}{x^2}+45a^2\sqrt{b}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-12*a^2*(A + 2*B*x) + 2*b^2*x^4*(4*A + 3*B*x)
+ a*b*x^2*(56*A + 27*B*x)))/x^2 + 60*a^(3/2)*A*b*Log[x] - 60*a^(3
/2)*A*b*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] + 45*a^2*Sqrt[b]*B*Log[b
*x + Sqrt[b]*Sqrt[a + b*x^2]])/24
```

Maple [A]   time = 0.013, size = 181, normalized size = 1.3

$$-\frac{A}{2\,ax^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Ab}{2\,a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Ab}{6}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,Ab}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+\frac{5\,abA}{2}\sqrt{bx^2+a}-\frac{B}{ax}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{bBx}{a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,bBx}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{15\,abBx}{8}\sqrt{bx^2+a}+\frac{15\,a^2B}{8}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(7/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(5/2)+5/6*A*b*(b
*x^2+a)^(3/2)-5/2*A*b*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+5/2*A*b*a*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(7/2)+B*b/a*x*(b*x^2
+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^(3/2)+15/8*B*b*a*x*(b*x^2+a)^(1/2)+
15/8*B*b^(1/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.275154, size = 1, normalized size = 0.01

$$\left[\frac{45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+60\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3\right.}{48\,x^2}\right.$$

$$-\frac{120\,A\sqrt{-a}abx^2\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3\right.}{48\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3
+ 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/2
4*(45*B*a^2*sqrt(-b)*x^2*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) +
 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
```

```
)/x^2) + (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2
 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/48*(120*A*sqrt
(-a)*a*b*x^2*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - 45*B*a^2*sqrt
(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(6*B*
b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x
- 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/24*(45*B*a^2*sqrt(-b)*x^2*arc
tan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - 60*A*sqrt(-a)*a*b*x^2*arcta
n(a/(sqrt(b*x^2 + a)*sqrt(-a))) + (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27
*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a
))/x^2]
```

Sympy [A] time = 16.7827, size = 279, normalized size = 1.98

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$
$$-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Ba^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Ba^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Ba^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{Bb^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqr
t(a/(b*x**2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1
)) + 2*A*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sq
rt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) - B*a
**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3/2)*b*x*sqrt(1 + b*x**2/a
) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b**2*x*
*3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqr
t(a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.236044, size = 296, normalized size = 2.1

$$\frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Ba^2\sqrt{b}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,Ab^2\right)x\right)x\right)\sqrt{bx^2+a}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aa^2b+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^3b-2\,Ba^4\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(B*x + A)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a
) - 15/8*B*a^2*sqrt(b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/
24*(56*A*a*b + (27*B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x
^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a
))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)^2
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (
a*(16*A + 9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.278379, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (
a*(16*A + 9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi in Sympy [A] time = 24.6885, size = 94, normalized size = 0.9

$$\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{3Ba^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a\left(16A+9Bx\right)\sqrt{a+bx^2}}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*x**2*sqrt(a + b*x**2)/(3*b) + 3*B*a**2*atanh(sqrt(b)*x/sqrt(a +
 b*x**2))/(8*b**(5/2)) + B*x**3*sqrt(a + b*x**2)/(4*b) - a*(16*A
+ 9*B*x)*sqrt(a + b*x**2)/(24*b**2)
```

Mathematica [A] time = 0.103583, size = 79, normalized size = 0.76

$$\frac{9a^2B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}\sqrt{a+bx^2}\left(-16aA-9aBx+8Abx^2+6bBx^3\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x
^3) + 9*a^2*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(24*b^(5/2))
```

Maple [A] time = 0.011, size = 96, normalized size = 0.9

$$\frac{Ax^2}{3\,b}\sqrt{bx^2+a}-\frac{2\,Aa}{3\,b^2}\sqrt{bx^2+a}+\frac{x^3B}{4\,b}\sqrt{bx^2+a}-\frac{3\,Bxa}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*A*x^2*(b*x^2+a)^(1/2)/b-2/3*A*a/b^2*(b*x^2+a)^(1/2)+1/4*B*x^3
*(b*x^2+a)^(1/2)/b-3/8*B*a/b^2*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(5/2
)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x + A)*x^3/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26066, size = 1, normalized size = 0.01

$$\left[\frac{9\,Ba^2\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(6\,Bbx^3+8\,Abx^2-9\,Bax-16\,Aa\right)\sqrt{bx^2+a}\sqrt{b}}{48\,b^{\frac{5}{2}}},\ \frac{9\,Ba^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x + A)*x^3/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)
) + 2*(6*B*b*x^3 + 8*A*b*x^2 - 9*B*a*x - 16*A*a)*sqrt(b*x^2 + a)*
sqrt(b))/b^(5/2), 1/24*(9*B*a^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (6*B*b*x^3 + 8*A*b*x^2 - 9*B*a*x - 16*A*a)*sqrt(b*x^2 + a)*sq
rt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 8.06697, size = 150, normalized size = 1.44

$$A\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**
2)/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - 3*B*a**(3/2)*x/(
8*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/
a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + B*x**5/(4*
sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.227241, size = 100, normalized size = 0.96

$$\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^2}\right)x-\frac{16\,Aa}{b^2}\right)-\frac{3\,Ba^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*
A*a/b^2) - 3/8*B*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2
)
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2
])/(6*b^2) - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2
))
```

Rubi [A] time = 0.169885, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2
])/(6*b^2) - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2
))
```

Rubi in Sympy [A] time = 14.7248, size = 73, normalized size = 0.9

$$-\frac{Aa\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}}+\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\sqrt{a+bx^2}\,(-3Abx+4Ba)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(3/2)) + B*x**2*sqrt
(a + b*x**2)/(3*b) - sqrt(a + b*x**2)*(-3*A*b*x + 4*B*a)/(6*b**2)
```

Mathematica [A] time = 0.0677468, size = 67, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}(bx(3A+2Bx)-4aB)-3aA\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + b*x*(3*A + 2*B*x)) - 3*a*A*Sqrt[b]*Log
[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(6*b^2)
```

Maple [A] time = 0.009, size = 75, normalized size = 0.9

$$\frac{Ax}{2\,b}\sqrt{bx^2+a}-\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Bx^2}{3\,b}\sqrt{bx^2+a}-\frac{2\,Ba}{3\,b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A*x/b*(b*x^2+a)^(1/2)-1/2*A*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))+1/3*B*x^2*(b*x^2+a)^(1/2)/b-2/3*B*a/b^2*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266668, size = 1, normalized size = 0.01

$$\left[\frac{3\,Aab\log\left(2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\,\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}\sqrt{b}}{12\,b^{\frac{5}{2}}},\ -\frac{3\,Aab\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}\sqrt{-b}}{6\,\sqrt{-b}b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*b*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))
 + 2*(2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a)*sqrt(b))/b^(5/
2), -1/6*(3*A*a*b*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2
 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 4.99128, size = 94, normalized size = 1.16

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+B\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a
))/(2*b**(3/2)) + B*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x
**2*sqrt(a + b*x**2)/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

GIAC/XCAS [A] time = 0.226875, size = 82, normalized size = 1.01

$$\frac{1}{6}\,\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*l
n(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+bx^2}(2A+Bx)}{2b} - \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.0803692, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sqrt{a+bx^2}(2A+Bx)}{2b} - \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 7.39284, size = 48, normalized size = 0.86

$$-\frac{Ba\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}} + \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -B*a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(3/2)) + (2*A + B*x)
*sqrt(a + b*x**2)/(2*b)
```

Mathematica [A] time = 0.0536122, size = 61, normalized size = 1.09

$$\sqrt{a+bx^2}\left(\frac{A}{b}+\frac{Bx}{2b}\right) - \frac{aB\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A/b + (B*x)/(2*b))*Sqrt[a + b*x^2] - (a*B*Log[b*x + Sqrt[b]*Sqrt
[a + b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.006, size = 55, normalized size = 1.

$$\frac{A}{b}\sqrt{bx^2+a} + \frac{Bx}{2\,b}\sqrt{bx^2+a} - \frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] A/b*(b*x^2+a)^(1/2)+1/2*B*x/b*(b*x^2+a)^(1/2)-1/2*B*a/b^(3/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261594, size = 1, normalized size = 0.02

$$\left[\frac{Ba\log\left(2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+2\sqrt{bx^2+a}(Bx+2A)\sqrt{b}}{4b^{\frac{3}{2}}},\ -\frac{Ba\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}(Bx+2A)\sqrt{-b}}{2\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)) + 2*
sqrt(b*x^2 + a)*(B*x + 2*A)*sqrt(b))/b^(3/2), -1/2*(B*a*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x + 2*A)*sqrt(-b))
/(sqrt(-b)*b)]
```

Sympy [A] time = 4.57541, size = 70, normalized size = 1.25

$$A\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, Tr
ue)) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x
/sqrt(a))/(2*b**(3/2))
```

GIAC/XCAS [A] time = 0.219447, size = 68, normalized size = 1.21

$$\frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2A}{b}\right)+\frac{Ba\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/sqrt(b*x^2 + a),x, algorithm="giac")

[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*ln(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(3/2)
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
Sqrt[b]
```

Rubi [A] time = 0.0477824, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
Sqrt[b]
```

Rubi in Sympy [A] time = 6.19711, size = 37, normalized size = 0.86

$$\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*atanh(sqrt(b)*x/sqrt(a + b*x**2))/sqrt(b) + B*sqrt(a + b*x**2)/
b
```

Mathematica [A] time = 0.032228, size = 46, normalized size = 1.07

$$\frac{A\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sq
rt[b]
```

Maple [A] time = 0.007, size = 37, normalized size = 0.9

$$A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{B}{b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268949, size = 1, normalized size = 0.02

$$\left[\frac{Ab\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+2\sqrt{bx^2+a}B\sqrt{b}}{2b^{\frac{3}{2}}}, \frac{Ab\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}B\sqrt{-b}}{\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*(A*b*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)) + 2
*sqrt(b*x^2 + a)*B*sqrt(b))/b^(3/2), (A*b*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) + sqrt(b*x^2 + a)*B*sqrt(-b))/(sqrt(-b)*b)]
```

Sympy [A] time = 1.35068, size = 102, normalized size = 2.37

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b
< 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)),
 (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) +
B*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, Tr
ue))
```

GIAC/XCAS [A] time = 0.227547, size = 53, normalized size = 1.23

$$-\frac{A\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -A*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a
)*B/b
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.132796, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 11.2993, size = 48, normalized size = 0.91

$$-\frac{A\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{B\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a) + B*atanh(sqrt(b)*x/sq
rt(a + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0574139, size = 67, normalized size = 1.26

$$-\frac{A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{A\log(x)}{\sqrt{a}}+\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (A*Log[x])/Sqrt[a] - (A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/Sqrt[a]
 + (B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.01, size = 52, normalized size = 1.

$$B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294268, size = 1, normalized size = 0.02

$$\left[\frac{B\sqrt{a}\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+A\sqrt{b}\log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)}{2\sqrt{a}\sqrt{b}}, \frac{2B\sqrt{a}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+A\sqrt{-b}\log\left(-\frac{(\right.}{}\right.}{2\sqrt{a}\sqrt{-b}}\right.$$

$$-\frac{2A\sqrt{b}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{2\sqrt{-a}\sqrt{b}}, \frac{B\sqrt{-a}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-b}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{\sqrt{-a}\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(a)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b
)) + A*sqrt(b)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)
/x^2))/(sqrt(a)*sqrt(b)), 1/2*(2*B*sqrt(a)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) + A*sqrt(-b)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*
x^2 + a)*a)/x^2))/(sqrt(a)*sqrt(-b)), -1/2*(2*A*sqrt(b)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) - B*sqrt(-a)*log(-2*sqrt(b*x^2 + a)*b*x -
 (2*b*x^2 + a)*sqrt(b)))/(sqrt(-a)*sqrt(b)), (B*sqrt(-a)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-b)*arctan(sqrt(-a)/sqrt(b*x^
2 + a)))/(sqrt(-a)*sqrt(-b))]
```

Sympy [A] time = 2.54756, size = 99, normalized size = 1.87

$$-\frac{A\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a > 0 \wedge b < 0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a > 0 \wedge b > 0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b > 0 \wedge a < 0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((sqrt(-a/b)*a
sin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x
```

```
*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt
(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
```

GIAC/XCAS [A] time = 0.223134, size = 78, normalized size = 1.47

$$\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{B\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*
ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/Sqrt[a]
```

Rubi [A] time = 0.121638, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/Sqrt[a]
```

Rubi in Sympy [A] time = 9.29236, size = 39, normalized size = 0.83

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(a*x) - B*atanh(sqrt(a + b*x**2)/sqrt(a))/sqr
t(a)
```

Mathematica [A] time = 0.0678633, size = 58, normalized size = 1.23

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{B\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*Log[x])/Sqrt[a] - (B*Log[a + Sq
rt[a]*Sqrt[a + b*x^2]])/Sqrt[a]
```

Maple [A] time = 0.01, size = 49, normalized size = 1.

$$-\frac{A}{ax}\sqrt{bx^2+a}-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -A*(b*x^2+a)^(1/2)/a/x-B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.255294, size = 1, normalized size = 0.02

$$\left[\frac{Bax\log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)-2\sqrt{bx^2+a}A\sqrt{a}}{2a^{\frac{3}{2}}x},-\frac{Bax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}A\sqrt{-a}}{\sqrt{-a}ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*x*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^
2) - 2*sqrt(b*x^2 + a)*A*sqrt(a))/(a^(3/2)*x), -(B*a*x*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*A*sqrt(-a))/(sqrt(-a)*a*
x)]
```

Sympy [A] time = 3.01679, size = 41, normalized size = 0.87

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/
sqrt(a)
```

GIAC/XCAS [A] time = 0.217755, size = 88, normalized size = 1.87

$$\frac{2B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="giac")

[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*
A*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] time = 0.192931, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi in Sympy [A] time = 15.1556, size = 60, normalized size = 0.83

$$-\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{Ab\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{B\sqrt{a+bx^2}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(2*a*x**2) + A*b*atanh(sqrt(a + b*x**2)/sqrt(
a))/(2*a**(3/2)) - B*sqrt(a + b*x**2)/(a*x)
```

Mathematica [A] time = 0.0795756, size = 72, normalized size = 1.

$$\frac{-\sqrt{a}\sqrt{a+bx^2}(A+2Bx)+Abx^2\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)-Abx^2\log(x)}{2a^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (-(Sqrt[a]*(A + 2*B*x)*Sqrt[a + b*x^2]) - A*b*x^2*Log[x] + A*b*x^
2*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(2*a^(3/2)*x^2)
```

Maple [A] time = 0.012, size = 68, normalized size = 0.9

$$-\frac{A}{2\,ax^2}\sqrt{bx^2+a}+\frac{Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)-B*(b*x^2+a)^(1/2)/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273409, size = 1, normalized size = 0.01

$$\left[\frac{Abx^2 \log\left(-\frac{(bx^2+2\,a)\sqrt{a}+2\sqrt{bx^2+a}a}{x^2}\right) - 2\sqrt{bx^2+a}(2\,Bx+A)\sqrt{a}}{4\,a^{\frac{3}{2}}x^2}, \frac{Abx^2 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - \sqrt{bx^2+a}(2\,Bx+A)\sqrt{-a}}{2\sqrt{-a}ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(A*b*x^2*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/
x^2) - 2*sqrt(b*x^2 + a)*(2*B*x + A)*sqrt(a))/(a^(3/2)*x^2), 1/2*
(A*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(2*B*
x + A)*sqrt(-a))/(sqrt(-a)*a*x^2)]
```

Sympy [A] time = 5.06845, size = 66, normalized size = 0.92

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt
(b)*x))/(2*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

GIAC/XCAS [A] time = 0.225065, size = 197, normalized size = 2.74

$$-\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a)
 + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*
B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a)
```

3.29 $\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{a+bx^2}(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a +
b*x^2])/(2*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*b^(5/2))
```

Rubi [A] time = 0.175091, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{a+bx^2}(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a +
b*x^2])/(2*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*b^(5/2))
```

Rubi in Sympy [A] time = 16.4023, size = 78, normalized size = 0.96

$$-\frac{3Ba\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{5}{2}}}-\frac{x^2\,(2A+2Bx)}{2b\sqrt{a+bx^2}}+\frac{(8A+6Bx)\,\sqrt{a+bx^2}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] -3*B*a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(5/2)) - x**2*(2*A
 + 2*B*x)/(2*b*sqrt(a + b*x**2)) + (8*A + 6*B*x)*sqrt(a + b*x**2)
/(4*b**2)
```

Mathematica [A] time = 0.139233, size = 75, normalized size = 0.93

$$\frac{a(4A+3Bx)+bx^2(2A+Bx)}{2b^2\sqrt{a+bx^2}}-\frac{3aB\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (b*x^2*(2*A + B*x) + a*(4*A + 3*B*x))/(2*b^2*Sqrt[a + b*x^2]) - (
3*a*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A] time = 0.01, size = 93, normalized size = 1.2

$$\frac{Ax^2}{b}\frac{1}{\sqrt{bx^2+a}}+2\frac{Aa}{b^2\sqrt{bx^2+a}}+\frac{x^3B}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,Bxa}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] A*x^2/b/(b*x^2+a)^(1/2)+2*A*a/b^2/(b*x^2+a)^(1/2)+1/2*B*x^3/b/(b*
x^2+a)^(1/2)+3/2*B*a/b^2*x/(b*x^2+a)^(1/2)-3/2*B*a/b^(5/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272456, size = 1, normalized size = 0.01

$$\left[\frac{2\left(Bbx^3+2\,Abx^2+3\,Bax+4\,Aa\right)\sqrt{bx^2+a}\sqrt{b}+3\left(Babx^2+Ba^2\right)\log\left(2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{4\,(b^3x^2+ab^2)\sqrt{b}},\frac{\left(Bbx^3+2\,Abx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b*x^3 + 2*A*b*x^2 + 3*B*a*x + 4*A*a)*sqrt(b*x^2 + a)*s
qrt(b) + 3*(B*a*b*x^2 + B*a^2)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x
^2 + a)*sqrt(b)))/((b^3*x^2 + a*b^2)*sqrt(b)), 1/2*((B*b*x^3 + 2*
A*b*x^2 + 3*B*a*x + 4*A*a)*sqrt(b*x^2 + a)*sqrt(-b) - 3*(B*a*b*x^
2 + B*a^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^3*x^2 + a*b^2)
*sqrt(-b))]
```

Sympy [A] time = 9.07681, size = 117, normalized size = 1.44

$$A\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**
2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b*
*2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)
) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.222166, size = 95, normalized size = 1.17

$$\frac{\left(\left(\frac{Bx}{b} + \frac{2A}{b}\right)x + \frac{3Ba}{b^2}\right)x + \frac{4Aa}{b^2}}{2\sqrt{bx^2 + a}} + \frac{3Ba\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 +
a) + 3/2*B*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.30 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2
+ (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.13668, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2
+ (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi in Sympy [A] time = 14.7082, size = 65, normalized size = 0.98

$$\frac{A\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}}+\frac{2B\sqrt{a+bx^2}}{b^2}-\frac{x\left(2A+2Bx\right)}{2b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(3/2) + 2*B*sqrt(a + b*x**
2)/b**2 - x*(2*A + 2*B*x)/(2*b*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.105024, size = 60, normalized size = 0.91

$$\frac{A\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{3/2}}+\frac{2aB+bx(Bx-A)}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B + b*x*(-A + B*x))/(b^2*Sqrt[a + b*x^2]) + (A*Log[b*x + Sqr
t[b]*Sqrt[a + b*x^2]])/b^(3/2)
```

Maple [A] time = 0.009, size = 72, normalized size = 1.1

$$-\frac{Ax}{b}\frac{1}{\sqrt{bx^2+a}}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Bx^2}{b}\frac{1}{\sqrt{bx^2+a}}+2\frac{Ba}{b^2\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -A*x/b/(b*x^2+a)^(1/2)+A/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+B*
x^2/b/(b*x^2+a)^(1/2)+2*B*a/b^2/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26142, size = 1, normalized size = 0.02

$$\left[\frac{2\left(Bbx^2 - Abx + 2\,Ba\right)\sqrt{bx^2+a}\sqrt{b} + \left(Ab^2x^2 + Aab\right)\log\left(-2\,\sqrt{bx^2+a}bx - \left(2\,bx^2+a\right)\sqrt{b}\right)}{2\left(b^3x^2+ab^2\right)\sqrt{b}}, \frac{\left(Bbx^2 - Abx + 2\,Ba\right)\sqrt{bx^2+\right.}{(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a)*sqrt(b) + (A*b^
2*x^2 + A*a*b)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)
))/((b^3*x^2 + a*b^2)*sqrt(b)), ((B*b*x^2 - A*b*x + 2*B*a)*sqrt(b
*x^2 + a)*sqrt(-b) + (A*b^2*x^2 + A*a*b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)))/((b^3*x^2 + a*b^2)*sqrt(-b))]
```

Sympy [A] time = 6.06385, size = 83, normalized size = 1.26

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x*
*2/a))) + B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt
(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

GIAC/XCAS [A] time = 0.220276, size = 78, normalized size = 1.18

$$\frac{\left(\frac{Bx}{b} - \frac{A}{b}\right)x + \frac{2\,Ba}{b^2}}{\sqrt{bx^2+a}} - \frac{A\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="giac")

[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*ln(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.31 $\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.0743529, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(3/2)
```

Rubi in Sympy [A] time = 7.00043, size = 41, normalized size = 0.85

$$\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(3/2) - (A + B*x)/(b*sqrt(
a + b*x**2))
```

Mathematica [A] time = 0.0601885, size = 53, normalized size = 1.1

$$\frac{-A-Bx}{b\sqrt{a+bx^2}}+\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-A - B*x)/(b*Sqrt[a + b*x^2]) + (B*Log[b*x + Sqrt[b]*Sqrt[a + b*
x^2]])/b^(3/2)
```

Maple [A] time = 0.006, size = 54, normalized size = 1.1

$$-\frac{A}{b}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx}{b}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -A/b/(b*x^2+a)^(1/2)-B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260172, size = 1, normalized size = 0.02

$$\left[-\frac{2\sqrt{bx^2+a}(Bx+A)\sqrt{b}-\left(Bbx^2+Ba\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{2\left(b^2x^2+ab\right)\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{bx^2+a}(Bx+A)\sqrt{-b}-\left(Bbx^2+Ba\right)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{\left(b^2x^2+ab\right)\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*(B*x + A)*sqrt(b) - (B*b*x^2 + B*a)*log(
-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^2*x^2 + a*b)
*sqrt(b)), -(sqrt(b*x^2 + a)*(B*x + A)*sqrt(-b) - (B*b*x^2 + B*a)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^2*x^2 + a*b)*sqrt(-b))]
```

Sympy [A] time = 5.96411, size = 66, normalized size = 1.38

$$A\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2
)), True)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*
sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.219365, size = 65, normalized size = 1.35

$$-\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}}-\frac{B\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*ln(abs(-sqrt(b)*x + sqrt(b*x^2
 + a)))/b^(3/2)
```

3.32 $\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rubi [A] time = 0.0326111, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(a + b*x^2)^(3/2),x]

[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rubi in Sympy [A] time = 4.26139, size = 22, normalized size = 0.79

$$-\frac{-Abx+Ba}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(b*x**2+a)**(3/2),x)

[Out] -(-A*b*x + B*a)/(a*b*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0283806, size = 27, normalized size = 0.96

$$\frac{Abx-aB}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]

[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Maple [A] time = 0.005, size = 26, normalized size = 0.9

$$\frac{Axb-Ba}{ab}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(b*x^2+a)^(3/2),x)

[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Maxima [A] time = 1.34291, size = 42, normalized size = 1.5

$$\frac{Ax}{\sqrt{bx^2+a}a}-\frac{B}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Fricas [A] time = 0.265056, size = 47, normalized size = 1.68

$$\frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] time = 4.23324, size = 46, normalized size = 1.64

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b
*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

GIAC/XCAS [A] time = 0.215193, size = 31, normalized size = 1.11

$$\frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

3.33 $\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/a^(3/2)
```

Rubi [A] time = 0.1346, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/a^(3/2)
```

Rubi in Sympy [A] time = 13.4371, size = 39, normalized size = 0.83

$$-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(3/2) + (A + B*x)/(a*sqrt(a
 + b*x**2))
```

Mathematica [A] time = 0.101972, size = 58, normalized size = 1.23

$$-\frac{A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{3/2}}+\frac{A\log(x)}{a^{3/2}}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) + (A*Log[x])/a^(3/2) - (A*Log[a + S
qrt[a]*Sqrt[a + b*x^2]])/a^(3/2)
```

Maple [A] time = 0.01, size = 60, normalized size = 1.3

$$\frac{Bx}{a}\frac{1}{\sqrt{bx^2+a}}+\frac{A}{a}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x/a/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259789, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}(Bx+A)\sqrt{a}+\left(Abx^2+Aa\right)\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)}{2\left(abx^2+a^2\right)\sqrt{a}}, \frac{\sqrt{bx^2+a}(Bx+A)\sqrt{-a}-\left(Abx^2+Aa\right)\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right.}{\left(abx^2+a^2\right)\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b*x^2 + a)*(B*x + A)*sqrt(a) + (A*b*x^2 + A*a)*log(-
((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a*b*x^2 + a
^2)*sqrt(a)), (sqrt(b*x^2 + a)*(B*x + A)*sqrt(-a) - (A*b*x^2 + A*
a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a*b*x^2 + a^2)*sqrt(-a))]
```

Sympy [A] time = 6.88998, size = 206, normalized size = 4.38

$$A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a
**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(s
qrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*
x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x*
*2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
+ B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.216938, size = 80, normalized size = 1.7

$$\frac{\frac{Bx}{a} + \frac{A}{a}}{\sqrt{bx^2 + a}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x
^2 + a))/sqrt(-a))/(sqrt(-a)*a)
```

3.34 $\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) -
 (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.212819, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) -
 (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rubi in Sympy [A] time = 20.0147, size = 60, normalized size = 0.86

$$-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -2*A*sqrt(a + b*x**2)/(a**2*x) - B*atanh(sqrt(a + b*x**2)/sqrt(a)
)/a**(3/2) + (A + B*x)/(a*x*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.161076, size = 73, normalized size = 1.04

$$\frac{\frac{-aA+aBx-2Abx^2}{x\sqrt{a+bx^2}}-\sqrt{a}B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a}B\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((-(a*A) + a*B*x - 2*A*b*x^2)/(x*Sqrt[a + b*x^2]) + Sqrt[a]*B*Log
[x] - Sqrt[a]*B*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/a^2
```

Maple [A] time = 0.013, size = 80, normalized size = 1.1

$$-\frac{A}{ax}\frac{1}{\sqrt{bx^2+a}}-2\frac{Axb}{a^2\sqrt{bx^2+a}}+\frac{B}{a}\frac{1}{\sqrt{bx^2+a}}-B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(1/2)-2*A*b/a^2*x/(b*x^2+a)^(1/2)+B/a/(b*x^2+a)^
(1/2)-B/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26437, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}\,a\sqrt{a}-\left(Babx^3+Ba^2x\right)\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\sqrt{bx^2+a}\,a}{x^2}\right)}{2\,(a^2bx^3+a^3x)\sqrt{a}},\ -\frac{\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}\,a\sqrt{-a}+\left(Babx^3+Ba^2x\right)\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{(a^2bx^3+a^3x)\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a)*sqrt(a) - (B*a
*b*x^3 + B*a^2*x)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)
*a)/x^2))/((a^2*b*x^3 + a^3*x)*sqrt(a)), -((2*A*b*x^2 - B*a*x + A
*a)*sqrt(b*x^2 + a)*sqrt(-a) + (B*a*b*x^3 + B*a^2*x)*arctan(sqrt(
-a)/sqrt(b*x^2 + a)))/((a^2*b*x^3 + a^3*x)*sqrt(-a))]
```

Sympy [A] time = 9.37371, size = 235, normalized size = 3.36

$$A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqr
t(a/(b*x**2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) +
2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*
```

```
b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(
7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)
*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
+ 2*a**(7/2)*b*x**2))
```

GIAC/XCAS [A] time = 0.222788, size = 130, normalized size = 1.86

$$-\frac{\frac{Abx}{a^2} - \frac{B}{a}}{\sqrt{bx^2 + a}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqr
t(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)*a)
```

3.35 $\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*
x^2) - (2*B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.303504, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*
x^2) - (2*B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 31.5056, size = 88, normalized size = 0.93

$$-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{3Ab\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -3*A*sqrt(a + b*x**2)/(2*a**2*x**2) + 3*A*b*atanh(sqrt(a + b*x**2
)/sqrt(a))/(2*a**(5/2)) - 2*B*sqrt(a + b*x**2)/(a**2*x) + (A + B*
x)/(a*x**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.266282, size = 83, normalized size = 0.87

$$\frac{-\frac{\sqrt{a}\left(a(A+2Bx)+bx^2(3A+4Bx)\right)}{x^2\sqrt{a+bx^2}}+3Ab\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)-3Ab\log(x)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[a]*(a*(A + 2*B*x) + b*x^2*(3*A + 4*B*x)))/(x^2*Sqrt[a +
b*x^2])) - 3*A*b*Log[x] + 3*A*b*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])
/(2*a^(5/2))
```

Maple [A] time = 0.011, size = 101, normalized size = 1.1

$$-\frac{A}{2\,a x^{2}}\frac{1}{\sqrt{b x^{2}+a}}-\frac{3\,A b}{2\,a^{2}}\frac{1}{\sqrt{b x^{2}+a}}+\frac{3\,A b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{5}{2}}-\frac{B}{a x}\frac{1}{\sqrt{b x^{2}+a}}-2\frac{b B x}{a^{2}\sqrt{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A*b/a^2/(b*x^2+a)^(1/2)+3/2*A*b/
a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-B/a/x/(b*x^2+a)^(1/
2)-2*B*b/a^2*x/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268861, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(4\,B b x^{3}+3\,A b x^{2}+2\,B a x+A a\right)\sqrt{b x^{2}+a}\sqrt{a}-3\left(A b^{2} x^{4}+A a b x^{2}\right)\log\left(-\frac{\left(b x^{2}+2\,a\right)\sqrt{a}+2\,\sqrt{b x^{2}+a}a}{x^{2}}\right)}{4\left(a^{2} b x^{4}+a^{3} x^{2}\right)\sqrt{a}},\right.$$
$$\left.-\frac{\left(4\,B b x^{3}+3\,A b x^{2}+2\,B a x+A a\right)\sqrt{b x^{2}+a}\sqrt{-a}-3\left(A b^{2} x^{4}+A a b x^{2}\right)\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2}+a}}\right)}{2\left(a^{2} b x^{4}+a^{3} x^{2}\right)\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(4*B*b*x^3 + 3*A*b*x^2 + 2*B*a*x + A*a)*sqrt(b*x^2 + a)*
sqrt(a) - 3*(A*b^2*x^4 + A*a*b*x^2)*log(-((b*x^2 + 2*a)*sqrt(a) +
 2*sqrt(b*x^2 + a)*a)/x^2))/((a^2*b*x^4 + a^3*x^2)*sqrt(a)), -1/2
*((4*B*b*x^3 + 3*A*b*x^2 + 2*B*a*x + A*a)*sqrt(b*x^2 + a)*sqrt(-a
) - 3*(A*b^2*x^4 + A*a*b*x^2)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(
(a^2*b*x^4 + a^3*x^2)*sqrt(-a))]
```

Sympy [A] time = 13.0872, size = 124, normalized size = 1.31

$$A\left(-\frac{1}{2a\sqrt{b}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(-\frac{1}{a\sqrt{b}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2
*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(
```

```
5/2))) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/
(a**2*sqrt(a/(b*x**2) + 1)))
```

GIAC/XCAS [A] time = 0.220413, size = 231, normalized size = 2.43

$$-\frac{\frac{Bbx}{a^2}+\frac{Ab}{a^2}}{\sqrt{bx^2+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x
 - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt
(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b
) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqr
t(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

3.36 $\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*S
qrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2
)
```

Rubi [A] time = 0.162611, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*S
qrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2
)
```

Rubi in Sympy [A] time = 16.1205, size = 73, normalized size = 0.92

$$\frac{B\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{5}{2}}}-\frac{x^2\left(2A+2Bx\right)}{6b\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{4A+6Bx}{6b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(5/2) - x**2*(2*A + 2*B*x)
/(6*b*(a + b*x**2)**(3/2)) - (4*A + 6*B*x)/(6*b**2*sqrt(a + b*x**
2))
```

Mathematica [A] time = 0.152609, size = 73, normalized size = 0.92

$$\frac{-\left(a+bx^2\right)\left(3A+4Bx\right)+aA+aBx}{3b^2\left(a+bx^2\right)^{3/2}}+\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*A + a*B*x - (3*A + 4*B*x)*(a + b*x^2))/(3*b^2*(a + b*x^2)^(3/2
)) + (B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [A] time = 0.012, size = 91, normalized size = 1.2

$$-\frac{Ax^{2}}{b}\left(bx^{2}+a\right)^{-\frac{3}{2}}-\frac{2\,Aa}{3\,b^{2}}\left(bx^{2}+a\right)^{-\frac{3}{2}}-\frac{x^{3}B}{3\,b}\left(bx^{2}+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^{2}}\frac{1}{\sqrt{bx^{2}+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -A*x^2/b/(b*x^2+a)^(3/2)-2/3*A*a/b^2/(b*x^2+a)^(3/2)-1/3*B*x^3/b/
(b*x^2+a)^(3/2)-B*x/b^2/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(x*b^(1/2)+(b
*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270001, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(4\,Bbx^{3}+3\,Abx^{2}+3\,Bax+2\,Aa\right)\sqrt{bx^{2}+a}\sqrt{b}-3\left(Bb^{2}x^{4}+2\,Babx^{2}+Ba^{2}\right)\log\left(-2\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)}{6\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)\sqrt{b}},\right.$$
$$\left.-\frac{\left(4\,Bbx^{3}+3\,Abx^{2}+3\,Bax+2\,Aa\right)\sqrt{bx^{2}+a}\sqrt{-b}-3\left(Bb^{2}x^{4}+2\,Babx^{2}+Ba^{2}\right)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)}{3\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(4*B*b*x^3 + 3*A*b*x^2 + 3*B*a*x + 2*A*a)*sqrt(b*x^2 + a
)*sqrt(b) - 3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*log(-2*sqrt(b*x^2
 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^4*x^4 + 2*a*b^3*x^2 + a^2
*b^2)*sqrt(b)), -1/3*((4*B*b*x^3 + 3*A*b*x^2 + 3*B*a*x + 2*A*a)*s
qrt(b*x^2 + a)*sqrt(-b) - 3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^4*x^4 + 2*a*b^3*x^2 + a^2*b^
2)*sqrt(-b))]
```

Sympy [A] time = 22.7279, size = 400, normalized size = 5.06

$$A\left(\begin{cases}-\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$
$$+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right.$$
$$\left.-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^{3}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a
 + b*x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*s
qrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a*
*(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(
39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*s
qrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*as
inh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a)
+ 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23
/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**
(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(3
9/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sq
rt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.228951, size = 95, normalized size = 1.2

$$-\frac{\left(\left(\frac{4\,Bx}{b}+\frac{3\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{2\,Aa}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{B\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a
)^(3/2) - B*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.37 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqr
t[a + b*x^2])
```

Rubi [A] time = 0.0951767, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqr
t[a + b*x^2])
```

Rubi in Sympy [A] time = 6.83382, size = 46, normalized size = 0.87

$$-\frac{2B}{3b^2\sqrt{a+bx^2}}-\frac{x^2\left(-Abx+Ba\right)}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] -2*B/(3*b**2*sqrt(a + b*x**2)) - x**2*(-A*b*x + B*a)/(3*a*b*(a +
b*x**2)**(3/2))
```

Mathematica [A] time = 0.0517857, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 41, normalized size = 0.8

$$\frac{Ax^3b^2-3\,Bx^2ab-2\,a^2B}{3\,ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

Maxima [A] time = 1.35864, size = 95, normalized size = 1.79

$$-\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b}-\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Ax}{3\,\sqrt{bx^2+a}ab}-\frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/
3*A*x/(sqrt(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [A] time = 0.255794, size = 85, normalized size = 1.6

$$\frac{\left(Ab^2x^3-3\,Babx^2-2\,Ba^2\right)\sqrt{bx^2+a}}{3\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^
4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [A] time = 19.1196, size = 141, normalized size = 2.66

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1
+ b*x**2/a)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b
**3*x**2*sqrt(a + b*x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2)
+ 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(5/2)), T
rue))
```

GIAC/XCAS [A] time = 0.218953, size = 49, normalized size = 0.92

$$\frac{\left(\frac{Ax}{a}-\frac{3\,B}{b}\right)x^2-\frac{2\,Ba}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

3.38 $\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2]
)
```

Rubi [A] time = 0.0721242, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2]
)
```

Rubi in Sympy [A] time = 5.8059, size = 36, normalized size = 0.77

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*x/(3*a*b*sqrt(a + b*x**2)) - (A + B*x)/(3*b*(a + b*x**2)**(3/2)
)
```

Mathematica [A] time = 0.0396952, size = 32, normalized size = 0.68

$$\frac{bBx^3-aA}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 29, normalized size = 0.6

$$-\frac{-bBx^3+Aa}{3\,ab}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

Maxima [A] time = 1.33132, size = 69, normalized size = 1.47

$$-\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^2+a}ab}-\frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) -
1/3*A/((b*x^2 + a)^(3/2)*b)
```

Fricas [A] time = 0.266372, size = 66, normalized size = 1.4

$$\frac{\left(Bbx^3-Aa\right)\sqrt{bx^2+a}}{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 +
a^3*b)
```

Sympy [A] time = 18.7961, size = 95, normalized size = 2.02

$$A\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*
x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2
)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.217458, size = 35, normalized size = 0.74

$$\frac{\frac{Bx^3}{a}-\frac{A}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*x/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

3.39 $\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a
+ b*x^2])
```

Rubi [A] time = 0.0433596, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a
+ b*x^2])
```

Rubi in Sympy [A] time = 5.17336, size = 42, normalized size = 0.82

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{-Abx+Ba}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] 2*A*x/(3*a**2*sqrt(a + b*x**2)) - (-A*b*x + B*a)/(3*a*b*(a + b*x*
*2)**(3/2))
```

Mathematica [A] time = 0.038876, size = 43, normalized size = 0.84

$$\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 40, normalized size = 0.8

$$\frac{2\,Ax^3b^2+3\,Axab-a^2B}{3\,a^2b}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Maxima [A] time = 1.33401, size = 65, normalized size = 1.27

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1
/3*B/((b*x^2 + a)^(3/2)*b)
```

Fricas [A] time = 0.24755, size = 84, normalized size = 1.65

$$\frac{\left(2\,Ab^2x^3+3\,Aabx-Ba^2\right)\sqrt{bx^2+a}}{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^
4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [A] time = 18.697, size = 146, normalized size = 2.86

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(
1 + b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(
5/2)*b*x**2*sqrt(1 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a
+ b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a*
*(5/2)), True))
```

GIAC/XCAS [A] time = 0.216269, size = 50, normalized size = 0.98

$$\frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

3.40 $\int \frac{A+Bx}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a +
 b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.218707, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a +
 b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi in Sympy [A] time = 26.6381, size = 65, normalized size = 0.86

$$-\frac{A\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(5/2) + (A + B*x)/(3*a*(a +
 b*x**2)**(3/2)) + (3*A + 2*B*x)/(3*a**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.296461, size = 79, normalized size = 1.04

$$\frac{\frac{\sqrt{a}\left(4aA+3aBx+3Abx^2+2bBx^3\right)}{\left(a+bx^2\right)^{3/2}}-3A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+3A\log(x)}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[a]*(4*a*A + 3*a*B*x + 3*A*b*x^2 + 2*b*B*x^3))/(a + b*x^2)^
(3/2) + 3*A*Log[x] - 3*A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(3*a^(
5/2))
```

Maple [A] time = 0.008, size = 92, normalized size = 1.2

$$\frac{Bx}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,Bx}{3\,a^2}\frac{1}{\sqrt{bx^2+a}}+\frac{A}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{A}{a^2}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B*x/a/(b*x^2+a)^(3/2)+2/3*B/a^2*x/(b*x^2+a)^(1/2)+1/3*A/a/(b*
x^2+a)^(3/2)+A/a^2/(b*x^2+a)^(1/2)-A/a^(5/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256535, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,Bbx^3+3\,Abx^2+3\,Bax+4\,Aa\right)\sqrt{bx^2+a}\sqrt{a}+3\left(Ab^2x^4+2\,Aabx^2+Aa^2\right)\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)}{6\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\sqrt{a}},\ \frac{\left(2\,Bbx^3+3\,A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*B*b*x^3 + 3*A*b*x^2 + 3*B*a*x + 4*A*a)*sqrt(b*x^2 + a)
*sqrt(a) + 3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*log(-((b*x^2 + 2*a
)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)*sqrt(a)), 1/3*((2*B*b*x^3 + 3*A*b*x^2 + 3*B*a*x + 4*A*a)
*sqrt(b*x^2 + a)*sqrt(-a) - 3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a^2*b^2*x^4 + 2*a^3*b*x^2 + a^
4)*sqrt(-a))]
```

Sympy [A] time = 29.0716, size = 840, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x
**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(
6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a*
*(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/
2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b
**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*
a**6*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a
**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x*
*2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*b**
2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
```

```
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*lo
g(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b
*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a**(7/2)*sqrt(1
+ b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x**3/(3
*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/
a)))
```

GIAC/XCAS [A] time = 0.221261, size = 111, normalized size = 1.46

$$\frac{\left(\left(\frac{2\,Bbx}{a^2}+\frac{3\,Ab}{a^2}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="giac")
```

```
[Out] 1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)
^(3/2) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqr
t(-a)*a^2)
```

3.41 $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt
[a + b*x^2]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.303975, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt
[a + b*x^2]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi in Sympy [A] time = 35.014, size = 88, normalized size = 0.85

$$-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -8*A*sqrt(a + b*x**2)/(3*a**3*x) - B*atanh(sqrt(a + b*x**2)/sqrt(
a))/a**(5/2) + (A + B*x)/(3*a*x*(a + b*x**2)**(3/2)) + (4*A + 3*B
*x)/(3*a**2*x*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.218526, size = 95, normalized size = 0.91

$$-\frac{B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{5/2}}+\frac{B\log(x)}{a^{5/2}}+\frac{a^2(4Bx-3A)+3abx^2(Bx-4A)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x))/(3*a
^3*x*(a + b*x^2)^(3/2)) + (B*Log[x])/a^(5/2) - (B*Log[a + Sqrt[a]
*Sqrt[a + b*x^2]])/a^(5/2)
```

Maple [A] time = 0.014, size = 112, normalized size = 1.1

$$-\frac{A}{ax}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{4\,Axb}{3\,a^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{8\,Axb}{3\,a^3}\frac{1}{\sqrt{bx^2+a}}+\frac{B}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{B}{a^2}\frac{1}{\sqrt{bx^2+a}}-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(3/2)-4/3*A*b/a^2*x/(b*x^2+a)^(3/2)-8/3*A*b/a^3*
x/(b*x^2+a)^(1/2)+1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B
/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281344, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(8\,Ab^2x^4-3\,Babx^3+12\,Aabx^2-4\,Ba^2x+3\,Aa^2\right)\sqrt{bx^2+a}\sqrt{a}-3\left(Bab^2x^5+2\,Ba^2bx^3+Ba^3x\right)\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\sqrt{bx^2+a}\,a}{x^2}\right)}{6\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{a}},\ -\frac{\left(8\,Ab^2x^4-3\,Babx^3+12\,Aabx^2-4\,Ba^2x+3\,Aa^2\right)\sqrt{bx^2+a}\sqrt{-a}+3\left(Bab^2x^5+2\,Ba^2bx^3+Ba^3x\right)\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x +
3*A*a^2)*sqrt(b*x^2 + a)*sqrt(a) - 3*(B*a*b^2*x^5 + 2*B*a^2*b*x^3
 + B*a^3*x)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^
2))/((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(a)), -1/3*((8*A*b^2
*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a^2)*sqrt(b*x
^2 + a)*sqrt(-a) + 3*(B*a*b^2*x^5 + 2*B*a^2*b*x^3 + B*a^3*x)*arct
an(sqrt(-a)/sqrt(b*x^2 + a)))/((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x
)*sqrt(-a))]
```

Sympy [A] time = 41.4101, size = 910, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2)
 + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**
(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2
+ 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2)
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3
*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
+ 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sq
rt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1
 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b*
*2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19
/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*
b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**
(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) -
 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*
a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b
**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*
b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

GIAC/XCAS [A] time = 0.219434, size = 161, normalized size = 1.55

$$-\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x
^2 + a)^(3/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a
))/(sqrt(-a)*a^2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)*a^2)
```

3.42 $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*
Sqrt[a + b*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[
a + b*x^2])/(3*a^3*x) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2*a^(7/2))
```

Rubi [A] time = 0.405455, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*
Sqrt[a + b*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[
a + b*x^2])/(3*a^3*x) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2*a^(7/2))
```

Rubi in Sympy [A] time = 48.7299, size = 119, normalized size = 0.92

$$-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}+\frac{5Ab\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -5*A*sqrt(a + b*x**2)/(2*a**3*x**2) + 5*A*b*atanh(sqrt(a + b*x**2
)/sqrt(a))/(2*a**(7/2)) - 8*B*sqrt(a + b*x**2)/(3*a**3*x) + (A +
B*x)/(3*a*x**2*(a + b*x**2)**(3/2)) + (5*A + 4*B*x)/(3*a**2*x**2*
sqrt(a + b*x**2))
```

Mathematica [A] time = 0.354827, size = 103, normalized size = 0.8

$$\frac{-\frac{\sqrt{a}\left(3a^2(A+2Bx)+4abx^2(5A+6Bx)+b^2x^4(15A+16Bx)\right)}{x^2\left(a+bx^2\right)^{3/2}}+15Ab\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)-15Ab\log(x)}{6a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-((Sqrt[a]*(3*a^2*(A + 2*B*x) + 4*a*b*x^2*(5*A + 6*B*x) + b^2*x^
4*(15*A + 16*B*x)))/(x^2*(a + b*x^2)^(3/2))) - 15*A*b*Log[x] + 15
*A*b*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(6*a^(7/2))
```

Maple [A] time = 0.012, size = 134, normalized size = 1.

$$-\frac{A}{2\,a x^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{5\,A b}{6\,a^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{5\,A b}{2\,a^{3}}\frac{1}{\sqrt{b x^{2}+a}}+\frac{5\,A b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{a x}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{4\,b B x}{3\,a^{2}}\left(b x^{2}+a\right)^{-\frac{3}{2}}-\frac{8\,b B x}{3\,a^{3}}\frac{1}{\sqrt{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A*b/a^2/(b*x^2+a)^(3/2)-5/2*A*b/
a^3/(b*x^2+a)^(1/2)+5/2*A*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x)-B/a/x/(b*x^2+a)^(3/2)-4/3*B*b/a^2*x/(b*x^2+a)^(3/2)-8/3*
B*b/a^3*x/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266771, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(16\,B b^{2} x^{5}+15\,A b^{2} x^{4}+24\,B a b x^{3}+20\,A a b x^{2}+6\,B a^{2} x+3\,A a^{2}\right)\sqrt{b x^{2}+a}\sqrt{a}-15\left(A b^{3} x^{6}+2\,A a b^{2} x^{4}+A a^{2} b x^{2}\right)\log\left(-\right.}{12\left(a^{3} b^{2} x^{6}+2\,a^{4} b x^{4}+a^{5} x^{2}\right)\sqrt{a}}\right.$$

$$-\frac{\left(16\,B b^{2} x^{5}+15\,A b^{2} x^{4}+24\,B a b x^{3}+20\,A a b x^{2}+6\,B a^{2} x+3\,A a^{2}\right)\sqrt{b x^{2}+a}\sqrt{-a}-15\left(A b^{3} x^{6}+2\,A a b^{2} x^{4}+A a^{2} b x^{2}\right)\arctan\left(\right.}{6\left(a^{3} b^{2} x^{6}+2\,a^{4} b x^{4}+a^{5} x^{2}\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(16*B*b^2*x^5 + 15*A*b^2*x^4 + 24*B*a*b*x^3 + 20*A*a*b*
x^2 + 6*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a)*sqrt(a) - 15*(A*b^3*x^
6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*log(-((b*x^2 + 2*a)*sqrt(a) + 2*
sqrt(b*x^2 + a)*a)/x^2))/((a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*s
qrt(a)), -1/6*((16*B*b^2*x^5 + 15*A*b^2*x^4 + 24*B*a*b*x^3 + 20*A
*a*b*x^2 + 6*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a)*sqrt(-a) - 15*(A*
b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*arctan(sqrt(-a)/sqrt(b*x^2
 + a)))/((a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*sqrt(-a))]
```

Sympy [A] time = 55.7557, size = 1034, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a*
*16*b*x**2*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b
*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**
16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
+ 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x
**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)
*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a
**15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a*
*15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36
*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*
x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**
8) + 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a
**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12
*a**(33/2)*b**3*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)
/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(1
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

GIAC/XCAS [A] time = 0.221876, size = 266, normalized size = 2.06

$$-\frac{\left(\left(\frac{5\,Bb^{2}x}{a^{3}}+\frac{6\,Ab^{2}}{a^{3}}\right)x+\frac{6\,Bb}{a^{2}}\right)x+\frac{7\,Ab}{a^{2}}}{3\,(bx^{2}+a)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}}+\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{3}Ab+2\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)Aab-2\,Ba^{2}\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2
)/(b*x^2 + a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))
/sqrt(-a))/(sqrt(-a)*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
+ 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sq
rt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2
 + a))^2 - a)^2*a^3)
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi [A] time = 0.045564, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]

[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi in Sympy [A] time = 5.94348, size = 29, normalized size = 1.07

$$-\frac{(-x+1)\sqrt{-x^2+1}}{2}-\frac{\sqrt{-x^2+1}}{2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-x)*x/(-x**2+1)**(1/2),x)

[Out] -(-x + 1)*sqrt(-x**2 + 1)/2 - sqrt(-x**2 + 1)/2 - asin(x)/2
```

Mathematica [A] time = 0.0344414, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]

[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

Maple [A] time = 0.01, size = 29, normalized size = 1.1

$$\frac{x}{2}\sqrt{-x^2+1}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x)

[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.5042, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A] time = 0.251631, size = 123, normalized size = 4.56

$$-\frac{2\,x^3-2\,x^2-2\left(x^2+2\,\sqrt{-x^2+1}-2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(x^3-2\,x^2-2\,x\right)\sqrt{-x^2+1}-2\,x}{2\left(x^2+2\,\sqrt{-x^2+1}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 - 2*x^2 - 2*(x^2 + 2*sqrt(-x^2 + 1) - 2)*arctan((sqrt
(-x^2 + 1) - 1)/x) - (x^3 - 2*x^2 - 2*x)*sqrt(-x^2 + 1) - 2*x)/(x
^2 + 2*sqrt(-x^2 + 1) - 2)
```

Sympy [A] time = 0.274267, size = 24, normalized size = 0.89

$$\frac{x\sqrt{-x^2+1}}{2}-\sqrt{-x^2+1}-\frac{\operatorname{asin}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)*x/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)/2 - sqrt(-x**2 + 1) - asin(x)/2
```

GIAC/XCAS [A] time = 0.212749, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)-\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)*x/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi [A] time = 0.049725, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - x^2)/Sqrt[1 - x^2],x]

[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi in Sympy [A] time = 7.11965, size = 29, normalized size = 1.07

$$-\frac{(-x+1)\sqrt{-x^2+1}}{2}-\frac{\sqrt{-x^2+1}}{2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+x)/(-x**2+1)**(1/2),x)

[Out] -(-x + 1)*sqrt(-x**2 + 1)/2 - sqrt(-x**2 + 1)/2 - asin(x)/2
```

Mathematica [A] time = 0.0126403, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - x^2)/Sqrt[1 - x^2],x]

[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

Maple [A] time = 0.007, size = 29, normalized size = 1.1

$$\frac{x}{2}\sqrt{-x^2+1}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+x)/(-x^2+1)^(1/2),x)

[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49413, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - x)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A] time = 0.231425, size = 123, normalized size = 4.56

$$-\frac{2x^3 - 2x^2 - 2\left(x^2 + 2\sqrt{-x^2+1} - 2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \left(x^3 - 2x^2 - 2x\right)\sqrt{-x^2+1} - 2x}{2\left(x^2 + 2\sqrt{-x^2+1} - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - x)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 - 2*x^2 - 2*(x^2 + 2*sqrt(-x^2 + 1) - 2)*arctan((sqrt
(-x^2 + 1) - 1)/x) - (x^3 - 2*x^2 - 2*x)*sqrt(-x^2 + 1) - 2*x)/(x
^2 + 2*sqrt(-x^2 + 1) - 2)
```

Sympy [A] time = 0.268054, size = 24, normalized size = 0.89

$$\frac{x\sqrt{-x^2+1}}{2} - \sqrt{-x^2+1} - \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+x)/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)/2 - sqrt(-x**2 + 1) - asin(x)/2
```

GIAC/XCAS [A] time = 0.219212, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - x)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$

Optimal. Leaf size=17

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rubi [A] time = 0.0222769, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x^2)/(-3 + x^2),x]

[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rubi in Sympy [A] time = 4.40597, size = 17, normalized size = 1.

$$x-2\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3)/(x**2-3),x)

[Out] x - 2*sqrt(3)*atanh(sqrt(3)*x/3)
```

Mathematica [A] time = 0.0165828, size = 33, normalized size = 1.94

$$x+\sqrt{3}\log\left(\sqrt{3}-x\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x^2)/(-3 + x^2),x]

[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$x-2\,\mathit{Artanh}\left(1/3\,x\sqrt{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3)/(x^2-3),x)

[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.48273, size = 30, normalized size = 1.76

$$\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 - 3),x, algorithm="maxima")

[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

Fricas [A] time = 0.226644, size = 35, normalized size = 2.06

$$\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 - 3),x, algorithm="fricas")

[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

Sympy [A] time = 0.076731, size = 27, normalized size = 1.59

$$x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3)/(x**2-3),x)

[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

GIAC/XCAS [A] time = 0.213023, size = 41, normalized size = 2.41

$$\sqrt{3}\ln\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 - 3),x, algorithm="giac")

[Out] sqrt(3)*ln(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$

Optimal. Leaf size=6

$$x - 2\tan^{-1}(x)$$

```
[Out] x - 2*ArcTan[x]
```

Rubi [A] time = 0.0150226, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Rubi in Sympy [A] time = 4.39052, size = 5, normalized size = 0.83

$$x - 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-1)/(x**2+1),x)
```

```
[Out] x - 2*atan(x)
```

Mathematica [A] time = 0.00666589, size = 6, normalized size = 1.

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Maple [A] time = 0.003, size = 7, normalized size = 1.2

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1),x)
```

```
[Out] x-2*arctan(x)
```

Maxima [A] time = 1.50244, size = 8, normalized size = 1.33

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(x)
```

Fricas [A] time = 0.228536, size = 8, normalized size = 1.33

$$x - 2 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(x)
```

Sympy [A] time = 0.07292, size = 5, normalized size = 0.83

$$x - 2 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/(x**2+1),x)
```

```
[Out] x - 2*atan(x)
```

GIAC/XCAS [A] time = 0.209032, size = 8, normalized size = 1.33

$$x - 2 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] x - 2*arctan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{16\sqrt{a+bx^2}(Ab-8aC)}{35ab^5}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3(a+bx^2)^{3/2}}-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2(a+bx^2)^{5/2}}-\frac{x^7(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*
a*B - (A*b - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a
*B - 6*(A*b - 8*a*C)*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a
*B - 8*(A*b - 8*a*C)*x))/(35*a*b^4*Sqrt[a + b*x^2]) - (16*(A*b -
8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.818364, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{16\sqrt{a+bx^2}(Ab-8aC)}{35ab^5}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3(a+bx^2)^{3/2}}-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2(a+bx^2)^{5/2}}-\frac{x^7(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*
a*B - (A*b - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a
*B - 6*(A*b - 8*a*C)*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a
*B - 8*(A*b - 8*a*C)*x))/(35*a*b^4*Sqrt[a + b*x^2]) - (16*(A*b -
8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/b^(9/2)
```

Rubi in Sympy [A] time = 59.1647, size = 192, normalized size = 0.9

$$\frac{B\,\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{9}{2}}}-\frac{x^7\left(Ba-x\left(Ab-Ca\right)\right)}{7ab\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{x^5\left(14Ba-x\left(2Ab-16Ca\right)\right)}{70ab^2\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{x^3\left(140Ba-x\left(24Ab-192Ca\right)\right)}{420ab^3\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x\left(840Ba-x\left(192Ab-1536Ca\right)\right)}{840ab^4\sqrt{a+bx^2}}-\frac{16\sqrt{a+bx^2}\left(Ab-8Ca\right)}{35ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] B*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(9/2) - x**7*(B*a - x*(A*b
 - C*a))/(7*a*b*(a + b*x**2)**(7/2)) - x**5*(14*B*a - x*(2*A*b -
16*C*a))/(70*a*b**2*(a + b*x**2)**(5/2)) - x**3*(140*B*a - x*(24*
A*b - 192*C*a))/(420*a*b**3*(a + b*x**2)**(3/2)) - x*(840*B*a - x
*(192*A*b - 1536*C*a))/(840*a*b**4*sqrt(a + b*x**2)) - 16*sqrt(a
+ b*x**2)*(A*b - 8*C*a)/(35*a*b**5)
```

Mathematica [A] time = 0.299105, size = 157, normalized size = 0.74

$$\frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(5x(24Cx-5B)-12A)+14ab^3x^4(x(60Cx-29B)-15A)+105\sqrt{b}B\left(a+\right.}{105b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2
*(-12*A + 5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B +
 60*C*x)) + b^4*x^6*(-105*A + x*(-176*B + 105*C*x)) + 105*Sqrt[b]
*B*(a + b*x^2)^(7/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(105*b^5
*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.049, size = 265, normalized size = 1.2

$$-\frac{Ax^6}{b}\left(bx^2+a\right)^{-\frac{7}{2}}-2\frac{aAx^4}{b^2\left(bx^2+a\right)^{7/2}}-\frac{8\,Aa^2x^2}{5\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{16\,Aa^3}{35\,b^4}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$-\frac{Bx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Bx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Bx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^4}\frac{1}{\sqrt{bx^2+a}}$$
$$+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{Cx^8}{b}\left(bx^2+a\right)^{-\frac{7}{2}}+8\frac{aCx^6}{b^2\left(bx^2+a\right)^{7/2}}$$
$$+16\frac{a^2Cx^4}{b^3\left(bx^2+a\right)^{7/2}}+\frac{64\,Ca^3x^2}{5\,b^4}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{128\,Ca^4}{35\,b^5}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -A*x^6/b/(b*x^2+a)^(7/2)-2*A*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*A*a^2/
b^3*x^2/(b*x^2+a)^(7/2)-16/35*A*a^3/b^4/(b*x^2+a)^(7/2)-1/7*B*x^7
/b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^3/(b
*x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))+C*x^8/b/(b*x^2+a)^(7/2)+8*C*a/b^2*x^6/(b*x^2+a)^(7/2
)+16*C*a^2/b^3*x^4/(b*x^2+a)^(7/2)+64/5*C*a^3/b^4*x^2/(b*x^2+a)^(
7/2)+128/35*C*a^4/b^5/(b*x^2+a)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^7/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292743, size = 1, normalized size = 0.

$$\left[\frac{2\left(105\,Cb^4x^8-176\,Bb^4x^7-406\,Bab^3x^5-350\,Ba^2b^2x^3+105\left(8\,Cab^3-Ab^4\right)x^6-105\,Ba^3bx+384\,Ca^4-48\,Aa^3b+210\,(8\right.}{210\,(b^9x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^7/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(2*(105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*
B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384
*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*
a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)*sqrt(b) + 105*(B*b^5*x^8
+ 4*B*a*b^4*x^6 + 6*B*a^2*b^3*x^4 + 4*B*a^3*b^2*x^2 + B*a^4*b)*lo
g(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^9*x^8 + 4*
a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*sqrt(b)), 1/
105*((105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2
*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^
4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^3*b
 - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)*sqrt(-b) + 105*(B*b^5*x^8 + 4*
B*a*b^4*x^6 + 6*B*a^2*b^3*x^4 + 4*B*a^3*b^2*x^2 + B*a^4*b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7
*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227557, size = 275, normalized size = 1.29

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b} - \frac{176\,B}{b}\right)x + \frac{105\,\left(8\,Ca^4b^7 - Aa^3b^8\right)}{a^3b^9}\right)x - \frac{406\,Ba}{b^2}\right)x + \frac{210\,\left(8\,Ca^5b^6 - Aa^4b^7\right)}{a^3b^9}\right)x - \frac{350\,Ba^2}{b^3}\right)x + \frac{168\,\left(8\,Ca^6b^5 - Aa^5b^6\right)}{a^3b^9}\right)x - \frac{105}{b}}{105\,\left(bx^2 + a\right)^{\frac{7}{2}}} - \frac{B\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^7/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b
^8)/(a^3*b^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)
/(a^3*b^9))*x - 350*B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/
(a^3*b^9))*x - 105*B*a^3/b^4)*x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a
^3*b^9))/(b*x^2 + a)^(7/2) - B*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(9/2)
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}-\frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}}-\frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}}$$

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*
B + 7*C*x))/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(1
05*b^3*(a + b*x^2)^(3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^
2]) + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.430999, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}-\frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}}-\frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*
B + 7*C*x))/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(1
05*b^3*(a + b*x^2)^(3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^
2]) + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi in Sympy [A] time = 42.4344, size = 133, normalized size = 0.89

$$\frac{C\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{9}{2}}}-\frac{x^4(12B+14Cx)}{70b^2(a+bx^2)^{\frac{5}{2}}}-\frac{x^2(96B+140Cx)}{420b^3(a+bx^2)^{\frac{3}{2}}}-\frac{192B+420Cx}{420b^4\sqrt{a+bx^2}}-\frac{x^6(Ba-x(Ab-Ca))}{7ab(a+bx^2)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] C*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(9/2) - x**4*(12*B + 14*C*
x)/(70*b**2*(a + b*x**2)**(5/2)) - x**2*(96*B + 140*C*x)/(420*b**
3*(a + b*x**2)**(3/2)) - (192*B + 420*C*x)/(420*b**4*sqrt(a + b*x
**2)) - x**6*(B*a - x*(A*b - C*a))/(7*a*b*(a + b*x**2)**(7/2))
```

Mathematica [A] time = 0.310559, size = 127, normalized size = 0.85

$$\frac{-3a^4(16B+35Cx)-14a^3bx^2(12B+25Cx)-14a^2b^2x^4(15B+29Cx)-ab^3x^6(105B+176Cx)+15Ab^4x^7}{105ab^4(a+bx^2)^{7/2}}+\frac{C\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (15*A*b^4*x^7 - 14*a^3*b*x^2*(12*B + 25*C*x) - 14*a^2*b^2*x^4*(15
*B + 29*C*x) - 3*a^4*(16*B + 35*C*x) - a*b^3*x^6*(105*B + 176*C*x
```

```
))/(105*a*b^4*(a + b*x^2)^(7/2)) + (C*Log[b*x + Sqrt[b]*Sqrt[a +
b*x^2]])/b^(9/2)
```

Maple [B] time = 0.018, size = 277, normalized size = 1.9

$$-\frac{Ax^5}{2b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{5\,aAx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15\,a^2Ax}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,aAx}{56\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Ax}{14\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Ax}{7\,ab^3}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx^6}{b}\left(bx^2+a\right)^{-\frac{7}{2}}-2\,\frac{aBx^4}{b^2\left(bx^2+a\right)^{7/2}}-\frac{8\,Bx^2a^2}{5\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{16\,Ba^3}{35\,b^4}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Cx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Cx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Cx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Cx}{b^4}\frac{1}{\sqrt{bx^2+a}}+C\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2*A*x^5/b/(b*x^2+a)^(7/2)-5/8*A*a/b^2*x^3/(b*x^2+a)^(7/2)-15/5
6*A*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*A*a/b^3*x/(b*x^2+a)^(5/2)+1/14
*A/b^3*x/(b*x^2+a)^(3/2)+1/7*A/a/b^3*x/(b*x^2+a)^(1/2)-B*x^6/b/(b
*x^2+a)^(7/2)-2*B*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*B*a^2/b^3*x^2/(b*
x^2+a)^(7/2)-16/35*B*a^3/b^4/(b*x^2+a)^(7/2)-1/7*C*x^7/b/(b*x^2+a
)^(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^(5/2)-1/3*C/b^3*x^3/(b*x^2+a)^(3/
2)-C/b^4*x/(b*x^2+a)^(1/2)+C/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28308, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(105\,Bab^3x^6+406\,Ca^2b^2x^5+210\,Ba^2b^2x^4+350\,Ca^3bx^3+\left(176\,Cab^3-15\,Ab^4\right)x^7+168\,Ba^3bx^2+105\,Ca^4x+48\,Ba^4\right)\cdots}{210\left(ab^8x^8+4\,a^2b^7x^6+6\,a^3b^6x^4\cdots\right.}\right.$$

$$-\frac{\left(105\,Bab^3x^6+406\,Ca^2b^2x^5+210\,Ba^2b^2x^4+350\,Ca^3bx^3+\left(176\,Cab^3-15\,Ab^4\right)x^7+168\,Ba^3bx^2+105\,Ca^4x+48\,Ba^4\right)\sqrt{b\cdots}}{105\left(ab^8x^8+4\,a^2b^7x^6+6\,a^3b^6x^4+4\,a^4b^5x^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/210*(2*(105*B*a*b^3*x^6 + 406*C*a^2*b^2*x^5 + 210*B*a^2*b^2*x
^4 + 350*C*a^3*b*x^3 + (176*C*a*b^3 - 15*A*b^4)*x^7 + 168*B*a^3*b
*x^2 + 105*C*a^4*x + 48*B*a^4)*sqrt(b*x^2 + a)*sqrt(b) - 105*(C*a
*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^2 + C*
a^5)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)*s
qrt(b)), -1/105*((105*B*a*b^3*x^6 + 406*C*a^2*b^2*x^5 + 210*B*a^2
```

```
*b^2*x^4 + 350*C*a^3*b*x^3 + (176*C*a*b^3 - 15*A*b^4)*x^7 + 168*B
*a^3*b*x^2 + 105*C*a^4*x + 48*B*a^4)*sqrt(b*x^2 + a)*sqrt(-b) - 1
05*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x
^2 + C*a^5)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^8*x^8 + 4*a
^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.221754, size = 186, normalized size = 1.24

$$-\frac{\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{\left(176\,Ca^3b^7-15\,Aa^2b^8\right)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\,(bx^2+a)^{\frac{7}{2}}}$$
$$-\frac{C\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b
^8)) + 406*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B
*a^2/b^3)*x + 105*C*a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2)
- C*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4(a+bx^2)^{3/2}}-\frac{x^4(6aC+Ab-5bBx)}{35ab^2(a+bx^2)^{5/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*
b + 6*a*C - 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*
a*C))/(105*b^4*(a + b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*S
qrt[a + b*x^2])
```

Rubi [A] time = 0.383377, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4(a+bx^2)^{3/2}}-\frac{x^4(6aC+Ab-5bBx)}{35ab^2(a+bx^2)^{5/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*
b + 6*a*C - 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*
a*C))/(105*b^4*(a + b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*S
qrt[a + b*x^2])
```

Rubi in Sympy [A] time = 27.0208, size = 117, normalized size = 0.89

$$\frac{4\left(Ab+6Ca\right)}{105b^4\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x^5\left(Ba-x\left(Ab-Ca\right)\right)}{7ab\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{x^4\left(Ab-5Bbx+6Ca\right)}{35ab^2\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{4\left(Ab+6Ca\right)}{35ab^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] 4*(A*b + 6*C*a)/(105*b**4*(a + b*x**2)**(3/2)) - x**5*(B*a - x*(A
*b - C*a))/(7*a*b*(a + b*x**2)**(7/2)) - x**4*(A*b - 5*B*b*x + 6*
C*a)/(35*a*b**2*(a + b*x**2)**(5/2)) - 4*(A*b + 6*C*a)/(35*a*b**4
*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.129502, size = 89, normalized size = 0.67

$$\frac{-48a^4C-8a^3b\left(A+21Cx^2\right)-14a^2b^2x^2\left(2A+15Cx^2\right)-35ab^3x^4\left(A+3Cx^2\right)+15b^4Bx^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b
^2*x^2*(2*A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a +
 b*x^2)^(7/2))
```

Maple [A] time = 0.009, size = 95, normalized size = 0.7

$$-\frac{-15\,Bx^{7}b^{4}+105\,Cx^{6}ab^{3}+35\,Aab^{3}x^{4}+210\,Ca^{2}b^{2}x^{4}+28\,Aa^{2}b^{2}x^{2}+168\,Ca^{3}bx^{2}+8\,Aa^{3}b+48\,Ca^{4}}{105\,ab^{4}}\left(bx^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^
2*x^4+28*A*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2
+a)^(7/2)/a/b^4
```

Maxima [A] time = 1.36102, size = 324, normalized size = 2.45

$$-\frac{Cx^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Bx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{2\,Cax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax^{4}}{3\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{5\,Bax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{8\,Ca^{2}x^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{4\,Aax^{2}}{15\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{Bx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Bx}{7\,\sqrt{bx^{2}+a}ab^{3}}+\frac{3\,Bax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Ba^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{16\,Ca^{3}}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{4}}-\frac{8\,Aa^{2}}{105\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^5/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) -
2*C*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*
b) - 5/8*B*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2
+ a)^(7/2)*b^3) - 4/15*A*a*x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x
/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56
*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2)
*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((b*x^2
 + a)^(7/2)*b^3)
```

Fricas [A] time = 0.263091, size = 185, normalized size = 1.4

$$\frac{\left(15\,Bb^{4}x^{7}-105\,Cab^{3}x^{6}-48\,Ca^{4}-8\,Aa^{3}b-35\,\left(6\,Ca^{2}b^{2}+Aab^{3}\right)x^{4}-28\,\left(6\,Ca^{3}b+Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\,(ab^{8}x^{8}+4\,a^{2}b^{7}x^{6}+6\,a^{3}b^{6}x^{4}+4\,a^{4}b^{5}x^{2}+a^{5}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^5/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35
*(6*C*a^2*b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sq
rt(b*x^2 + a)/(a*b^8*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*
b^5*x^2 + a^5*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.231022, size = 151, normalized size = 1.14

$$\frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7C}{b}\right)x^2-\frac{7\left(6Ca^4b^2+Aa^3b^3\right)}{a^3b^4}\right)x^2-\frac{28\left(6Ca^5b+Aa^4b^2\right)}{a^3b^4}\right)x^2-\frac{8\left(6Ca^6+Aa^5b\right)}{a^3b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^5/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a
^3*b^4))*x^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C
*a^6 + A*a^5*b)/(a^3*b^4))/(b*x^2 + a)^(7/2)
```

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*
a*B + (2*A*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B +
 3*(2*A*b + 5*a*C)*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5
*a*C)*x)/(35*a^2*b^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.446106, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*
a*B + (2*A*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B +
 3*(2*A*b + 5*a*C)*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5
*a*C)*x)/(35*a^2*b^3*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 31.0389, size = 128, normalized size = 0.86

$$-\frac{8B}{105ab^3\sqrt{a+bx^2}}-\frac{x^4\left(Ba-x\left(Ab-Ca\right)\right)}{7ab\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{x^3\left(2Ab-4Bbx+5Ca\right)}{35ab^2\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{x^2\left(4Ba-x\left(6Ab+15Ca\right)\right)}{105a^2b^2\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] -8*B/(105*a*b**3*sqrt(a + b*x**2)) - x**4*(B*a - x*(A*b - C*a))/(
7*a*b*(a + b*x**2)**(7/2)) - x**3*(2*A*b - 4*B*b*x + 5*C*a)/(35*a
*b**2*(a + b*x**2)**(5/2)) - x**2*(4*B*a - x*(6*A*b + 15*C*a))/(1
05*a**2*b**2*(a + b*x**2)**(3/2))
```

Mathematica [A] time = 0.121039, size = 78, normalized size = 0.52

$$\frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+3ab^3x^5\left(7A+5Cx^2\right)+6Ab^4x^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 6*A*b^4*x^7 + 3*a
*b^3*x^5*(7*A + 5*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.009, size = 76, normalized size = 0.5

$$\frac{6\,A b^{4} x^{7}+15\,C a b^{3} x^{7}+21\,A x^{5} a b^{3}-35\,B x^{4} a^{2} b^{2}-28\,B a^{3} x^{2} b-8\,a^{4} B}{105\,a^{2} b^{3}}\left(b x^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4
-28*B*a^3*b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

Maxima [A] time = 1.37174, size = 342, normalized size = 2.3

$$-\frac{C x^{5}}{2\,(b x^{2}+a)^{\frac{7}{2}} b}-\frac{B x^{4}}{3\,(b x^{2}+a)^{\frac{7}{2}} b}-\frac{5\,C a x^{3}}{8\,(b x^{2}+a)^{\frac{7}{2}} b^{2}}-\frac{A x^{3}}{4\,(b x^{2}+a)^{\frac{7}{2}} b}-\frac{4\,B a x^{2}}{15\,(b x^{2}+a)^{\frac{7}{2}} b^{2}}+\frac{C x}{14\,(b x^{2}+a)^{\frac{3}{2}} b^{3}}+\frac{C x}{7\,\sqrt{b x^{2}+a}\,a b^{3}}+\frac{3\,C a x}{56\,(b x^{2}+a)^{\frac{5}{2}} b^{3}}-\frac{15\,C a^{2} x}{56\,(b x^{2}+a)^{\frac{7}{2}} b^{3}}+\frac{3\,A x}{140\,(b x^{2}+a)^{\frac{5}{2}} b^{2}}+\frac{2\,A x}{35\,\sqrt{b x^{2}+a}\,a^{2} b^{2}}+\frac{A x}{35\,(b x^{2}+a)^{\frac{3}{2}} a b^{2}}-\frac{3\,A a x}{28\,(b x^{2}+a)^{\frac{7}{2}} b^{2}}-\frac{8\,B a^{2}}{105\,(b x^{2}+a)^{\frac{7}{2}} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b
) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^
(7/2)*b) - 4/15*B*a*x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^
2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/
((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) +
 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^
2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2
+ a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [A] time = 0.278414, size = 165, normalized size = 1.11

$$\frac{\left(21\,A a b^{3} x^{5}-35\,B a^{2} b^{2} x^{4}+3\,\left(5\,C a b^{3}+2\,A b^{4}\right) x^{7}-28\,B a^{3} b x^{2}-8\,B a^{4}\right)\sqrt{b x^{2}+a}}{105\,\left(a^{2} b^{7} x^{8}+4\,a^{3} b^{6} x^{6}+6\,a^{4} b^{5} x^{4}+4\,a^{5} b^{4} x^{2}+a^{6} b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4
)*x^7 - 28*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 +
4*a^3*b^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.21702, size = 109, normalized size = 0.73

$$\frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{\left(5\,Ca^2b^3+2\,Aab^4\right)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) -
35*B/b)*x^2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3(a+bx^2)^{3/2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2(a+bx^2)^{5/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*
B + (3*A*b + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b
+ 4*a*C) - 3*b*B*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a
^2*b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.358691, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3(a+bx^2)^{3/2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2(a+bx^2)^{5/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*
B + (3*A*b + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b
+ 4*a*C) - 3*b*B*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a
^2*b^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 29.0823, size = 124, normalized size = 0.89

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{x^3(Ba-x(Ab-Ca))}{7ab(a+bx^2)^{\frac{7}{2}}}-\frac{x^2(3Ab-3Bbx+4Ca)}{35ab^2(a+bx^2)^{\frac{5}{2}}}-\frac{6Ab+6Bbx+8Ca}{105ab^3(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] 2*B*x/(35*a**2*b**2*sqrt(a + b*x**2)) - x**3*(B*a - x*(A*b - C*a)
)/(7*a*b*(a + b*x**2)**(7/2)) - x**2*(3*A*b - 3*B*b*x + 4*C*a)/(3
5*a*b**2*(a + b*x**2)**(5/2)) - (6*A*b + 6*B*b*x + 8*C*a)/(105*a*
b**3*(a + b*x**2)**(3/2))
```

Mathematica [A] time = 0.0960272, size = 84, normalized size = 0.6

$$\frac{-8a^4C-2a^3b\left(3A+14Cx^2\right)-7a^2b^2x^2\left(3A+5Cx^2\right)+21ab^3Bx^5+6b^4Bx^7}{105a^2b^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5
*C*x^2) - 2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2
))
```

Maple [A] time = 0.011, size = 85, normalized size = 0.6

$$-\frac{-6\,Bb^{4}x^{7}-21\,Bx^{5}ab^{3}+35\,Cx^{4}a^{2}b^{2}+21\,Aa^{2}b^{2}x^{2}+28\,Ca^{3}bx^{2}+6\,Aa^{3}b+8\,Ca^{4}}{105\,a^{2}b^{3}}\left(bx^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2
*x^2+28*C*a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

Maxima [A] time = 1.38504, size = 242, normalized size = 1.74

$$-\frac{Cx^{4}}{3\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Bx^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{4\,Cax^{2}}{15\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{3\,Bx}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Bx}{35\,\sqrt{bx^{2}+a}a^{2}b^{2}}+\frac{Bx}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Bax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{8\,Ca^{2}}{105\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{2\,Aa}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^3/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b
) - 4/15*C*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)
^(7/2)*b) + 3/140*B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*
x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a
*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2/((b*x^2 + a)^(7/2)*b^3)
- 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [A] time = 0.285538, size = 177, normalized size = 1.27

$$\frac{\left(6\,Bb^{4}x^{7}+21\,Bab^{3}x^{5}-35\,Ca^{2}b^{2}x^{4}-8\,Ca^{4}-6\,Aa^{3}b-7\left(4\,Ca^{3}b+3\,Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\left(a^{2}b^{7}x^{8}+4\,a^{3}b^{6}x^{6}+6\,a^{4}b^{5}x^{4}+4\,a^{5}b^{4}x^{2}+a^{6}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^3/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4
- 6*A*a^3*b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a
^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*
b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22317, size = 128, normalized size = 0.92

$$\frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\left(4\,Ca^4b+3\,Aa^3b^2\right)}{a^3b^3}\right)x^2-\frac{2\left(4\,Ca^5+3\,Aa^4b\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^3/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b
 + 3*A*a^3*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3
))/(b*x^2 + a)^(7/2)
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\,(a+bx^2)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B +
 (4*A*b + 3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*
C)*x)/(105*a^2*b^2*(a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(10
5*a^3*b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.297266, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\,(a+bx^2)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B +
 (4*A*b + 3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*
C)*x)/(105*a^2*b^2*(a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(10
5*a^3*b^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 23.5693, size = 126, normalized size = 0.91

$$-\frac{x^2\,(Ba-x\,(Ab-Ca))}{7ab\,(a+bx^2)^{\frac{7}{2}}}-\frac{2Ba+x\,(4Ab+3Ca)}{35ab^2\,(a+bx^2)^{\frac{5}{2}}}+\frac{x\,(4Ab+3Ca)}{105a^2b^2\,(a+bx^2)^{\frac{3}{2}}}+\frac{2x\,(4Ab+3Ca)}{105a^3b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] -x**2*(B*a - x*(A*b - C*a))/(7*a*b*(a + b*x**2)**(7/2)) - (2*B*a
+ x*(4*A*b + 3*C*a))/(35*a*b**2*(a + b*x**2)**(5/2)) + x*(4*A*b +
 3*C*a)/(105*a**2*b**2*(a + b*x**2)**(3/2)) + 2*x*(4*A*b + 3*C*a)
/(105*a**3*b**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.107597, size = 87, normalized size = 0.63

$$\frac{-6a^4B-21a^3bBx^2+7a^2b^2x^3\left(5A+3Cx^2\right)+2ab^3x^5\left(14A+3Cx^2\right)+8Ab^4x^7}{105a^3b^2\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3
*C*x^2) + 2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^
(7/2))
```

Maple [A] time = 0.009, size = 88, normalized size = 0.6

$$\frac{8\,Ab^4x^7+6\,Cab^3x^7+28\,Aab^3x^5+21\,Ca^2b^2x^5+35\,Ax^3a^2b^2-21\,Bx^2a^3b-6\,Ba^4}{105\,a^3b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+
35*A*a^2*b^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

Maxima [A] time = 1.36151, size = 266, normalized size = 1.91

$$-\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx^2}{5\,(bx^2+a)^{\frac{7}{2}}b}+\frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2}+\frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{2\,Ba}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b
) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x
^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(s
qrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/3
5*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [A] time = 0.3116, size = 181, normalized size = 1.3

$$\frac{\left(35\,Aa^2b^2x^3+2\left(3\,Cab^3+4\,Ab^4\right)x^7-21\,Ba^3bx^2+7\left(3\,Ca^2b^2+4\,Aab^3\right)x^5-6\,Ba^4\right)\sqrt{bx^2+a}}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*
b*x^2 + 7*(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a
)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 +
a^7*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219227, size = 127, normalized size = 0.91

$$\frac{\left(\left(x^{2}\left(\frac{2\,\left(3\,Cab^{4}+4\,Ab^{5}\right)x^{2}}{a^{3}b^{3}}+\frac{7\,\left(3\,Ca^{2}b^{3}+4\,Aab^{4}\right)}{a^{3}b^{3}}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^{2}-\frac{6\,Ba}{b^{2}}}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*
b^3 + 4*A*a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2
)/(b*x^2 + a)^(7/2)
```

3.53 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=119

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\,(a+bx^2)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2
*a*C - b*B*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(
a + b*x^2)^(3/2)) + (8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.182183, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\,(a+bx^2)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2
*a*C - b*B*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(
a + b*x^2)^(3/2)) + (8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 21.3302, size = 107, normalized size = 0.9

$$\frac{4Bx}{105a^2b\,(a+bx^2)^{\frac{3}{2}}}+\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{x^2\,(Ab+Bbx-Ca)}{7ab\,(a+bx^2)^{\frac{7}{2}}}-\frac{5Ab+4Bbx+2Ca}{35ab^2\,(a+bx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] 4*B*x/(105*a**2*b*(a + b*x**2)**(3/2)) + 8*B*x/(105*a**3*b*sqrt(a
 + b*x**2)) + x**2*(A*b + B*b*x - C*a)/(7*a*b*(a + b*x**2)**(7/2)
) - (5*A*b + 4*B*b*x + 2*C*a)/(35*a*b**2*(a + b*x**2)**(5/2))
```

Mathematica [A] time = 0.078417, size = 75, normalized size = 0.63

$$\frac{-6a^4C-3a^3b\left(5A+7Cx^2\right)+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*C + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5 + 8*b^4*B*x^7 - 3*a
^3*b*(5*A + 7*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.008, size = 73, normalized size = 0.6

$$-\frac{-8\,b^4Bx^7-28\,Bb^3x^5a-35\,Bx^3a^2b^2+21\,Cx^2a^3b+15\,Aa^3b+6\,Ca^4}{105\,a^3b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x
^2+15*A*a^3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

Maxima [A] time = 1.34971, size = 166, normalized size = 1.39

$$-\frac{Cx^2}{5\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{2\,Ca}{35\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{A}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b)
+ 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2
)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 +
a)^(7/2)*b^2) - 1/7*A/((b*x^2 + a)^(7/2)*b)
```

Fricas [A] time = 0.36146, size = 161, normalized size = 1.35

$$\frac{\left(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b\right)\sqrt{bx^2+a}}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3
*b*x^2 - 6*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219849, size = 111, normalized size = 0.93

$$\frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3}+\frac{7\,Bb}{a^2}\right)x^2+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^2-\frac{3\,(2\,Ca^4b+5\,Aa^3b^2)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b
)*x^2 - 3*(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

3.54 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\,(a+bx^2)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\,(a+bx^2)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)
*x)/(35*a^2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b
*(a + b*x^2)^(3/2)) + (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.153129, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\,(a+bx^2)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\,(a+bx^2)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```


```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)
*x)/(35*a^2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b
*(a + b*x^2)^(3/2)) + (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x
^2])
```

Rubi in Sympy [A] time = 13.633, size = 110, normalized size = 0.87

$$-\frac{Ba-x\,(Ab-Ca)}{7ab\,(a+bx^2)^{\frac{7}{2}}}+\frac{x\,(6Ab+Ca)}{35a^2b\,(a+bx^2)^{\frac{5}{2}}}+\frac{4x\,(6Ab+Ca)}{105a^3b\,(a+bx^2)^{\frac{3}{2}}}+\frac{8x\,(6Ab+Ca)}{105a^4b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] -(B*a - x*(A*b - C*a))/(7*a*b*(a + b*x**2)**(7/2)) + x*(6*A*b + C
*a)/(35*a**2*b*(a + b*x**2)**(5/2)) + 4*x*(6*A*b + C*a)/(105*a**3
*b*(a + b*x**2)**(3/2)) + 8*x*(6*A*b + C*a)/(105*a**4*b*sqrt(a +
b*x**2))
```

Mathematica [A] time = 0.0919634, size = 92, normalized size = 0.72

$$\frac{-15a^4B+35a^3bx\left(3A+Cx^2\right)+14a^2b^2x^3\left(15A+2Cx^2\right)+8ab^3x^5\left(21A+Cx^2\right)+48Ab^4x^7}{105a^4b\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^
5*(21*A + C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a
 + b*x^2)^(7/2))
```

Maple [A] time = 0.007, size = 96, normalized size = 0.8

$$\frac{48\,Ab^{4}x^{7}+8\,Cab^{3}x^{7}+168\,Aab^{3}x^{5}+28\,Ca^{2}b^{2}x^{5}+210\,Aa^{2}b^{2}x^{3}+35\,Ca^{3}bx^{3}+105\,Axa^{3}b-15\,Ba^{4}}{105\,a^{4}b}\left(bx^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^
5+210*A*a^2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2
+a)^(7/2)/a^4/b
```

Maxima [A] time = 1.48432, size = 207, normalized size = 1.63

$$\frac{16\,Ax}{35\,\sqrt{bx^{2}+a}a^{4}}+\frac{8\,Ax}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,Ax}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{Ax}{7\,(bx^{2}+a)^{\frac{7}{2}}a}-\frac{Cx}{7\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^{2}+a}a^{3}b}+\frac{4\,Cx}{105\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b}+\frac{Cx}{35\,(bx^{2}+a)^{\frac{5}{2}}ab}-\frac{B}{7\,(bx^{2}+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3
) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)
*a) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*
a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 +
 a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2)*b)
```

Fricas [A] time = 0.342734, size = 185, normalized size = 1.46

$$\frac{\left(8\left(Cab^{3}+6\,Ab^{4}\right)x^{7}+105\,Aa^{3}bx+28\left(Ca^{2}b^{2}+6\,Aab^{3}\right)x^{5}-15\,Ba^{4}+35\left(Ca^{3}b+6\,Aa^{2}b^{2}\right)x^{3}\right)\sqrt{bx^{2}+a}}{105\left(a^{4}b^{5}x^{8}+4\,a^{5}b^{4}x^{6}+6\,a^{6}b^{3}x^{4}+4\,a^{7}b^{2}x^{2}+a^{8}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2
+ 6*A*a*b^3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqr
t(b*x^2 + a)/(a^4*b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7
*b^2*x^2 + a^8*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22074, size = 151, normalized size = 1.19

$$\frac{\left(\left(4\,x^{2}\left(\frac{2\,\left(Cab^{5}+6\,Ab^{6}\right)x^{2}}{a^{4}b^{3}}+\frac{7\,\left(Ca^{2}b^{4}+6\,Aab^{5}\right)}{a^{4}b^{3}}\right)+\frac{35\,\left(Ca^{3}b^{3}+6\,Aa^{2}b^{4}\right)}{a^{4}b^{3}}\right)x^{2}+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^
4 + 6*A*a*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3
))*x^2 + 105*A/a)*x - 15*B/b)/(b*x^2 + a)^(7/2)
```

3.55 $\int \frac{A+Bx+Cx^2}{x(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35
*a^2*(a + b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3
/2)) + (35*A + 16*B*x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.418346, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35
*a^2*(a + b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3
/2)) + (35*A + 16*B*x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi in Sympy [A] time = 61.4015, size = 121, normalized size = 0.88

$$-\frac{A\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{9}{2}}}+\frac{Ab+Bbx-Ca}{7ab\left(a+bx^2\right)^{\frac{7}{2}}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{105A+48Bx}{105a^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(9/2) + (A*b + B*b*x - C*a)
/(7*a*b*(a + b*x**2)**(7/2)) + (7*A + 6*B*x)/(35*a**2*(a + b*x**2
)**(5/2)) + (35*A + 24*B*x)/(105*a**3*(a + b*x**2)**(3/2)) + (105
*A + 48*B*x)/(105*a**4*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.791321, size = 131, normalized size = 0.95

$$-\frac{A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{9/2}}+\frac{A\log(x)}{a^{9/2}}+\frac{-15a^4C+a^3b(176A+105Bx)+14a^2b^2x^2(29A+15Bx)+14ab^3x^4(25A+12Bx)+3b^4x^6(35A+16Bx)}{105a^4b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A
+ 15*B*x) + 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/
```

```
(105*a^4*b*(a + b*x^2)^(7/2)) + (A*Log[x])/a^(9/2) - (A*Log[a + S
qrt[a]*Sqrt[a + b*x^2]])/a^(9/2)
```

Maple [A] time = 0.011, size = 169, normalized size = 1.2

$$\frac{Bx}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{6\,Bx}{35\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{8\,Bx}{35\,a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{16\,Bx}{35\,a^4}\frac{1}{\sqrt{bx^2+a}}+\frac{A}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{A}{5\,a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{A}{3\,a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{A}{a^4}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}-\frac{C}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*B*x/a/(b*x^2+a)^(7/2)+6/35*B/a^2*x/(b*x^2+a)^(5/2)+8/35*B/a^3
*x/(b*x^2+a)^(3/2)+16/35*B/a^4*x/(b*x^2+a)^(1/2)+1/7*A/a/(b*x^2+a
)^(7/2)+1/5*A/a^2/(b*x^2+a)^(5/2)+1/3*A/a^3/(b*x^2+a)^(3/2)+A/a^4
/(b*x^2+a)^(1/2)-A/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-
1/7*C/b/(b*x^2+a)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.327987, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,Bb^4x^7+105\,Ab^4x^6+168\,Bab^3x^5+350\,Aab^3x^4+210\,Ba^2b^2x^3+406\,Aa^2b^2x^2+105\,Ba^3bx-15\,Ca^4+176\,Aa^3b\right)\sqrt{bx^2}}{210\left(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x),x, algorithm="fricas")
```

```
[Out] [1/210*(2*(48*B*b^4*x^7 + 105*A*b^4*x^6 + 168*B*a*b^3*x^5 + 350*A
*a*b^3*x^4 + 210*B*a^2*b^2*x^3 + 406*A*a^2*b^2*x^2 + 105*B*a^3*b*
x - 15*C*a^4 + 176*A*a^3*b)*sqrt(b*x^2 + a)*sqrt(a) + 105*(A*b^5*
x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2 + A*a^4*b
)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a^4*
b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)*
sqrt(a)), 1/105*((48*B*b^4*x^7 + 105*A*b^4*x^6 + 168*B*a*b^3*x^5
+ 350*A*a*b^3*x^4 + 210*B*a^2*b^2*x^3 + 406*A*a^2*b^2*x^2 + 105*B
*a^3*b*x - 15*C*a^4 + 176*A*a^3*b)*sqrt(b*x^2 + a)*sqrt(-a) - 105
*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2 +
 A*a^4*b)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a^4*b^5*x^8 + 4*a^5
*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)*sqrt(-a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223839, size = 205, normalized size = 1.49

$$\frac{\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)x-\frac{15\,Ca^{14}b^2-176\,Aa^{13}b^3}{a^{14}b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x
 + 350*A*b^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*
x - (15*C*a^14*b^2 - 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2
) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^4)
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/
a - 6*C)*x)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a -
8*C)*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)
*x)/(35*a^4*Sqrt[a + b*x^2]) - (A*Sqrt[a + b*x^2])/(a^5*x) - (B*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.698497, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/
a - 6*C)*x)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a -
8*C)*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)
*x)/(35*a^4*Sqrt[a + b*x^2]) - (A*Sqrt[a + b*x^2])/(a^5*x) - (B*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rubi in Sympy [A] time = 72.8151, size = 163, normalized size = 0.87

$$-\frac{A}{ax\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{9}{2}}}+\frac{Ba-x\left(8Ab-Ca\right)}{7a^2\left(a+bx^2\right)^{\frac{7}{2}}}+\frac{7Ba-x\left(48Ab-6Ca\right)}{35a^3\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{35Ba-x\left(192Ab-24Ca\right)}{105a^4\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{105Ba-x\left(384Ab-48Ca\right)}{105a^5\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -A/(a*x*(a + b*x**2)**(7/2)) - B*atanh(sqrt(a + b*x**2)/sqrt(a))/
a**(9/2) + (B*a - x*(8*A*b - C*a))/(7*a**2*(a + b*x**2)**(7/2)) +
 (7*B*a - x*(48*A*b - 6*C*a))/(35*a**3*(a + b*x**2)**(5/2)) + (35
*B*a - x*(192*A*b - 24*C*a))/(105*a**4*(a + b*x**2)**(3/2)) + (10
5*B*a - x*(384*A*b - 48*C*a))/(105*a**5*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.347678, size = 158, normalized size = 0.84

$$-\frac{B\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{9/2}}+\frac{B\log(x)}{a^{9/2}}$$
$$+\frac{a^4(x(176B+105Cx)-105A)+14a^3bx^2(x(29B+15Cx)-60A)+14a^2b^2x^4(x(25B+12Cx)-120A)+3ab^3x^6(x(35B+16Cx}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 1
4*a^3*b*x^2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x
*(35*B + 16*C*x)) + a^4*(-105*A + x*(176*B + 105*C*x)))/(105*a^5*
x*(a + b*x^2)^(7/2)) + (B*Log[x])/a^(9/2) - (B*Log[a + Sqrt[a]*Sq
rt[a + b*x^2]])/a^(9/2)
```

Maple [A] time = 0.014, size = 240, normalized size = 1.3

$$\frac{Cx}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{6Cx}{35a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{8Cx}{35a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{16Cx}{35a^4}\frac{1}{\sqrt{bx^2+a}}$$
$$-\frac{A}{ax}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{8Axb}{7a^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{48Axb}{35a^3}\left(bx^2+a\right)^{-\frac{5}{2}}$$
$$-\frac{64Axb}{35a^4}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{128Axb}{35a^5}\frac{1}{\sqrt{bx^2+a}}+\frac{B}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{B}{5a^2}\left(bx^2+a\right)^{-\frac{5}{2}}$$
$$+\frac{B}{3a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{B}{a^4}\frac{1}{\sqrt{bx^2+a}}-B\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*C*x/a/(b*x^2+a)^(7/2)+6/35*C/a^2*x/(b*x^2+a)^(5/2)+8/35*C/a^3
*x/(b*x^2+a)^(3/2)+16/35*C/a^4*x/(b*x^2+a)^(1/2)-A/a/x/(b*x^2+a)^
(7/2)-8/7*A*b/a^2*x/(b*x^2+a)^(7/2)-48/35*A*b/a^3*x/(b*x^2+a)^(5/
2)-64/35*A*b/a^4*x/(b*x^2+a)^(3/2)-128/35*A*b/a^5*x/(b*x^2+a)^(1/
2)+1/7*B/a/(b*x^2+a)^(7/2)+1/5*B/a^2/(b*x^2+a)^(5/2)+1/3*B/a^3/(b
*x^2+a)^(3/2)+B/a^4/(b*x^2+a)^(1/2)-B/a^(9/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.333533, size = 1, normalized size = 0.01

$$\left[\frac{2\left(105\,Bab^3x^7+350\,Ba^2b^2x^5+48\left(Cab^3-8\,Ab^4\right)x^8+406\,Ba^3bx^3+168\left(Ca^2b^2-8\,Aab^3\right)x^6+176\,Ba^4x-105\,Aa^4+210}{210\left(a^5b^4x^9+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(2*(105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*
A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 +
176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(
C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a) + 105*(B*a*b^4*x^
9 + 4*B*a^2*b^3*x^7 + 6*B*a^3*b^2*x^5 + 4*B*a^4*b*x^3 + B*a^5*x)*
log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a^5*b^
4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)*sqrt
(a)), 1/105*((105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 -
 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6
 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 10
5*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a) - 105*(B*a*b^
4*x^9 + 4*B*a^2*b^3*x^7 + 6*B*a^3*b^2*x^5 + 4*B*a^4*b*x^3 + B*a^5
*x)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a^5*b^4*x^9 + 4*a^6*b^3*x
^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)*sqrt(-a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223566, size = 323, normalized size = 1.72

$$\frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4} + \frac{(16\,Ca^{20}b^6 - 93\,Aa^{19}b^7)x}{a^{24}b^3}\right) + \frac{28\,(2\,Ca^{21}b^5 - 11\,Aa^{20}b^6)}{a^{24}b^3}\right)x + \frac{350\,Bb^2}{a^3}\right)x + \frac{210\,(Ca^{22}b^4 - 5\,Aa^{21}b^5)}{a^{24}b^3}\right)x + \frac{406\,Bb}{a^2}\right)x + \frac{105\,(Ca^{23}\cdots}{}}{105\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 - a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x
/(a^24*b^3)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x +
350*B*b^2/a^3)*x + 210*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x
+ 406*B*b/a^2)*x + 105*(C*a^23*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x
+ 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((sqrt(b)*x - sqr
t(b*x^2 + a))^2 - a)*a^4)
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(a*((A*b)/a - C) + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b
- a*C) + 13*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C)
 + 87*b*B*x)/(105*a^4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93
*b*B*x)/(35*a^5*Sqrt[a + b*x^2]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2
) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*b - 2*a*C)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rubi [A] time = 0.866946, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{-aC+Ab+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A*b - a*C + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C)
 + 13*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*
b*B*x)/(105*a^4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x
)/(35*a^5*Sqrt[a + b*x^2]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B
*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*b - 2*a*C)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/(2*a^(11/2))
```

Rubi in Sympy [A] time = 85.8284, size = 207, normalized size = 0.95

$$-\frac{A}{2ax^2\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{128B\sqrt{a+bx^2}}{35a^5x}+\frac{2Ba-x\left(9Ab-2Ca\right)}{14a^2x\left(a+bx^2\right)^{\frac{7}{2}}}+\frac{16Ba-x\left(63Ab-14Ca\right)}{70a^3x\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{96Ba-x\left(315Ab-70Ca\right)}{210a^4x\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{384Ba-x\left(945Ab-210Ca\right)}{210a^5x\sqrt{a+bx^2}}+\frac{(9Ab-2Ca)\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] -A/(2*a*x**2*(a + b*x**2)**(7/2)) - 128*B*sqrt(a + b*x**2)/(35*a*
*5*x) + (2*B*a - x*(9*A*b - 2*C*a))/(14*a**2*x*(a + b*x**2)**(7/2
)) + (16*B*a - x*(63*A*b - 14*C*a))/(70*a**3*x*(a + b*x**2)**(5/2
)) + (96*B*a - x*(315*A*b - 70*C*a))/(210*a**4*x*(a + b*x**2)**(3
/2)) + (384*B*a - x*(945*A*b - 210*C*a))/(210*a**5*x*sqrt(a + b*x
**2)) + (9*A*b - 2*C*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(2*a**(11
/2))
```

Mathematica [A] time = 0.492398, size = 183, normalized size = 0.84

$$-\frac{\frac{\sqrt{a}\left(a^4\left(105A+210Bx-352Cx^2\right)+4a^3bx^2(396A+7x(60B-29Cx))+14a^2b^2x^4(261A+10x(24B-5Cx))+42ab^3x^6(75A+x(64B-5Cx))+3b^4x^8(315A+256Bx)\right)}{x^2\left(a+bx^2\right)^{7/2}}+105}{210a^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-((Sqrt[a]*(3*b^4*x^8*(315*A + 256*B*x) + a^4*(105*A + 210*B*x -
 352*C*x^2) + 4*a^3*b*x^2*(396*A + 7*x*(60*B - 29*C*x)) + 14*a^2*
b^2*x^4*(261*A + 10*x*(24*B - 5*C*x)) + 42*a*b^3*x^6*(75*A + x*(6
4*B - 5*C*x))))/(x^2*(a + b*x^2)^(7/2))) + 105*(-9*A*b + 2*a*C)*L
og[x] + 105*(9*A*b - 2*a*C)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(21
0*a^(11/2))
```

Maple [A] time = 0.016, size = 288, normalized size = 1.3

$$-\frac{A}{2ax^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{9Ab}{14a^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{9Ab}{10a^3}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{3Ab}{2a^4}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{9Ab}{2a^5}\frac{1}{\sqrt{bx^2+a}}+\frac{9Ab}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{11}{2}}-\frac{B}{ax}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{8bBx}{7a^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{48bBx}{35a^3}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{64bBx}{35a^4}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{128bBx}{35a^5}\frac{1}{\sqrt{bx^2+a}}+\frac{C}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{C}{5a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{C}{3a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{C}{a^4}\frac{1}{\sqrt{bx^2+a}}-C\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(7/2)-9/14*A*b/a^2/(b*x^2+a)^(7/2)-9/10*A*
b/a^3/(b*x^2+a)^(5/2)-3/2*A*b/a^4/(b*x^2+a)^(3/2)-9/2*A*b/a^5/(b*
x^2+a)^(1/2)+9/2*A*b/a^(11/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)-B/a/x/(b*x^2+a)^(7/2)-8/7*B*b/a^2*x/(b*x^2+a)^(7/2)-48/35*B*b/
a^3*x/(b*x^2+a)^(5/2)-64/35*B*b/a^4*x/(b*x^2+a)^(3/2)-128/35*B*b/
a^5*x/(b*x^2+a)^(1/2)+1/7*C/a/(b*x^2+a)^(7/2)+1/5*C/a^2/(b*x^2+a)
^(5/2)+1/3*C/a^3/(b*x^2+a)^(3/2)+C/a^4/(b*x^2+a)^(1/2)-C/a^(9/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.37926, size = 1, normalized size = 0.

$$\left[-\frac{2\left(768\,Bb^4x^9+2688\,Bab^3x^7+3360\,Ba^2b^2x^5-105\left(2\,Cab^3-9\,Ab^4\right)x^8+1680\,Ba^3bx^3-350\left(2\,Ca^2b^2-9\,Aab^3\right)x^6+210\right.}{}\right.$$

$$-\frac{\left(768\,Bb^4x^9+2688\,Bab^3x^7+3360\,Ba^2b^2x^5-105\left(2\,Cab^3-9\,Ab^4\right)x^8+1680\,Ba^3bx^3-350\left(2\,Ca^2b^2-9\,Aab^3\right)x^6+210\,Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/420*(2*(768*B*b^4*x^9 + 2688*B*a*b^3*x^7 + 3360*B*a^2*b^2*x^5
 - 105*(2*C*a*b^3 - 9*A*b^4)*x^8 + 1680*B*a^3*b*x^3 - 350*(2*C*a^
2*b^2 - 9*A*a*b^3)*x^6 + 210*B*a^4*x + 105*A*a^4 - 406*(2*C*a^3*b
 - 9*A*a^2*b^2)*x^4 - 176*(2*C*a^4 - 9*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)*sqrt(a) + 105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 -
9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b
 - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*log(-((b*x^2 + 2
*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2))/((a^5*b^4*x^10 + 4*a^6*b
^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2)*sqrt(a)), -1/210*
((768*B*b^4*x^9 + 2688*B*a*b^3*x^7 + 3360*B*a^2*b^2*x^5 - 105*(2*
C*a*b^3 - 9*A*b^4)*x^8 + 1680*B*a^3*b*x^3 - 350*(2*C*a^2*b^2 - 9*
A*a*b^3)*x^6 + 210*B*a^4*x + 105*A*a^4 - 406*(2*C*a^3*b - 9*A*a^2
*b^2)*x^4 - 176*(2*C*a^4 - 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-
a) + 105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4
)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^
3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*arctan(sqrt(-a)/sqrt(b*x^
2 + a)))/((a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b
*x^4 + a^9*x^2)*sqrt(-a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.23069, size = 439, normalized size = 2.

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\left(Ca^{24}b^6-4\,Aa^{23}b^7\right)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\left(10\,Ca^{25}b^5-39\,Aa^{24}b^6\right)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\left(29\,Ca^{26}b^4-108\,Aa^{25}b^5\right)}{a^{28}b^3}\right)x+}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$
$$+\frac{\left(2\,Ca-9\,Ab\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(9/2)*x^3),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(
a^28*b^3))*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b
^6)/(a^28*b^3))*x + 1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A
*a^25*b^5)/(a^28*b^3))*x + 420*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 29
1*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7/2) + (2*C*a - 9*A*b)*arc
tan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^5) + ((s
qrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a
))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*
sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

3.58 $\int \frac{A(cx)^m}{a+bx^2} dx$

Optimal. Leaf size=45

$$\frac{A(cx)^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*c*(1 + m))
```

Rubi [A] time = 0.0552703, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{A(cx)^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*c*(1 + m))
```

Rubi in Sympy [A] time = 7.12858, size = 34, normalized size = 0.76

$$\frac{A\,(cx)^{m+1} {}_2F_1\left(\begin{matrix} 1, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| -\frac{bx^2}{a}\right)}{ac\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(A*(c*x)**m/(b*x**2+a),x)
```

```
[Out] A*(c*x)**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(
a*c*(m + 1))
```

Mathematica [A] time = 0.0259577, size = 43, normalized size = 0.96

$$\frac{Ax(cx)^m {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+1}{2}+1; -\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*
x^2)/a)])/(a*(1 + m))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{A\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(A*(c*x)^m/(b*x^2+a),x)

[Out] int(A*(c*x)^m/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$A\int \frac{(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m*A/(b*x^2 + a),x, algorithm="maxima")

[Out] A*integrate((c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m A}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m*A/(b*x^2 + a),x, algorithm="fricas")

[Out] integral((c*x)^m*A/(b*x^2 + a), x)
```

Sympy [A] time = 2.26141, size = 97, normalized size = 2.16

$$A\left(\frac{c^m m x x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^m x x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(A*(c*x)**m/(b*x**2+a),x)

[Out] A*(c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)
*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b
*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamm
a(m/2 + 3/2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m A}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m*A/(b*x^2 + a),x, algorithm="giac")

[Out] integrate((c*x)^m*A/(b*x^2 + a), x)
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (
2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rubi [A] time = 0.135594, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (
2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rubi in Sympy [A] time = 15.158, size = 68, normalized size = 0.75

$$\frac{A\,(cx)^{m+1}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ac\,(m+1)}+\frac{B\,(cx)^{m+2}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ac^2\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*(c*x)**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(
a*c*(m + 1)) + B*(c*x)**(m + 2)*hyper((1, m/2 + 1), (m/2 + 2,), -
b*x**2/a)/(a*c**2*(m + 2))
```

Mathematica [A] time = 0.0819656, size = 82, normalized size = 0.9

$$\frac{x(cx)^m\left(A(m+2)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+B(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)\right)}{a(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(
(b*x^2)/a)] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2
, -((b*x^2)/a)]))/(a*(1 + m)*(2 + m))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{(cx)^m (Bx + A)}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx + A)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx + A)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [A] time = 6.03401, size = 192, normalized size = 2.11

$$\frac{Ac^m mxx^m \left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m xx^m \left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Bc^m mx^2x^m \left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right)\left(\frac{m}{2} + 1\right)}{4a\left(\frac{m}{2} + 2\right)} + \frac{Bc^m x^2x^m \left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right)\left(\frac{m}{2} + 1\right)}{2a\left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gam
ma(m/2 + 3/2)) + B*c**m*m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*gamma(m/2 + 2)) + B*c**m*x*
*2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
+ 1)/(2*a*gamma(m/2 + 2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx + A)\,(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="giac")

[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.60 $\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=76

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 +
 m))
```

Rubi [A] time = 0.123953, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 +
 m))
```

Rubi in Sympy [A] time = 14.3618, size = 56, normalized size = 0.74

$$\frac{C\,(cx)^{m+1}}{bc\,(m+1)}+\frac{(cx)^{m+1}\,(Ab-Ca)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{abc\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

```
[Out] C*(c*x)**(m + 1)/(b*c*(m + 1)) + (c*x)**(m + 1)*(A*b - C*a)*hyper
((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*b*c*(m + 1))
```

Mathematica [A] time = 0.0684895, size = 58, normalized size = 0.76

$$-\frac{x(cx)^m\left((aC-Ab)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)-aC\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] -((x*(c*x)^m*(-(a*C) + (-(A*b) + a*C)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(cx)^m\left(Cx^2+A\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + A)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [A] time = 7.88841, size = 204, normalized size = 2.68

$$\frac{Ac^m mxx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m xx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Cc^m mx^3x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right)\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3Cc^m x^3x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right)\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gam
ma(m/2 + 3/2)) + C*c**m*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*C
*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*
gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a),x, algorithm="giac")

[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

3.61 $\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=121

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 +
 m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2
, -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rubi [A] time = 0.279132, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 +
 m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2
, -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rubi in Sympy [A] time = 39.393, size = 90, normalized size = 0.74

$$\frac{B\,(cx)^{m+2}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ac^2\,(m+2)}+\frac{C\,(cx)^{m+1}}{bc\,(m+1)}+\frac{(cx)^{m+1}\,(Ab-Ca)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{abc\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] B*(c*x)**(m + 2)*hyper((1, m/2 + 1), (m/2 + 2,), -b*x**2/a)/(a*c*
*2*(m + 2)) + C*(c*x)**(m + 1)/(b*c*(m + 1)) + (c*x)**(m + 1)*(A*
b - C*a)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*b*c*(m
 + 1))
```

Mathematica [A] time = 0.103921, size = 100, normalized size = 0.83

$$\frac{x(cx)^m\left(bB(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)-(m+2)\left((aC-Ab)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)-aC\right)\right)}{ab(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
-((b*x^2)/a)] - (2 + m)*(-(a*C) + (-(A*b) + a*C)*Hypergeometric2F
1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])))/(a*b*(1 + m)*(2 + m))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(cx)^m (Cx^2 + Bx + A)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [A] time = 9.67938, size = 298, normalized size = 2.46

$$\frac{Ac^m mxx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m xx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m mx^2x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^2x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m mx^3x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Cc^m x^3x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gam
ma(m/2 + 3/2)) + B*c**m*m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*gamma(m/2 + 2)) + B*c**m*x*
*2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
```

```
+ 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**3*x**m*lerchphi(b*x**2*ex
p_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 +
5/2)) + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.62 $\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^
7)/7 + (b*C*x^8)/8 + (b*D*x^9)/9
```

Rubi [A] time = 0.147377, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^
7)/7 + (b*C*x^8)/8 + (b*D*x^9)/9
```

Rubi in Sympy [A] time = 22.3874, size = 60, normalized size = 0.92

$$\frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9}+x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 +
 D*a/7) + x**6*(A*b/6 + C*a/6)
```

Mathematica [A] time = 0.0292464, size = 65, normalized size = 1.

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^
7)/7 + (b*C*x^8)/8 + (b*D*x^9)/9
```

Maple [A] time = 0.004, size = 54, normalized size = 0.8

$$\frac{aAx^4}{4}+\frac{aBx^5}{5}+\frac{(Ab+aC)\,x^6}{6}+\frac{(Bb+aD)\,x^7}{7}+\frac{bCx^8}{8}+\frac{bDx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b
*C*x^8+1/9*b*D*x^9
```

Maxima [A] time = 1.35269, size = 72, normalized size = 1.11

$$\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7+\frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^3,x, algorithm="maxima")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1
/6*(C*a + A*b)*x^6 + 1/4*A*a*x^4
```

Fricas [A] time = 0.251345, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9bD+\frac{1}{8}x^8bC+\frac{1}{7}x^7aD+\frac{1}{7}x^7bB+\frac{1}{6}x^6aC+\frac{1}{6}x^6bA+\frac{1}{5}x^5aB+\frac{1}{4}x^4aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b*D + 1/8*x^8*b*C + 1/7*x^7*a*D + 1/7*x^7*b*B + 1/6*x^6*a
*C + 1/6*x^6*b*A + 1/5*x^5*a*B + 1/4*x^4*a*A
```

Sympy [A] time = 0.053357, size = 60, normalized size = 0.92

$$\frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9}+x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 +
 D*a/7) + x**6*(A*b/6 + C*a/6)
```

GIAC/XCAS [A] time = 0.21116, size = 77, normalized size = 1.18

$$\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,Dax^7+\frac{1}{7}\,Bbx^7+\frac{1}{6}\,Cax^6+\frac{1}{6}\,Abx^6+\frac{1}{5}\,Bax^5+\frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^3,x, algorithm="giac")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x
^6 + 1/6*A*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

3.63 $\int x^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^
6)/6 + (b*C*x^7)/7 + (b*D*x^8)/8
```

Rubi [A] time = 0.147951, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^
6)/6 + (b*C*x^7)/7 + (b*D*x^8)/8
```

Rubi in Sympy [A] time = 25.7079, size = 60, normalized size = 0.92

$$\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Cbx^7}{7}+\frac{Dbx^8}{8}+x^6\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 +
 D*a/6) + x**5*(A*b/5 + C*a/5)
```

Mathematica [A] time = 0.0295779, size = 65, normalized size = 1.

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^
6)/6 + (b*C*x^7)/7 + (b*D*x^8)/8
```

Maple [A] time = 0.002, size = 54, normalized size = 0.8

$$\frac{aAx^3}{3}+\frac{aBx^4}{4}+\frac{(Ab+aC)\,x^5}{5}+\frac{(Bb+aD)\,x^6}{6}+\frac{bCx^7}{7}+\frac{bDx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b
*C*x^7+1/8*b*D*x^8
```

Maxima [A] time = 1.34727, size = 72, normalized size = 1.11

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}(Da + Bb)x^6 + \frac{1}{4}Bax^4 + \frac{1}{5}(Ca + Ab)x^5 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1
/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 0.258405, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8bD + \frac{1}{7}x^7bC + \frac{1}{6}x^6aD + \frac{1}{6}x^6bB + \frac{1}{5}x^5aC + \frac{1}{5}x^5bA + \frac{1}{4}x^4aB + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b*D + 1/7*x^7*b*C + 1/6*x^6*a*D + 1/6*x^6*b*B + 1/5*x^5*a
*C + 1/5*x^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A
```

Sympy [A] time = 0.053457, size = 60, normalized size = 0.92

$$\frac{Aax^3}{3} + \frac{Bax^4}{4} + \frac{Cbx^7}{7} + \frac{Dbx^8}{8} + x^6\left(\frac{Bb}{6} + \frac{Da}{6}\right) + x^5\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 +
 D*a/6) + x**5*(A*b/5 + C*a/5)
```

GIAC/XCAS [A] time = 0.212686, size = 77, normalized size = 1.18

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}Dax^6 + \frac{1}{6}Bbx^6 + \frac{1}{5}Cax^5 + \frac{1}{5}Abx^5 + \frac{1}{4}Bax^4 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x
^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.64 $\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^
5)/5 + (b*C*x^6)/6 + (b*D*x^7)/7
```

Rubi [A] time = 0.123128, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^
5)/5 + (b*C*x^6)/6 + (b*D*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa\int x\,dx+\frac{Bax^3}{3}+\frac{Cbx^6}{6}+\frac{Dbx^7}{7}+x^5\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*Integral(x, x) + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*
(B*b/5 + D*a/5) + x**4*(A*b/4 + C*a/4)
```

Mathematica [A] time = 0.0193494, size = 65, normalized size = 1.

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^
5)/5 + (b*C*x^6)/6 + (b*D*x^7)/7
```

Maple [A] time = 0.002, size = 54, normalized size = 0.8

$$\frac{aAx^2}{2}+\frac{aBx^3}{3}+\frac{(Ab+aC)\,x^4}{4}+\frac{(Bb+aD)\,x^5}{5}+\frac{bCx^6}{6}+\frac{bDx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b
*C*x^6+1/7*b*D*x^7
```

Maxima [A] time = 1.35071, size = 72, normalized size = 1.11

$$\frac{1}{7}Dbx^7 + \frac{1}{6}Cbx^6 + \frac{1}{5}(Da + Bb)x^5 + \frac{1}{3}Bax^3 + \frac{1}{4}(Ca + Ab)x^4 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1
/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 0.22847, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7bD + \frac{1}{6}x^6bC + \frac{1}{5}x^5aD + \frac{1}{5}x^5bB + \frac{1}{4}x^4aC + \frac{1}{4}x^4bA + \frac{1}{3}x^3aB + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*D + 1/6*x^6*b*C + 1/5*x^5*a*D + 1/5*x^5*b*B + 1/4*x^4*a
*C + 1/4*x^4*b*A + 1/3*x^3*a*B + 1/2*x^2*a*A
```

Sympy [A] time = 0.052273, size = 60, normalized size = 0.92

$$\frac{Aax^2}{2} + \frac{Bax^3}{3} + \frac{Cbx^6}{6} + \frac{Dbx^7}{7} + x^5\left(\frac{Bb}{5} + \frac{Da}{5}\right) + x^4\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 +
 D*a/5) + x**4*(A*b/4 + C*a/4)
```

GIAC/XCAS [A] time = 0.211772, size = 77, normalized size = 1.18

$$\frac{1}{7}Dbx^7 + \frac{1}{6}Cbx^6 + \frac{1}{5}Dax^5 + \frac{1}{5}Bbx^5 + \frac{1}{4}Cax^4 + \frac{1}{4}Abx^4 + \frac{1}{3}Bax^3 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)*x,x, algorithm="giac")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x
^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.65 $\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 +
 (b*C*x^5)/5 + (b*D*x^6)/6
```

Rubi [A] time = 0.0906429, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 +
 (b*C*x^5)/5 + (b*D*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Ba\int x\,dx+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+a\int A\,dx+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] B*a*Integral(x, x) + C*b*x**5/5 + D*b*x**6/6 + a*Integral(A, x) +
 x**4*(B*b/4 + D*a/4) + x**3*(A*b/3 + C*a/3)
```

Mathematica [A] time = 0.0167431, size = 60, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 +
 (b*C*x^5)/5 + (b*D*x^6)/6
```

Maple [A] time = 0.003, size = 51, normalized size = 0.9

$$aAx+\frac{Bax^2}{2}+\frac{(Ab+aC)\,x^3}{3}+\frac{(Bb+aD)\,x^4}{4}+\frac{bCx^5}{5}+\frac{bDx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5
+1/6*b*D*x^6
```

Maxima [A] time = 1.34506, size = 68, normalized size = 1.13

$$\frac{1}{6}Dbx^6 + \frac{1}{5}Cbx^5 + \frac{1}{4}(Da + Bb)x^4 + \frac{1}{2}Bax^2 + \frac{1}{3}(Ca + Ab)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1
/3*(C*a + A*b)*x^3 + A*a*x
```

Fricas [A] time = 0.228372, size = 1, normalized size = 0.02

$$\frac{1}{6}x^6bD + \frac{1}{5}x^5bC + \frac{1}{4}x^4aD + \frac{1}{4}x^4bB + \frac{1}{3}x^3aC + \frac{1}{3}x^3bA + \frac{1}{2}x^2aB + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b*D + 1/5*x^5*b*C + 1/4*x^4*a*D + 1/4*x^4*b*B + 1/3*x^3*a
*C + 1/3*x^3*b*A + 1/2*x^2*a*B + x*a*A
```

Sympy [A] time = 0.050122, size = 56, normalized size = 0.93

$$Aax + \frac{Bax^2}{2} + \frac{Cbx^5}{5} + \frac{Dbx^6}{6} + x^4\left(\frac{Bb}{4} + \frac{Da}{4}\right) + x^3\left(\frac{Ab}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/
4) + x**3*(A*b/3 + C*a/3)
```

GIAC/XCAS [A] time = 0.21818, size = 73, normalized size = 1.22

$$\frac{1}{6}Dbx^6 + \frac{1}{5}Cbx^5 + \frac{1}{4}Dax^4 + \frac{1}{4}Bbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Abx^3 + \frac{1}{2}Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x
^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 +
 (b*D*x^5)/5 + a*A*Log[x]
```

Rubi [A] time = 0.080597, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 +
 (b*D*x^5)/5 + a*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa\log(x)+\frac{Cbx^4}{4}+\frac{Dbx^5}{5}+a\int B\,dx+x^3\left(\frac{Bb}{3}+\frac{Da}{3}\right)+(Ab+Ca)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + C*b*x**4/4 + D*b*x**5/5 + a*Integral(B, x) + x**3*(B
*b/3 + D*a/3) + (A*b + C*a)*Integral(x, x)
```

Mathematica [A] time = 0.0298128, size = 56, normalized size = 1.

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 +
 (b*D*x^5)/5 + a*A*Log[x]
```

Maple [A] time = 0.006, size = 53, normalized size = 1.

$$\frac{bDx^5}{5}+\frac{bCx^4}{4}+\frac{bBx^3}{3}+\frac{Dx^3a}{3}+\frac{Ax^2b}{2}+\frac{Cx^2a}{2}+Bxa+aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/5*b*D*x^5+1/4*b*C*x^4+1/3*b*B*x^3+1/3*D*x^3*a+1/2*A*x^2*b+1/2*C
*x^2*a+B*x*a+a*A*ln(x)
```

Maxima [A] time = 1.34488, size = 65, normalized size = 1.16

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}(Da + Bb)x^3 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x,x, algorithm="maxima")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*
a + A*b)*x^2 + A*a*log(x)
```

Fricas [A] time = 0.246517, size = 65, normalized size = 1.16

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}(Da + Bb)x^3 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x,x, algorithm="fricas")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*
a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] time = 0.596622, size = 54, normalized size = 0.96

$$Aa\log(x) + Bax + \frac{Cbx^4}{4} + \frac{Dbx^5}{5} + x^3\left(\frac{Bb}{3} + \frac{Da}{3}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/
3) + x**2*(A*b/2 + C*a/2)
```

GIAC/XCAS [A] time = 0.225917, size = 72, normalized size = 1.29

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Dax^3 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x
^2 + 1/2*A*b*x^2 + B*a*x + A*a*ln(abs(x))
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=54

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 +
(b*D*x^4)/4 + a*B*Log[x]
```

Rubi [A] time = 0.0991602, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 +
(b*D*x^4)/4 + a*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{x}+Ba\log(x)+\frac{Cbx^3}{3}+\frac{Dbx^4}{4}+(Bb+Da)\int x\,dx+\frac{(Ab+Ca)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + (B*b + D*a)*Integ
ral(x, x) + (A*b + C*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.0592605, size = 54, normalized size = 1.

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 +
(b*D*x^4)/4 + a*B*Log[x]
```

Maple [A] time = 0.01, size = 50, normalized size = 0.9

$$\frac{bDx^4}{4}+\frac{bCx^3}{3}+\frac{Bbx^2}{2}+\frac{Dx^2a}{2}+Axb+Cxa+aB\ln(x)-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/4*b*D*x^4+1/3*b*C*x^3+1/2*B*b*x^2+1/2*D*x^2*a+A*x*b+C*x*a+a*B*l
n(x)-a*A/x
```

Maxima [A] time = 1.34712, size = 65, normalized size = 1.2

$$\frac{1}{4}\,Dbx^4 + \frac{1}{3}\,Cbx^3 + \frac{1}{2}\,(Da + Bb)x^2 + Ba\log\left(x\right) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C
*a + A*b)*x - A*a/x
```

Fricas [A] time = 0.256657, size = 74, normalized size = 1.37

$$\frac{3\,Dbx^5 + 4\,Cbx^4 + 6\,(Da + Bb)x^3 + 12\,Bax\log\left(x\right) + 12\,(Ca + Ab)x^2 - 12\,Aa}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x)
 + 12*(C*a + A*b)*x^2 - 12*A*a)/x
```

Sympy [A] time = 0.622533, size = 49, normalized size = 0.91

$$-\frac{Aa}{x} + Ba\log\left(x\right) + \frac{Cbx^3}{3} + \frac{Dbx^4}{4} + x^2\left(\frac{Bb}{2} + \frac{Da}{2}\right) + x\left(Ab + Ca\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a
/2) + x*(A*b + C*a)
```

GIAC/XCAS [A] time = 0.214611, size = 68, normalized size = 1.26

$$\frac{1}{4}\,Dbx^4 + \frac{1}{3}\,Cbx^3 + \frac{1}{2}\,Dax^2 + \frac{1}{2}\,Bbx^2 + Cax + Abx + Ba\ln\left(|x|\right) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A
*b*x + B*a*ln(abs(x)) - A*a/x
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal. Leaf size=54

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3
)/3 + (A*b + a*C)*Log[x]
```

Rubi [A] time = 0.0990235, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3
)/3 + (A*b + a*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{2x^2}-\frac{Ba}{x}+Cb\int x\,dx+\frac{Dbx^3}{3}+(Ab+Ca)\log(x)+\frac{(Bb+Da)\int B\,dx}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) - B*a/x + C*b*Integral(x, x) + D*b*x**3/3 + (A*b +
C*a)*log(x) + (B*b + D*a)*Integral(B, x)/B
```

Mathematica [A] time = 0.0524996, size = 51, normalized size = 0.94

$$\log(x)(aC+Ab)-\frac{a\left(A+2Bx-2Dx^3\right)}{2x^2}+\frac{1}{6}bx\left(6B+3Cx+2Dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^
2) + (A*b + a*C)*Log[x]
```

Maple [A] time = 0.009, size = 48, normalized size = 0.9

$$\frac{bDx^3}{3}+\frac{bCx^2}{2}+bBx+Dxa+A\ln(x)\,b+C\ln(x)\,a-\frac{Aa}{2\,x^2}-\frac{Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/3*b*D*x^3+1/2*b*C*x^2+b*B*x+D*x*a+A*ln(x)*b+C*ln(x)*a-1/2*a*A/x
^2-a*B/x
```

Maxima [A] time = 1.34612, size = 65, normalized size = 1.2

$$\frac{1}{3}Dbx^3 + \frac{1}{2}Cbx^2 + (Da + Bb)x + (Ca + Ab)\log(x) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) -
1/2*(2*B*a*x + A*a)/x^2
```

Fricas [A] time = 0.263849, size = 74, normalized size = 1.37

$$\frac{2Dbx^5 + 3Cbx^4 + 6(Da + Bb)x^3 + 6(Ca + Ab)x^2\log(x) - 6Bax - 3Aa}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^
2*log(x) - 6*B*a*x - 3*A*a)/x^2
```

Sympy [A] time = 0.850776, size = 49, normalized size = 0.91

$$\frac{Cbx^2}{2} + \frac{Dbx^3}{3} + x(Bb + Da) + (Ab + Ca)\log(x) - \frac{Aa + 2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) - (A
*a + 2*B*a*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.214842, size = 65, normalized size = 1.2

$$\frac{1}{3}Dbx^3 + \frac{1}{2}Cbx^2 + Dax + Bbx + (Ca + Ab)\ln(|x|) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*ln(abs(x)
) - 1/2*(2*B*a*x + A*a)/x^2
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2
)/2 + (b*B + a*D)*Log[x]
```

Rubi [A] time = 0.100487, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2
)/2 + (b*B + a*D)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{3x^3}-\frac{Ba}{2x^2}+Db\int x\,dx+b\int C\,dx+(Bb+Da)\log(x)-\frac{Ab+Ca}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] -A*a/(3*x**3) - B*a/(2*x**2) + D*b*Integral(x, x) + b*Integral(C,
 x) + (B*b + D*a)*log(x) - (A*b + C*a)/x
```

Mathematica [A] time = 0.0327333, size = 55, normalized size = 1.02

$$\frac{-aC-Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*
x^2)/2 + (b*B + a*D)*Log[x]
```

Maple [A] time = 0.01, size = 51, normalized size = 0.9

$$\frac{bDx^2}{2}+bCx+Bb\ln(x)+D\ln(x)a-\frac{Aa}{3x^3}-\frac{Ba}{2x^2}-\frac{Ab}{x}-\frac{aC}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/2*b*D*x^2+b*C*x+B*b*ln(x)+D*ln(x)*a-1/3*a*A/x^3-1/2*a*B/x^2-1/x
*A*b-1/x*a*C
```

Maxima [A] time = 1.34776, size = 66, normalized size = 1.22

$$\frac{1}{2}Dbx^2 + Cbx + (Da + Bb)\log(x) - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a
+ A*b)*x^2 + 2*A*a)/x^3
```

Fricas [A] time = 0.223168, size = 74, normalized size = 1.37

$$\frac{3Dbx^5 + 6Cbx^4 + 6(Da + Bb)x^3\log(x) - 3Bax - 6(Ca + Ab)x^2 - 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x -
 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] time = 1.68682, size = 53, normalized size = 0.98

$$Cbx + \frac{Dbx^2}{2} + (Bb + Da)\log(x) - \frac{2Aa + 3Bax + x^2(6Ab + 6Ca)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) - (2*A*a + 3*B*a*x + x**2
*(6*A*b + 6*C*a))/(6*x**3)
```

GIAC/XCAS [A] time = 0.211784, size = 68, normalized size = 1.26

$$\frac{1}{2}Dbx^2 + Cbx + (Da + Bb)\ln(|x|) - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*ln(abs(x)) - 1/6*(3*B*a*x + 6*(
C*a + A*b)*x^2 + 2*A*a)/x^3
```

3.70 $\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=109

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b
*B + a*D)*x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9
)/9 + (b^2*C*x^10)/10 + (b^2*D*x^11)/11
```

Rubi [A] time = 0.258931, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b
*B + a*D)*x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9
)/9 + (b^2*C*x^10)/10 + (b^2*D*x^11)/11
```

Rubi in Sympy [A] time = 35.6268, size = 100, normalized size = 0.92

$$\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+\frac{ax^7\left(2Bb+Da\right)}{7}+\frac{ax^6\left(2Ab+Ca\right)}{6}+\frac{bx^9\left(Bb+2Da\right)}{9}+\frac{bx^8\left(Ab+2Ca\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11
 + a*x**7*(2*B*b + D*a)/7 + a*x**6*(2*A*b + C*a)/6 + b*x**9*(B*b
+ 2*D*a)/9 + b*x**8*(A*b + 2*C*a)/8
```

Mathematica [A] time = 0.0853129, size = 98, normalized size = 0.9

$$a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))+\frac{b^2x^8\left(495A+4x\left(110B+99Cx+90Dx^2\right)\right)}{3960}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(
495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A +
 x*(72*B + 7*x*(9*C + 8*D*x))))/252
```

Maple [A] time = 0.003, size = 102, normalized size = 0.9

$$\frac{b^2 D x^{11}}{11}+\frac{b^2 C x^{10}}{10}+\frac{\left(b^2 B+2 abD\right) x^9}{9}+\frac{\left(b^2 A+2 abC\right) x^8}{8}+\frac{\left(2 abB+a^2 D\right) x^7}{7}+\frac{\left(2 abA+a^2 C\right) x^6}{6}+\frac{a^2 B x^5}{5}+\frac{a^2 A x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^
2+2*C*a*b)*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/
5*a^2*B*x^5+1/4*a^2*A*x^4
```

Maxima [A] time = 1.34693, size = 136, normalized size = 1.25

$$\frac{1}{11} Db^2 x^{11}+\frac{1}{10} Cb^2 x^{10}+\frac{1}{9}\left(2 Dab+Bb^2\right) x^9+\frac{1}{8}\left(2 Cab+Ab^2\right) x^8+\frac{1}{5} Ba^2 x^5+\frac{1}{7}\left(Da^2+2 Bab\right) x^7+\frac{1}{4} Aa^2 x^4+\frac{1}{6}\left(Ca^2+2 Aab\right) x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1
/8*(2*C*a*b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*
x^7 + 1/4*A*a^2*x^4 + 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Fricas [A] time = 0.205721, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}b^2D+\frac{1}{10}x^{10}b^2C+\frac{2}{9}x^9baD+\frac{1}{9}x^9b^2B+\frac{1}{4}x^8baC+\frac{1}{8}x^8b^2A+\frac{1}{7}x^7a^2D+\frac{2}{7}x^7baB+\frac{1}{6}x^6a^2C+\frac{1}{3}x^6baA+\frac{1}{5}x^5a^2B+\frac{1}{4}x^4a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2*D + 1/10*x^10*b^2*C + 2/9*x^9*b*a*D + 1/9*x^9*b^2*B
 + 1/4*x^8*b*a*C + 1/8*x^8*b^2*A + 1/7*x^7*a^2*D + 2/7*x^7*b*a*B
+ 1/6*x^6*a^2*C + 1/3*x^6*b*a*A + 1/5*x^5*a^2*B + 1/4*x^4*a^2*A
```

Sympy [A] time = 0.073911, size = 110, normalized size = 1.01

$$\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11
 + x**9*(B*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7
*(2*B*a*b/7 + D*a**2/7) + x**6*(A*a*b/3 + C*a**2/6)
```

GIAC/XCAS [A] time = 0.208021, size = 142, normalized size = 1.3

$$\frac{1}{11} Db^2x^{11} + \frac{1}{10} Cb^2x^{10} + \frac{2}{9} Dabx^9 + \frac{1}{9} Bb^2x^9 + \frac{1}{4} Cabx^8 + \frac{1}{8} Ab^2x^8 + \frac{1}{7} Da^2x^7 + \frac{2}{7} Babx^7 + \frac{1}{6} Ca^2x^6 + \frac{1}{3} Aabx^6 + \frac{1}{5} Ba^2x^5 + \frac{1}{4} Aa^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9
 + 1/4*C*a*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7
+ 1/6*C*a^2*x^6 + 1/3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

3.71 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\, dx$

Optimal. Leaf size=109

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b
*B + a*D)*x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8
)/8 + (b^2*C*x^9)/9 + (b^2*D*x^10)/10
```

Rubi [A] time = 0.258367, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b
*B + a*D)*x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8
)/8 + (b^2*C*x^9)/9 + (b^2*D*x^10)/10
```

Rubi in Sympy [A] time = 39.589, size = 100, normalized size = 0.92

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Cb^2x^9}{9}+\frac{Db^2x^{10}}{10}+\frac{ax^6\left(2Bb+Da\right)}{6}+\frac{ax^5\left(2Ab+Ca\right)}{5}+\frac{bx^8\left(Bb+2Da\right)}{8}+\frac{bx^7\left(Ab+2Ca\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 +
 a*x**6*(2*B*b + D*a)/6 + a*x**5*(2*A*b + C*a)/5 + b*x**8*(B*b +
2*D*a)/8 + b*x**7*(A*b + 2*C*a)/7
```

Mathematica [A] time = 0.0970729, size = 92, normalized size = 0.84

$$\frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7x(45B+4x(10C+9Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(16
8*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*
B + 4*x*(10*C + 9*D*x))))/2520
```

Maple [A] time = 0.002, size = 102, normalized size = 0.9

$$\frac{b^2Dx^{10}}{10}+\frac{b^2Cx^9}{9}+\frac{(b^2B+2\,abD)\,x^8}{8}+\frac{(b^2A+2\,abC)\,x^7}{7}+\frac{(2\,abB+a^2D)\,x^6}{6}+\frac{(2\,abA+a^2C)\,x^5}{5}+\frac{a^2Bx^4}{4}+\frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+
2*C*a*b)*x^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*
a^2*B*x^4+1/3*a^2*A*x^3
```

Maxima [A] time = 1.33383, size = 136, normalized size = 1.25

$$\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)\,x^8+\frac{1}{7}\,(2\,Cab+Ab^2)\,x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,(Da^2+2\,Bab)\,x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7
*(2*C*a*b + A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^
6 + 1/3*A*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A] time = 0.203419, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}b^2D+\frac{1}{9}x^9b^2C+\frac{1}{4}x^8baD+\frac{1}{8}x^8b^2B+\frac{2}{7}x^7baC+\frac{1}{7}x^7b^2A+\frac{1}{6}x^6a^2D+\frac{1}{3}x^6baB+\frac{1}{5}x^5a^2C+\frac{2}{5}x^5baA+\frac{1}{4}x^4a^2B+\frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2*D + 1/9*x^9*b^2*C + 1/4*x^8*b*a*D + 1/8*x^8*b^2*B +
 2/7*x^7*b*a*C + 1/7*x^7*b^2*A + 1/6*x^6*a^2*D + 1/3*x^6*b*a*B +
1/5*x^5*a^2*C + 2/5*x^5*b*a*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.073936, size = 110, normalized size = 1.01

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Cb^2x^9}{9}+\frac{Db^2x^{10}}{10}+x^8\left(\frac{Bb^2}{8}+\frac{Dab}{4}\right)+x^7\left(\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^6\left(\frac{Bab}{3}+\frac{Da^2}{6}\right)+x^5\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 +
 x**8*(B*b**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(
```

```
B*a*b/3 + D*a**2/6) + x**5*(2*A*a*b/5 + C*a**2/5)
```

GIAC/XCAS [A] time = 0.239381, size = 142, normalized size = 1.3

$$\frac{1}{10}Db^2x^{10} + \frac{1}{9}Cb^2x^9 + \frac{1}{4}Dabx^8 + \frac{1}{8}Bb^2x^8 + \frac{2}{7}Cabx^7 + \frac{1}{7}Ab^2x^7 + \frac{1}{6}Da^2x^6 + \frac{1}{3}Babx^6 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{4}Ba^2x^4 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 +
 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 +
1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.72 $\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*
x^6)/3 + (b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9
+ (A*(a + b*x^2)^3)/(6*b)
```

Rubi [A] time = 0.316991, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*
x^6)/3 + (b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9
+ (A*(a + b*x^2)^3)/(6*b)
```

Rubi in Sympy [A] time = 40.1159, size = 94, normalized size = 0.9

$$\frac{A\left(a+bx^2\right)^3}{6b}+\frac{Ba^2x^3}{3}+\frac{Ca^2x^4}{4}+\frac{Cabx^6}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+\frac{ax^5\left(2Bb+Da\right)}{5}+\frac{bx^7\left(Bb+2Da\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*(a + b*x**2)**3/(6*b) + B*a**2*x**3/3 + C*a**2*x**4/4 + C*a*b*x
**6/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + a*x**5*(2*B*b + D*a)/5 +
b*x**7*(B*b + 2*D*a)/7
```

Mathematica [A] time = 0.0961104, size = 92, normalized size = 0.88

$$\frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+x(72B+7x(9C+8Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(1
05*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*
B + 7*x*(9*C + 8*D*x))))/2520
```

Maple [A] time = 0.003, size = 102, normalized size = 1.

$$\frac{b^2Dx^9}{9}+\frac{b^2Cx^8}{8}+\frac{\left(b^2B+2\,abD\right)x^7}{7}+\frac{\left(b^2A+2\,abC\right)x^6}{6}+\frac{\left(2\,abB+a^2D\right)x^5}{5}+\frac{\left(2\,abA+a^2C\right)x^4}{4}+\frac{a^2Bx^3}{3}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*
C*a*b)*x^6+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^
2*B*x^3+1/2*a^2*A*x^2
```

Maxima [A] time = 1.34595, size = 136, normalized size = 1.31

$$\frac{1}{9}Db^2x^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{7}\left(2Dab + Bb^2\right)x^7 + \frac{1}{6}\left(2Cab + Ab^2\right)x^6 + \frac{1}{3}Ba^2x^3 + \frac{1}{5}\left(Da^2 + 2Bab\right)x^5 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}\left(Ca^2 + 2Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(
2*C*a*b + A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5
+ 1/2*A*a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A] time = 0.204869, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9b^2D + \frac{1}{8}x^8b^2C + \frac{2}{7}x^7baD + \frac{1}{7}x^7b^2B + \frac{1}{3}x^6baC + \frac{1}{6}x^6b^2A + \frac{1}{5}x^5a^2D + \frac{2}{5}x^5baB + \frac{1}{4}x^4a^2C + \frac{1}{2}x^4baA + \frac{1}{3}x^3a^2B + \frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2*D + 1/8*x^8*b^2*C + 2/7*x^7*b*a*D + 1/7*x^7*b^2*B + 1
/3*x^6*b*a*C + 1/6*x^6*b^2*A + 1/5*x^5*a^2*D + 2/5*x^5*b*a*B + 1/
4*x^4*a^2*C + 1/2*x^4*b*a*A + 1/3*x^3*a^2*B + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.073285, size = 110, normalized size = 1.06

$$\frac{Aa^2x^2}{2} + \frac{Ba^2x^3}{3} + \frac{Cb^2x^8}{8} + \frac{Db^2x^9}{9} + x^7\left(\frac{Bb^2}{7} + \frac{2Dab}{7}\right) + x^6\left(\frac{Ab^2}{6} + \frac{Cab}{3}\right) + x^5\left(\frac{2Bab}{5} + \frac{Da^2}{5}\right) + x^4\left(\frac{Aab}{2} + \frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x
**7*(B*b**2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*
B*a*b/5 + D*a**2/5) + x**4*(A*a*b/2 + C*a**2/4)
```

GIAC/XCAS [A] time = 0.220532, size = 142, normalized size = 1.37

$$\frac{1}{9}Db^2x^9 + \frac{1}{8}Cb^2x^8 + \frac{2}{7}Dabx^7 + \frac{1}{7}Bb^2x^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{5}Da^2x^5 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2*x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1
/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/
4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.73 $\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx$

Optimal. Leaf size=99

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B(a+bx^2)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a
*C)*x^5)/5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(
a + b*x^2)^3)/(6*b)
```

Rubi [A] time = 0.182586, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B(a+bx^2)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a
*C)*x^5)/5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(
a + b*x^2)^3)/(6*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B(a+bx^2)^3}{6b}+\frac{Cb^2x^7}{7}+\frac{Da^2x^4}{4}+\frac{Dabx^6}{3}+\frac{Db^2x^8}{8}+a^2\int A\,dx+\frac{ax^3(2Ab+Ca)}{3}+\frac{bx^5(Ab+2Ca)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] B*(a + b*x**2)**3/(6*b) + C*b**2*x**7/7 + D*a**2*x**4/4 + D*a*b*x
**6/3 + D*b**2*x**8/8 + a**2*Integral(A, x) + a*x**3*(2*A*b + C*a
)/3 + b*x**5*(A*b + 2*C*a)/5
```

Mathematica [A] time = 0.0839187, size = 88, normalized size = 0.89

$$\frac{1}{840}\left(70a^2x(12A+x(6B+x(4C+3Dx)))+28abx^3(20A+x(15B+2x(6C+5Dx)))+b^2x^5(168A+5x(28B+3x(8C+7Dx)))\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A +
 x*(15*B + 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x
*(8*C + 7*D*x))))/840
```

Maple [A] time = 0.001, size = 99, normalized size = 1.

$$\frac{b^2Dx^8}{8}+\frac{b^2Cx^7}{7}+\frac{\left(b^2B+2\,abD\right)x^6}{6}+\frac{\left(b^2A+2\,abC\right)x^5}{5}+\frac{\left(2\,abB+a^2D\right)x^4}{4}+\frac{\left(2\,abA+a^2C\right)x^3}{3}+\frac{Bx^2a^2}{2}+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*
C*a*b)*x^5+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*
x^2*a^2+a^2*A*x
```

Maxima [A] time = 1.33552, size = 132, normalized size = 1.33

$$\frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(
2*C*a*b + A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4
+ A*a^2*x + 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Fricas [A] time = 0.200348, size = 1, normalized size = 0.01

$$\frac{1}{8}x^8b^2D+\frac{1}{7}x^7b^2C+\frac{1}{3}x^6baD+\frac{1}{6}x^6b^2B+\frac{2}{5}x^5baC+\frac{1}{5}x^5b^2A+\frac{1}{4}x^4a^2D+\frac{1}{2}x^4baB+\frac{1}{3}x^3a^2C+\frac{2}{3}x^3baA+\frac{1}{2}x^2a^2B+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2*D + 1/7*x^7*b^2*C + 1/3*x^6*b*a*D + 1/6*x^6*b^2*B + 2
/5*x^5*b*a*C + 1/5*x^5*b^2*A + 1/4*x^4*a^2*D + 1/2*x^4*b*a*B + 1/
3*x^3*a^2*C + 2/3*x^3*b*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

Sympy [A] time = 0.071296, size = 107, normalized size = 1.08

$$Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8}+x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(
B*b**2/6 + D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2
```

```
+ D*a**2/4) + x**3*(2*A*a*b/3 + C*a**2/3)
```

GIAC/XCAS [A] time = 0.221864, size = 138, normalized size = 1.39

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{3}Dabx^6 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{4}Da^2x^4 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2
/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/
3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.74 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=92

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (
b*(b*B + 2*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b)
+ a^2*A*Log[x]
```

Rubi [A] time = 0.152401, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (
b*(b*B + 2*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b)
+ a^2*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa^2\log(x)+\frac{Cb^2x^6}{6}+\frac{Db^2x^7}{7}+a^2\int B\,dx+\frac{ax^3(2Bb+Da)}{3}+a(2Ab+Ca)\int x\,dx+\frac{bx^5(Bb+2Da)}{5}+\frac{bx^4(Ab+2Ca)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + C*b**2*x**6/6 + D*b**2*x**7/7 + a**2*Integral(B,
x) + a*x**3*(2*B*b + D*a)/3 + a*(2*A*b + C*a)*Integral(x, x) + b*
x**5*(B*b + 2*D*a)/5 + b*x**4*(A*b + 2*C*a)/4
```

Mathematica [A] time = 0.0963594, size = 88, normalized size = 0.96

$$\frac{1}{420}x\left(70a^2(6B+x(3C+2Dx))+14abx(30A+x(20B+3x(5C+4Dx)))+b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right)+a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3
*x*(5*C + 4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*
x)))))/420 + a^2*A*Log[x]
```

Maple [A] time = 0.005, size = 100, normalized size = 1.1

$$\frac{b^2Dx^7}{7}+\frac{Cb^2x^6}{6}+\frac{b^2Bx^5}{5}+\frac{2\,Dx^5ab}{5}+\frac{Ab^2x^4}{4}+\frac{Cx^4ab}{2}+\frac{2\,Bx^3ab}{3}+\frac{Dx^3a^2}{3}+aAbx^2+\frac{Cx^2a^2}{2}+Bxa^2+a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/7*b^2*D*x^7+1/6*C*b^2*x^6+1/5*b^2*B*x^5+2/5*D*x^5*a*b+1/4*A*b^2
*x^4+1/2*C*x^4*a*b+2/3*B*x^3*a*b+1/3*D*x^3*a^2+a*A*b*x^2+1/2*C*x^
2*a^2+B*x*a^2+a^2*A*ln(x)
```

Maxima [A] time = 1.34855, size = 130, normalized size = 1.41

$$\frac{1}{7}\,Db^2x^7+\frac{1}{6}\,Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(
2*C*a*b + A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^
2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2
```

Fricas [A] time = 0.221195, size = 130, normalized size = 1.41

$$\frac{1}{7}\,Db^2x^7+\frac{1}{6}\,Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(
2*C*a*b + A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^
2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A] time = 0.721516, size = 104, normalized size = 1.13

$$Aa^2\log(x)+Ba^2x+\frac{Cb^2x^6}{6}+\frac{Db^2x^7}{7}+x^5\left(\frac{Bb^2}{5}+\frac{2Dab}{5}\right)+x^4\left(\frac{Ab^2}{4}+\frac{Cab}{2}\right)+x^3\left(\frac{2Bab}{3}+\frac{Da^2}{3}\right)+x^2\left(Aab+\frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(
B*b**2/5 + 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b
```

```
/3 + D*a**2/3) + x**2*(A*a*b + C*a**2/2)
```

GIAC/XCAS [A] time = 0.230497, size = 135, normalized size = 1.47

$$\frac{1}{7}\,Db^2x^7 + \frac{1}{6}\,Cb^2x^6 + \frac{2}{5}\,Dabx^5 + \frac{1}{5}\,Bb^2x^5 + \frac{1}{2}\,Cabx^4 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{3}\,Da^2x^3 + \frac{2}{3}\,Babx^3 + \frac{1}{2}\,Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1
/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/
2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*ln(abs(x))
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2} dx$

Optimal. Leaf size=90

$$-\frac{a^2A}{x} + a^2B\log(x) + \frac{1}{3}bx^3(2aC + Ab) + ax(aC + 2Ab) + abBx^2 + \frac{D\left(a + bx^2\right)^3}{6b} + \frac{1}{4}b^2Bx^4 + \frac{1}{5}b^2Cx^5$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x
^3)/3 + (b^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) +
 a^2*B*Log[x]
```

Rubi [A] time = 0.197665, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{a^2A}{x} + a^2B\log(x) + \frac{1}{3}bx^3(2aC + Ab) + ax(aC + 2Ab) + abBx^2 + \frac{D\left(a + bx^2\right)^3}{6b} + \frac{1}{4}b^2Bx^4 + \frac{1}{5}b^2Cx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x
^3)/3 + (b^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) +
 a^2*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{x} + Ba^2\log(x) + \frac{Cb^2x^5}{5} + \frac{Db^2x^6}{6} + a(2Bb + Da)\int x\,dx + \frac{bx^4(Bb + 2Da)}{4} + \frac{bx^3(Ab + 2Ca)}{3} + \frac{a(2Ab + Ca)\int C\,dx}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + a*(2*
B*b + D*a)*Integral(x, x) + b*x**4*(B*b + 2*D*a)/4 + b*x**3*(A*b
+ 2*C*a)/3 + a*(2*A*b + C*a)*Integral(C, x)/C
```

Mathematica [A] time = 0.138605, size = 88, normalized size = 0.98

$$a^2\left(-\frac{A}{x} + Cx + \frac{Dx^2}{2}\right) + a^2B\log(x) + \frac{1}{6}abx(12A + x(6B + x(4C + 3Dx))) + \frac{1}{60}b^2x^3(20A + x(15B + 2x(6C + 5Dx)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C +
 3*D*x))))/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60
 + a^2*B*Log[x]
```

Maple [A] time = 0.01, size = 98, normalized size = 1.1

$$\frac{Db^2x^6}{6}+\frac{b^2Cx^5}{5}+\frac{b^2Bx^4}{4}+\frac{Dx^4ab}{2}+\frac{Ax^3b^2}{3}+\frac{2\,Cx^3ab}{3}+Bx^2ab+\frac{Dx^2a^2}{2}+2\,Axab+Cxa^2+a^2B\ln\left(x\right)-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/6*D*b^2*x^6+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*x^4*a*b+1/3*A*x^3
*b^2+2/3*C*x^3*a*b+B*x^2*a*b+1/2*D*x^2*a^2+2*A*x*a*b+C*x*a^2+a^2*
B*ln(x)-a^2*A/x
```

Maxima [A] time = 1.35618, size = 130, normalized size = 1.44

$$\frac{1}{6}\,Db^2x^6+\frac{1}{5}\,Cb^2x^5+\frac{1}{4}\,\left(2\,Dab+Bb^2\right)x^4+\frac{1}{3}\,\left(2\,Cab+Ab^2\right)x^3 + Ba^2\log\left(x\right)+\frac{1}{2}\,\left(Da^2+2\,Bab\right)x^2-\frac{Aa^2}{x}+\left(Ca^2+2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(
2*C*a*b + A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 -
 A*a^2/x + (C*a^2 + 2*A*a*b)*x
```

Fricas [A] time = 0.222919, size = 139, normalized size = 1.54

$$\frac{10\,Db^2x^7+12\,Cb^2x^6+15\,\left(2\,Dab+Bb^2\right)x^5+20\,\left(2\,Cab+Ab^2\right)x^4+60\,Ba^2x\log\left(x\right)+30\,\left(Da^2+2\,Bab\right)x^3-60\,Aa^2+60\,(Ca}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20
*(2*C*a*b + A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)
*x^3 - 60*A*a^2 + 60*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A] time = 0.758291, size = 99, normalized size = 1.1

$$-\frac{Aa^2}{x}+Ba^2\log\left(x\right)+\frac{Cb^2x^5}{5}+\frac{Db^2x^6}{6}+x^4\left(\frac{Bb^2}{4}+\frac{Dab}{2}\right) + x^3\left(\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x^2\left(Bab+\frac{Da^2}{2}\right)+x\left(2Aab+Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*
(B*b**2/4 + D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b
+ D*a**2/2) + x*(2*A*a*b + C*a**2)
```

GIAC/XCAS [A] time = 0.224227, size = 132, normalized size = 1.47

$$\frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{2}\,Dabx^4 + \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Cabx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\ln\left(|x|\right) - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2
/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*
x + 2*A*a*b*x + B*a^2*ln(abs(x)) - A*a^2/x
```

3.76 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*
C)*x^2)/2 + (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)
/5 + a*(2*A*b + a*C)*Log[x]
```

Rubi [A] time = 0.186014, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*
C)*x^2)/2 + (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)
/5 + a*(2*A*b + a*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{2x^2}-\frac{Ba^2}{x}+\frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a\left(2Ab+Ca\right)\log\left(x\right)+\frac{bx^3\left(Bb+2Da\right)}{3}+b\left(Ab+2Ca\right)\int x\,dx+\frac{a\left(2Bb+Da\right)\int D\,dx}{D}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) - B*a**2/x + C*b**2*x**4/4 + D*b**2*x**5/5 + a*(
2*A*b + C*a)*log(x) + b*x**3*(B*b + 2*D*a)/3 + b*(A*b + 2*C*a)*In
tegral(x, x) + a*(2*B*b + D*a)*Integral(D, x)/D
```

Mathematica [A] time = 0.0850998, size = 87, normalized size = 0.89

$$-\frac{a^2\left(A+2Bx-2Dx^3\right)}{2x^2}+a\log(x)(aC+2Ab)+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (a*b*x*(6*B + x*(3*C + 2*D
*x)))/3 + (b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*
(2*A*b + a*C)*Log[x]
```

Maple [A] time = 0.01, size = 97, normalized size = 1.

$$\frac{b^2Dx^5}{5}+\frac{b^2Cx^4}{4}+\frac{Bb^2x^3}{3}+\frac{2\,Dx^3ab}{3}+\frac{Ax^2b^2}{2}+Cx^2ab+2\,Bxab+Dxa^2+2\,A\ln\left(x\right)ab+C\ln\left(x\right)a^2-\frac{Aa^2}{2\,x^2}-\frac{a^2B}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*B*b^2*x^3+2/3*D*x^3*a*b+1/2*A*x^2
*b^2+C*x^2*a*b+2*B*x*a*b+D*x*a^2+2*A*ln(x)*a*b+C*ln(x)*a^2-1/2*a^
2*A/x^2-a^2*B/x
```

Maxima [A] time = 1.35124, size = 130, normalized size = 1.33

$$\frac{1}{5}\,Db^2x^5+\frac{1}{4}\,Cb^2x^4+\frac{1}{3}\left(2\,Dab+Bb^2\right)x^3+\frac{1}{2}\left(2\,Cab+Ab^2\right)x^2+\left(Da^2+2\,Bab\right)x+\left(Ca^2+2\,Aab\right)\log\left(x\right)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(
2*C*a*b + A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*lo
g(x) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

Fricas [A] time = 0.219041, size = 139, normalized size = 1.42

$$\frac{12\,Db^2x^7+15\,Cb^2x^6+20\left(2\,Dab+Bb^2\right)x^5+30\left(2\,Cab+Ab^2\right)x^4-60\,Ba^2x+60\left(Da^2+2\,Bab\right)x^3+60\left(Ca^2+2\,Aab\right)x^2\log}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30
*(2*C*a*b + A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 +
60*(C*a^2 + 2*A*a*b)*x^2*log(x) - 30*A*a^2)/x^2
```

Sympy [A] time = 1.02945, size = 99, normalized size = 1.01

$$\frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a\left(2Ab+Ca\right)\log\left(x\right)+x^3\left(\frac{Bb^2}{3}+\frac{2Dab}{3}\right)+x^2\left(\frac{Ab^2}{2}+Cab\right)+x\left(2Bab+Da^2\right)-\frac{Aa^2+2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*
b**2/3 + 2*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a*
*2) - (A*a**2 + 2*B*a**2*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.223473, size = 131, normalized size = 1.34

$$\frac{1}{5}Db^2x^5 + \frac{1}{4}Cb^2x^4 + \frac{2}{3}Dabx^3 + \frac{1}{3}Bb^2x^3 + Cabx^2 + \frac{1}{2}Ab^2x^2 + Da^2x + 2Babx + \left(Ca^2 + 2Aab\right)\ln\left(|x|\right) - \frac{2Ba^2x + Aa^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C
*a*b*x^2 + 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b
)*ln(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b
 + 2*a*C)*x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^
4)/4 + a*(2*b*B + a*D)*Log[x]
```

Rubi [A] time = 0.18807, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b
 + 2*a*C)*x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^
4)/4 + a*(2*b*B + a*D)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{3x^3}-\frac{Ba^2}{2x^2}+\frac{Cb^2x^3}{3}+\frac{Db^2x^4}{4}+a\left(2Bb+Da\right)\log\left(x\right)-\frac{a\left(2Ab+Ca\right)}{x}+b\left(Bb+2Da\right)\int x\,dx+\frac{b\left(Ab+2Ca\right)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) - B*a**2/(2*x**2) + C*b**2*x**3/3 + D*b**2*x**4/
4 + a*(2*B*b + D*a)*log(x) - a*(2*A*b + C*a)/x + b*(B*b + 2*D*a)*
Integral(x, x) + b*(A*b + 2*C*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.103423, size = 83, normalized size = 0.85

$$-\frac{a^2(2A+3x(B+2Cx))}{6x^3}-\frac{2aAb}{x}+a\log(x)(aD+2bB)+abx(2C+Dx)+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/
(6*x^3) + (b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*
B + a*D)*Log[x]
```

Maple [A] time = 0.011, size = 97, normalized size = 1.

$$\frac{b^2Dx^4}{4}+\frac{b^2Cx^3}{3}+\frac{Bb^2x^2}{2}+Dx^2ab+Axb^2+2\,Cxab+2\,B\ln(x)\,ab+D\ln(x)\,a^2-\frac{Aa^2}{3\,x^3}-\frac{a^2B}{2\,x^2}-2\,\frac{abA}{x}-\frac{a^2C}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/4*b^2*D*x^4+1/3*b^2*C*x^3+1/2*B*b^2*x^2+D*x^2*a*b+A*x*b^2+2*C*x
*a*b+2*B*ln(x)*a*b+D*ln(x)*a^2-1/3*a^2*A/x^3-1/2*a^2*B/x^2-2*a/x*
A*b-a^2/x*C
```

Maxima [A] time = 1.35465, size = 131, normalized size = 1.34

$$\frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + \frac{1}{2}\left(2\,Dab + Bb^2\right)x^2 + \left(2\,Cab + Ab^2\right)x + \left(Da^2 + 2\,Bab\right)\log\left(x\right) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\left(Ca^2 + 2\,Aab\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*
a*b + A*b^2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*
a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Fricas [A] time = 0.233908, size = 139, normalized size = 1.42

$$\frac{3\,Db^2x^7 + 4\,Cb^2x^6 + 6\left(2\,Dab + Bb^2\right)x^5 + 12\left(2\,Cab + Ab^2\right)x^4 + 12\left(Da^2 + 2\,Bab\right)x^3\log\left(x\right) - 6\,Ba^2x - 4\,Aa^2 - 12\,\left(Ca^2 + 2\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2
*C*a*b + A*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x
 - 4*A*a^2 - 12*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] time = 1.98604, size = 99, normalized size = 1.01

$$\frac{Cb^2x^3}{3} + \frac{Db^2x^4}{4} + a\left(2Bb + Da\right)\log\left(x\right) + x^2\left(\frac{Bb^2}{2} + Dab\right) + x\left(Ab^2 + 2Cab\right) - \frac{2Aa^2 + 3Ba^2x + x^2\left(12Aab + 6Ca^2\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*
b**2/2 + D*a*b) + x*(A*b**2 + 2*C*a*b) - (2*A*a**2 + 3*B*a**2*x +
 x**2*(12*A*a*b + 6*C*a**2))/(6*x**3)
```

GIAC/XCAS [A] time = 0.226316, size = 131, normalized size = 1.34

$$\frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + Dabx^2 + \frac{1}{2}Bb^2x^2 + 2\,Cabx + Ab^2x + \left(Da^2 + 2\,Bab\right)\ln\left(|x|\right) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\left(Ca^2 + 2\,Aab\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a
*b*x + A*b^2*x + (D*a^2 + 2*B*a*b)*ln(abs(x)) - 1/6*(3*B*a^2*x +
2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

3.78 $\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*
(3*b*B + a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*
D)*x^9)/3 + (b^2*(A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11
)/11 + (b^3*C*x^12)/12 + (b^3*D*x^13)/13
```

Rubi [A] time = 0.372913, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*
(3*b*B + a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*
D)*x^9)/3 + (b^2*(A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11
)/11 + (b^3*C*x^12)/12 + (b^3*D*x^13)/13
```

Rubi in Sympy [A] time = 46.2808, size = 139, normalized size = 0.93

$$\frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13}+\frac{a^2x^7\left(3Bb+Da\right)}{7}+\frac{a^2x^6\left(3Ab+Ca\right)}{6}+\frac{abx^9\left(Bb+Da\right)}{3}+\frac{3abx^8\left(Ab+Ca\right)}{8}+\frac{b^2x^{11}\left(Bb+3Da\right)}{11}+\frac{b^2x^{10}\left(Ab+3Ca\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13
 + a**2*x**7*(3*B*b + D*a)/7 + a**2*x**6*(3*A*b + C*a)/6 + a*b*x*
*9*(B*b + D*a)/3 + 3*a*b*x**8*(A*b + C*a)/8 + b**2*x**11*(B*b + 3
*D*a)/11 + b**2*x**10*(A*b + 3*C*a)/10
```

Mathematica [A] time = 0.0580414, size = 149, normalized size = 1.

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*
(3*b*B + a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*
D)*x^9)/3 + (b^2*(A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11
)/11 + (b^3*C*x^12)/12 + (b^3*D*x^13)/13
```

Maple [A] time = 0.002, size = 150, normalized size = 1.

$$\frac{b^3Dx^{13}}{13}+\frac{b^3Cx^{12}}{12}+\frac{\left(b^3B+3\,ab^2D\right)x^{11}}{11}+\frac{\left(Ab^3+3\,ab^2C\right)x^{10}}{10}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^9}{9}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^8}{8}+\frac{\left(3\,a^2bB+a^3D\right)x^7}{7}+\frac{\left(3\,Aa^2b+a^3C\right)x^6}{6}+\frac{a^3Bx^5}{5}+\frac{a^3Ax^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+1/11*(B*b^3+3*D*a*b^2)*x^11+1/10*
(A*b^3+3*C*a*b^2)*x^10+1/9*(3*B*a*b^2+3*D*a^2*b)*x^9+1/8*(3*A*a*b
^2+3*C*a^2*b)*x^8+1/7*(3*B*a^2*b+D*a^3)*x^7+1/6*(3*A*a^2*b+C*a^3)
*x^6+1/5*a^3*B*x^5+1/4*a^3*A*x^4
```

Maxima [A] time = 1.34667, size = 196, normalized size = 1.32

$$\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\left(3\,Dab^2+Bb^3\right)x^{11}+\frac{1}{10}\left(3\,Cab^2+Ab^3\right)x^{10}+\frac{1}{3}\left(Da^2b+Bab^2\right)x^9+\frac{1}{5}\,Ba^3x^5+\frac{3}{8}\left(Ca^2b+Aab^2\right)x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\left(Da^3+3\,Ba^2b\right)x^7+\frac{1}{6}\left(Ca^3+3\,Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11
 + 1/10*(3*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 +
1/5*B*a^3*x^5 + 3/8*(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7
*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6*(C*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A] time = 0.202101, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}b^3D+\frac{1}{12}x^{12}b^3C+\frac{3}{11}x^{11}b^2aD+\frac{1}{11}x^{11}b^3B+\frac{3}{10}x^{10}b^2aC+\frac{1}{10}x^{10}b^3A+\frac{1}{3}x^9ba^2D+\frac{1}{3}x^9b^2aB+\frac{3}{8}x^8ba^2C+\frac{3}{8}x^8b^2aA+\frac{1}{7}x^7a^3D+\frac{3}{7}x^7ba^2B+\frac{1}{6}x^6a^3C+\frac{1}{2}x^6ba^2A+\frac{1}{5}x^5a^3B+\frac{1}{4}x^4a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3*D + 1/12*x^12*b^3*C + 3/11*x^11*b^2*a*D + 1/11*x^11
*b^3*B + 3/10*x^10*b^2*a*C + 1/10*x^10*b^3*A + 1/3*x^9*b*a^2*D +
1/3*x^9*b^2*a*B + 3/8*x^8*b*a^2*C + 3/8*x^8*b^2*a*A + 1/7*x^7*a^3
*D + 3/7*x^7*b*a^2*B + 1/6*x^6*a^3*C + 1/2*x^6*b*a^2*A + 1/5*x^5*
a^3*B + 1/4*x^4*a^3*A
```

Sympy [A] time = 0.086389, size = 163, normalized size = 1.09

$$\frac{Aa^3x^4}{4} + \frac{Ba^3x^5}{5} + \frac{Cb^3x^{12}}{12} + \frac{Db^3x^{13}}{13} + x^{11}\left(\frac{Bb^3}{11} + \frac{3Dab^2}{11}\right) + x^{10}\left(\frac{Ab^3}{10} + \frac{3Cab^2}{10}\right) + x^9\left(\frac{Bab^2}{3} + \frac{Da^2b}{3}\right) + x^8\left(\frac{3Aab^2}{8} + \frac{3Ca^2b}{8}\right) + x^7\left(\frac{3Ba^2b}{7} + \frac{Da^3}{7}\right) + x^6\left(\frac{Aa^2b}{2} + \frac{Ca^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13
 + x**11*(B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b
**2/10) + x**9*(B*a*b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3
*C*a**2*b/8) + x**7*(3*B*a**2*b/7 + D*a**3/7) + x**6*(A*a**2*b/2
+ C*a**3/6)
```

GIAC/XCAS [A] time = 0.221632, size = 207, normalized size = 1.39

$$\frac{1}{13}Db^3x^{13} + \frac{1}{12}Cb^3x^{12} + \frac{3}{11}Dab^2x^{11} + \frac{1}{11}Bb^3x^{11} + \frac{3}{10}Cab^2x^{10} + \frac{1}{10}Ab^3x^{10} + \frac{1}{3}Da^2bx^9 + \frac{1}{3}Bab^2x^9 + \frac{3}{8}Ca^2bx^8 + \frac{3}{8}Aab^2x^8 + \frac{1}{7}Da^3x^7 + \frac{3}{7}Ba^2bx^7 + \frac{1}{6}Ca^3x^6 + \frac{1}{2}Aa^2bx^6 + \frac{1}{5}Ba^3x^5 + \frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^3,x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^
3*x^11 + 3/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 +
1/3*B*a*b^2*x^9 + 3/8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x
^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^
3*x^5 + 1/4*A*a^3*x^4
```

3.79 $\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*
(3*b*B + a*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B +
a*D)*x^8)/8 + (b^2*(A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10
)/10 + (b^3*C*x^11)/11 + (b^3*D*x^12)/12
```

Rubi [A] time = 0.368424, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*
(3*b*B + a*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B +
a*D)*x^8)/8 + (b^2*(A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10
)/10 + (b^3*C*x^11)/11 + (b^3*D*x^12)/12
```

Rubi in Sympy [A] time = 50.0379, size = 141, normalized size = 0.95

$$\frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+\frac{a^2x^6\left(3Bb+Da\right)}{6}+\frac{a^2x^5\left(3Ab+Ca\right)}{5}+\frac{3abx^8\left(Bb+Da\right)}{8}+\frac{3abx^7\left(Ab+Ca\right)}{7}+\frac{b^2x^{10}\left(Bb+3Da\right)}{10}+\frac{b^2x^9\left(Ab+3Ca\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12
 + a**2*x**6*(3*B*b + D*a)/6 + a**2*x**5*(3*A*b + C*a)/5 + 3*a*b*
x**8*(B*b + D*a)/8 + 3*a*b*x**7*(A*b + C*a)/7 + b**2*x**10*(B*b +
 3*D*a)/10 + b**2*x**9*(A*b + 3*C*a)/9
```

Mathematica [A] time = 0.155142, size = 125, normalized size = 0.84

$$\frac{462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A+5x(28B+3x(8C+7Dx)))+33ab^2x^7(360A+7x(45B+4x(10C+9}{27720}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^
3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5
```

```
*x*(28*B + 3*x*(8*C + 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B
+ 4*x*(10*C + 9*D*x))))/27720
```

Maple [A] time = 0.002, size = 150, normalized size = 1.

$$\frac{b^3Dx^{12}}{12}+\frac{b^3Cx^{11}}{11}+\frac{\left(b^3B+3\,ab^2D\right)x^{10}}{10}+\frac{\left(Ab^3+3\,ab^2C\right)x^9}{9}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^8}{8}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^7}{7}+\frac{\left(3\,a^2bB+a^3D\right)x^6}{6}+\frac{\left(3\,Aa^2b+a^3C\right)x^5}{5}+\frac{a^3Bx^4}{4}+\frac{a^3Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+1/10*(B*b^3+3*D*a*b^2)*x^10+1/9*(
A*b^3+3*C*a*b^2)*x^9+1/8*(3*B*a*b^2+3*D*a^2*b)*x^8+1/7*(3*A*a*b^2
+3*C*a^2*b)*x^7+1/6*(3*B*a^2*b+D*a^3)*x^6+1/5*(3*A*a^2*b+C*a^3)*x
^5+1/4*a^3*B*x^4+1/3*a^3*A*x^3
```

Maxima [A] time = 1.35328, size = 196, normalized size = 1.32

$$\frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{1}{10}\left(3\,Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3\,Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}\,Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}\,Aa^3x^3+\frac{1}{6}\left(Da^3+3\,Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3\,Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10
 + 1/9*(3*C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/
4*B*a^3*x^4 + 3/7*(C*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(
D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(C*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A] time = 0.207658, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}b^3D+\frac{1}{11}x^{11}b^3C+\frac{3}{10}x^{10}b^2aD+\frac{1}{10}x^{10}b^3B+\frac{1}{3}x^9b^2aC+\frac{1}{9}x^9b^3A+\frac{3}{8}x^8ba^2D+\frac{3}{8}x^8b^2aB+\frac{3}{7}x^7ba^2C+\frac{3}{7}x^7b^2aA+\frac{1}{6}x^6a^3D+\frac{1}{2}x^6ba^2B+\frac{1}{5}x^5a^3C+\frac{3}{5}x^5ba^2A+\frac{1}{4}x^4a^3B+\frac{1}{3}x^3a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^3*D + 1/11*x^11*b^3*C + 3/10*x^10*b^2*a*D + 1/10*x^10
*b^3*B + 1/3*x^9*b^2*a*C + 1/9*x^9*b^3*A + 3/8*x^8*b*a^2*D + 3/8*
x^8*b^2*a*B + 3/7*x^7*b*a^2*C + 3/7*x^7*b^2*a*A + 1/6*x^6*a^3*D +
 1/2*x^6*b*a^2*B + 1/5*x^5*a^3*C + 3/5*x^5*b*a^2*A + 1/4*x^4*a^3*
B + 1/3*x^3*a^3*A
```

Sympy [A] time = 0.086901, size = 165, normalized size = 1.11

$$\frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\left(\frac{3Aab^2}{7}+\frac{3Ca^2b}{7}\right)+x^6\left(\frac{Ba^2b}{2}+\frac{Da^3}{6}\right)+x^5\left(\frac{3Aa^2b}{5}+\frac{Ca^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12
 + x**10*(B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/
3) + x**8*(3*B*a*b**2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*
C*a**2*b/7) + x**6*(B*a**2*b/2 + D*a**3/6) + x**5*(3*A*a**2*b/5 +
 C*a**3/5)
```

GIAC/XCAS [A] time = 0.222928, size = 207, normalized size = 1.39

$$\frac{1}{12}Db^3x^{12} + \frac{1}{11}Cb^3x^{11} + \frac{3}{10}Dab^2x^{10} + \frac{1}{10}Bb^3x^{10} + \frac{1}{3}Cab^2x^9 + \frac{1}{9}Ab^3x^9 + \frac{3}{8}Da^2bx^8 + \frac{3}{8}Bab^2x^8 + \frac{3}{7}Ca^2bx^7 + \frac{3}{7}Aab^2x^7 + \frac{1}{6}Da^3x^6 + \frac{1}{2}Ba^2bx^6 + \frac{1}{5}Ca^3x^5 + \frac{3}{5}Aa^2bx^5 + \frac{1}{4}Ba^3x^4 + \frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x^2,x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^
3*x^10 + 1/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*
B*a*b^2*x^8 + 3/7*C*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 +
 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^
4 + 1/3*A*a^3*x^3
```

3.80 $\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=138

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}$$

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*
b*C*x^6)/2 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2
*(b*B + 3*a*D)*x^9)/9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a
 + b*x^2)^4)/(8*b)
```

Rubi [A] time = 0.404802, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*
b*C*x^6)/2 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2
*(b*B + 3*a*D)*x^9)/9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a
 + b*x^2)^4)/(8*b)
```

Rubi in Sympy [A] time = 46.6828, size = 129, normalized size = 0.93

$$\frac{A\left(a+bx^2\right)^4}{8b}+\frac{Ba^3x^3}{3}+\frac{Ca^3x^4}{4}+\frac{Ca^2bx^6}{2}+\frac{3Cab^2x^8}{8}+\frac{Cb^3x^{10}}{10}+\frac{Db^3x^{11}}{11}+\frac{a^2x^5\left(3Bb+Da\right)}{5}+\frac{3abx^7\left(Bb+Da\right)}{7}+\frac{b^2x^9\left(Bb+3Da\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*(a + b*x**2)**4/(8*b) + B*a**3*x**3/3 + C*a**3*x**4/4 + C*a**2*
b*x**6/2 + 3*C*a*b**2*x**8/8 + C*b**3*x**10/10 + D*b**3*x**11/11
+ a**2*x**5*(3*B*b + D*a)/5 + 3*a*b*x**7*(B*b + D*a)/7 + b**2*x**
9*(B*b + 3*D*a)/9
```

Mathematica [A] time = 0.145892, size = 124, normalized size = 0.9

$$\frac{462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105A+2x(42B+5x(7C+6Dx)))+165ab^2x^6(84A+x(72B+7x(9C+8\ldots}{27720}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^
2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A +
```

```
2*x*(42*B + 5*x*(7*C + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B +
 7*x*(9*C + 8*D*x))))/27720
```

Maple [A] time = 0.003, size = 150, normalized size = 1.1

$$\frac{b^3Dx^{11}}{11}+\frac{b^3Cx^{10}}{10}+\frac{\left(b^3B+3\,ab^2D\right)x^9}{9}+\frac{\left(Ab^3+3\,ab^2C\right)x^8}{8}+\frac{\left(3\,ab^2B+3\,a^2bD\right)x^7}{7}+\frac{\left(3\,ab^2A+3\,a^2bC\right)x^6}{6}+\frac{\left(3\,a^2bB+a^3D\right)x^5}{5}+\frac{\left(3\,Aa^2b+a^3C\right)x^4}{4}+\frac{a^3Bx^3}{3}+\frac{a^3Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+1/9*(B*b^3+3*D*a*b^2)*x^9+1/8*(A*
b^3+3*C*a*b^2)*x^8+1/7*(3*B*a*b^2+3*D*a^2*b)*x^7+1/6*(3*A*a*b^2+3
*C*a^2*b)*x^6+1/5*(3*B*a^2*b+D*a^3)*x^5+1/4*(3*A*a^2*b+C*a^3)*x^4
+1/3*a^3*B*x^3+1/2*a^3*A*x^2
```

Maxima [A] time = 1.3862, size = 196, normalized size = 1.42

$$\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\left(3\,Dab^2+Bb^3\right)x^9+\frac{1}{8}\left(3\,Cab^2+Ab^3\right)x^8+\frac{3}{7}\left(Da^2b+Bab^2\right)x^7+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\left(Ca^2b+Aab^2\right)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\left(Da^3+3\,Ba^2b\right)x^5+\frac{1}{4}\left(Ca^3+3\,Aa^2b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x,x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 +
 1/8*(3*C*a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*
B*a^3*x^3 + 1/2*(C*a^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*
a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*a^3 + 3*A*a^2*b)*x^4
```

Fricas [A] time = 0.198592, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}b^3D+\frac{1}{10}x^{10}b^3C+\frac{1}{3}x^9b^2aD+\frac{1}{9}x^9b^3B+\frac{3}{8}x^8b^2aC+\frac{1}{8}x^8b^3A+\frac{3}{7}x^7ba^2D+\frac{3}{7}x^7b^2aB+\frac{1}{2}x^6ba^2C+\frac{1}{2}x^6b^2aA+\frac{1}{5}x^5a^3D+\frac{3}{5}x^5ba^2B+\frac{1}{4}x^4a^3C+\frac{3}{4}x^4ba^2A+\frac{1}{3}x^3a^3B+\frac{1}{2}x^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^3*D + 1/10*x^10*b^3*C + 1/3*x^9*b^2*a*D + 1/9*x^9*b^3
*B + 3/8*x^8*b^2*a*C + 1/8*x^8*b^3*A + 3/7*x^7*b*a^2*D + 3/7*x^7*
b^2*a*B + 1/2*x^6*b*a^2*C + 1/2*x^6*b^2*a*A + 1/5*x^5*a^3*D + 3/5
*x^5*b*a^2*B + 1/4*x^4*a^3*C + 3/4*x^4*b*a^2*A + 1/3*x^3*a^3*B +
1/2*x^2*a^3*A
```

Sympy [A] time = 0.086342, size = 163, normalized size = 1.18

$$\frac{Aa^3x^2}{2}+\frac{Ba^3x^3}{3}+\frac{Cb^3x^{10}}{10}+\frac{Db^3x^{11}}{11}+x^9\left(\frac{Bb^3}{9}+\frac{Dab^2}{3}\right)+x^8\left(\frac{Ab^3}{8}+\frac{3Cab^2}{8}\right)+x^7\left(\frac{3Bab^2}{7}+\frac{3Da^2b}{7}\right)+x^6\left(\frac{Aab^2}{2}+\frac{Ca^2b}{2}\right)+x^5\left(\frac{3Ba^2b}{5}+\frac{Da^3}{5}\right)+x^4\left(\frac{3Aa^2b}{4}+\frac{Ca^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11
 + x**9*(B*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8)
+ x**7*(3*B*a*b**2/7 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*
b/2) + x**5*(3*B*a**2*b/5 + D*a**3/5) + x**4*(3*A*a**2*b/4 + C*a*
*3/4)
```

GIAC/XCAS [A] time = 0.239498, size = 207, normalized size = 1.5

$$\frac{1}{11}Db^3x^{11} + \frac{1}{10}Cb^3x^{10} + \frac{1}{3}Dab^2x^9 + \frac{1}{9}Bb^3x^9 + \frac{3}{8}Cab^2x^8 + \frac{1}{8}Ab^3x^8 + \frac{3}{7}Da^2bx^7 + \frac{3}{7}Bab^2x^7 + \frac{1}{2}Ca^2bx^6 + \frac{1}{2}Aab^2x^6 + \frac{1}{5}Da^3x^5 + \frac{3}{5}Ba^2bx^5 + \frac{1}{4}Ca^3x^4 + \frac{3}{4}Aa^2bx^4 + \frac{1}{3}Ba^3x^3 + \frac{1}{2}Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3*x,x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x
^9 + 3/8*C*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*
b^2*x^7 + 1/2*C*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5
*B*a^2*b*x^5 + 1/4*C*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 +
1/2*A*a^3*x^2
```

3.81 $\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=133

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b
 + a*C)*x^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3
*a*b^2*D*x^8)/8 + (b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2
)^4)/(8*b)
```

Rubi [A] time = 0.249969, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b
 + a*C)*x^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3
*a*b^2*D*x^8)/8 + (b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2
)^4)/(8*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\left(a+bx^2\right)^4}{8b}+\frac{Cb^3x^9}{9}+\frac{Da^3x^4}{4}+\frac{Da^2bx^6}{2}+\frac{3Dab^2x^8}{8}+\frac{Db^3x^{10}}{10}+a^3\int A\,dx+\frac{a^2x^3\left(3Ab+Ca\right)}{3}+\frac{3abx^5\left(Ab+Ca\right)}{5}+\frac{b^2x^7\left(Ab+3Ca\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] B*(a + b*x**2)**4/(8*b) + C*b**3*x**9/9 + D*a**3*x**4/4 + D*a**2*
b*x**6/2 + 3*D*a*b**2*x**8/8 + D*b**3*x**10/10 + a**3*Integral(A,
 x) + a**2*x**3*(3*A*b + C*a)/3 + 3*a*b*x**5*(A*b + C*a)/5 + b**2
*x**7*(A*b + 3*C*a)/7
```

Mathematica [A] time = 0.137867, size = 121, normalized size = 0.91

$$\frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+5x(28B+3x(8C+7Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20
*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28
```

```
*B + 3*x*(8*C + 7*D*x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C
 + 9*D*x))))/2520
```

Maple [A] time = 0.003, size = 147, normalized size = 1.1

$$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\frac{(b^3B+3\,ab^2D)\,x^8}{8}+\frac{(Ab^3+3\,ab^2C)\,x^7}{7}+\frac{(3\,ab^2B+3\,a^2bD)\,x^6}{6}+\frac{(3\,ab^2A+3\,a^2bC)\,x^5}{5}+\frac{(3\,a^2bB+a^3D)\,x^4}{4}+\frac{(3\,Aa^2b+a^3C)\,x^3}{3}+\frac{a^3Bx^2}{2}+a^3Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+1/8*(B*b^3+3*D*a*b^2)*x^8+1/7*(A*b^
3+3*C*a*b^2)*x^7+1/6*(3*B*a*b^2+3*D*a^2*b)*x^6+1/5*(3*A*a*b^2+3*C
*a^2*b)*x^5+1/4*(3*B*a^2*b+D*a^3)*x^4+1/3*(3*A*a^2*b+C*a^3)*x^3+1
/2*a^3*B*x^2+a^3*A*x
```

Maxima [A] time = 1.42694, size = 192, normalized size = 1.44

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)\,x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)\,x^7+\frac{1}{2}\,(Da^2b+Bab^2)\,x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)\,x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)\,x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1
/7*(3*C*a*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*
a^3*x^2 + 3/5*(C*a^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*
B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*A*a^2*b)*x^3
```

Fricas [A] time = 0.196794, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}b^3D+\frac{1}{9}x^9b^3C+\frac{3}{8}x^8b^2aD+\frac{1}{8}x^8b^3B+\frac{3}{7}x^7b^2aC+\frac{1}{7}x^7b^3A+\frac{1}{2}x^6ba^2D+\frac{1}{2}x^6b^2aB+\frac{3}{5}x^5ba^2C+\frac{3}{5}x^5b^2aA+\frac{1}{4}x^4a^3D+\frac{3}{4}x^4ba^2B+\frac{1}{3}x^3a^3C+x^3ba^2A+\frac{1}{2}x^2a^3B+xa^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^3*D + 1/9*x^9*b^3*C + 3/8*x^8*b^2*a*D + 1/8*x^8*b^3*B
 + 3/7*x^7*b^2*a*C + 1/7*x^7*b^3*A + 1/2*x^6*b*a^2*D + 1/2*x^6*b^
2*a*B + 3/5*x^5*b*a^2*C + 3/5*x^5*b^2*a*A + 1/4*x^4*a^3*D + 3/4*x
^4*b*a^2*B + 1/3*x^3*a^3*C + x^3*b*a^2*A + 1/2*x^2*a^3*B + x*a^3*
A
```

Sympy [A] time = 0.08537, size = 158, normalized size = 1.19

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10}+x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right)+x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right)+x^4\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right)+x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8
*(B*b**3/8 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**
6*(B*a*b**2/2 + D*a**2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5)
+ x**4*(3*B*a**2*b/4 + D*a**3/4) + x**3*(A*a**2*b + C*a**3/3)
```

GIAC/XCAS [A] time = 0.222359, size = 201, normalized size = 1.51

$$\frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Da^3x^4 + \frac{3}{4}Ba^2bx^4 + \frac{1}{3}Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8
 + 3/7*C*a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^
2*x^6 + 3/5*C*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B
*a^2*b*x^4 + 1/3*C*a^3*x^3 + A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*
x
```

3.82 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=129

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C\left(a+bx^2\right)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*
b^2*x^4)/4 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*
B + 3*a*D)*x^7)/7 + (b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3
*A*Log[x]
```

Rubi [A] time = 0.198339, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C\left(a+bx^2\right)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*
b^2*x^4)/4 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*
B + 3*a*D)*x^7)/7 + (b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3
*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa^3\log(x)+\frac{Cb^3x^8}{8}+\frac{Db^3x^9}{9}+a^3\int B\,dx+\frac{a^2x^3\left(3Bb+Da\right)}{3}+a^2\left(3Ab+Ca\right)\int x\,dx+\frac{3abx^5\left(Bb+Da\right)}{5}+\frac{3abx^4\left(Ab+Ca\right)}{4}+\frac{b^2x^7\left(Bb+3Da\right)}{7}+\frac{b^2x^6\left(Ab+3Ca\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + C*b**3*x**8/8 + D*b**3*x**9/9 + a**3*Integral(B,
x) + a**2*x**3*(3*B*b + D*a)/3 + a**2*(3*A*b + C*a)*Integral(x, x
) + 3*a*b*x**5*(B*b + D*a)/5 + 3*a*b*x**4*(A*b + C*a)/4 + b**2*x*
*7*(B*b + 3*D*a)/7 + b**2*x**6*(A*b + 3*C*a)/6
```

Mathematica [A] time = 0.205435, size = 121, normalized size = 0.94

$$a^3A\log(x)+\frac{x\left(420a^3(6B+x(3C+2Dx))+126a^2bx(30A+x(20B+3x(5C+4Dx)))+18ab^2x^3(105A+2x(42B+5x(7C+6Dx)))+5b^3x\right.}{2520}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B
 + 3*x*(5*C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7
*C + 6*D*x))) + 5*b^3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))
/2520 + a^3*A*Log[x]
```

Maple [A] time = 0.006, size = 148, normalized size = 1.2

$$\frac{b^3Dx^9}{9}+\frac{Cb^3x^8}{8}+\frac{Bx^7b^3}{7}+\frac{3\,Dx^7ab^2}{7}+\frac{Ab^3x^6}{6}+\frac{Cx^6ab^2}{2}+\frac{3\,Bx^5ab^2}{5}+\frac{3\,Dx^5a^2b}{5}+\frac{3\,aAb^2x^4}{4}+\frac{3\,Cx^4a^2b}{4}+Bx^3a^2b+\frac{Dx^3a^3}{3}+\frac{3\,a^2Abx^2}{2}+\frac{Cx^2a^3}{2}+a^3Bx+a^3A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/9*b^3*D*x^9+1/8*C*b^3*x^8+1/7*B*x^7*b^3+3/7*D*x^7*a*b^2+1/6*A*b
^3*x^6+1/2*C*x^6*a*b^2+3/5*B*x^5*a*b^2+3/5*D*x^5*a^2*b+3/4*a*A*b^
2*x^4+3/4*C*x^4*a^2*b+B*x^3*a^2*b+1/3*D*x^3*a^3+3/2*a^2*A*b*x^2+1
/2*C*x^2*a^3+a^3*B*x+a^3*A*ln(x)
```

Maxima [A] time = 1.33249, size = 189, normalized size = 1.47

$$\frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{1}{7}\left(3\,Dab^2+Bb^3\right)x^7+\frac{1}{6}\left(3\,Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5+Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4+Aa^3\log(x)+\frac{1}{3}\left(Da^3+3\,Ba^2b\right)x^3+\frac{1}{2}\left(Ca^3+3\,Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6
*(3*C*a*b^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x
+ 3/4*(C*a^2*b + A*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a
^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a^2*b)*x^2
```

Fricas [A] time = 0.228935, size = 189, normalized size = 1.47

$$\frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{1}{7}\left(3\,Dab^2+Bb^3\right)x^7+\frac{1}{6}\left(3\,Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5+Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4+Aa^3\log(x)+\frac{1}{3}\left(Da^3+3\,Ba^2b\right)x^3+\frac{1}{2}\left(Ca^3+3\,Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6
*(3*C*a*b^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x
+ 3/4*(C*a^2*b + A*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a
^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a^2*b)*x^2
```

Sympy [A] time = 0.872435, size = 158, normalized size = 1.22

$$Aa^3\log(x)+Ba^3x+\frac{Cb^3x^8}{8}+\frac{Db^3x^9}{9}+x^7\left(\frac{Bb^3}{7}+\frac{3Dab^2}{7}\right)+x^6\left(\frac{Ab^3}{6}+\frac{Cab^2}{2}\right)+x^5\left(\frac{3Bab^2}{5}+\frac{3Da^2b}{5}\right)+x^4\left(\frac{3Aab^2}{4}+\frac{3Ca^2b}{4}\right)+x^3\left(Ba^2b+\frac{Da^3}{3}\right)+x^2\left(\frac{3Aa^2b}{2}+\frac{Ca^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(
B*b**3/7 + 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3
*B*a*b**2/5 + 3*D*a**2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4)
+ x**3*(B*a**2*b + D*a**3/3) + x**2*(3*A*a**2*b/2 + C*a**3/2)
```

GIAC/XCAS [A] time = 0.220736, size = 200, normalized size = 1.55

$$\frac{1}{9}Db^3x^9 + \frac{1}{8}Cb^3x^8 + \frac{3}{7}Dab^2x^7 + \frac{1}{7}Bb^3x^7 + \frac{1}{2}Cab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{3}{5}Da^2bx^5 + \frac{3}{5}Bab^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{3}{4}Aab^2x^4 + \frac{1}{3}Da^3x^3 + Ba^2bx^3 + \frac{1}{2}Ca^3x^2 + \frac{3}{2}Aa^2bx^2 + Ba^3x + Aa^3\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 +
 1/2*C*a*b^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*
x^5 + 3/4*C*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b
*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a^2*b*x^2 + B*a^3*x + A*a^3*ln(abs(x
))
```

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D\left(a+bx^2\right)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b
 + a*C)*x^3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^
3*B*x^6)/6 + (b^3*C*x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[
x]
```

Rubi [A] time = 0.266317, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D\left(a+bx^2\right)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b
 + a*C)*x^3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^
3*B*x^6)/6 + (b^3*C*x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[
x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^3}{x}+Ba^3\log(x)+\frac{Cb^3x^7}{7}+\frac{Db^3x^8}{8}+a^2(3Bb+Da)\int x\,dx+\frac{3abx^4(Bb+Da)}{4}+abx^3(Ab+Ca)+\frac{b^2x^6(Bb+3Da)}{6}+\frac{b^2x^5(Ab+3Ca)}{5}+\frac{a^2(3Ab+Ca)\int C\,dx}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + a**2*
(3*B*b + D*a)*Integral(x, x) + 3*a*b*x**4*(B*b + D*a)/4 + a*b*x**
3*(A*b + C*a) + b**2*x**6*(B*b + 3*D*a)/6 + b**2*x**5*(A*b + 3*C*
a)/5 + a**2*(3*A*b + C*a)*Integral(C, x)/C
```

Mathematica [A] time = 0.272209, size = 123, normalized size = 0.99

$$a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^3B\log(x)+\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))+\frac{1}{840}b^3x^5(168A+5x(28B+3x(8C+7Dx)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C
 + 3*D*x))))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))
)/20 + (b^3*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a
^3*B*Log[x]
```

Maple [A] time = 0.01, size = 145, normalized size = 1.2

$$\frac{Db^3x^8}{8}+\frac{b^3Cx^7}{7}+\frac{Bx^6b^3}{6}+\frac{Dx^6ab^2}{2}+\frac{Ax^5b^3}{5}+\frac{3\,Cx^5ab^2}{5}+\frac{3\,Bx^4ab^2}{4}+\frac{3\,Dx^4a^2b}{4}+Ax^3ab^2+Cx^3a^2b+\frac{3\,Bx^2a^2b}{2}+\frac{Dx^2a^3}{2}+3\,Aa^2bx+Cxa^3+a^3B\ln(x)-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/8*D*b^3*x^8+1/7*b^3*C*x^7+1/6*B*x^6*b^3+1/2*D*x^6*a*b^2+1/5*A*x
^5*b^3+3/5*C*x^5*a*b^2+3/4*B*x^4*a*b^2+3/4*D*x^4*a^2*b+A*x^3*a*b^
2+C*x^3*a^2*b+3/2*B*x^2*a^2*b+1/2*D*x^2*a^3+3*A*a^2*b*x+C*x*a^3+a
^3*B*ln(x)-a^3*A/x
```

Maxima [A] time = 1.33969, size = 188, normalized size = 1.52

$$\frac{1}{8}\,Db^3x^8+\frac{1}{7}\,Cb^3x^7+\frac{1}{6}\left(3\,Dab^2+Bb^3\right)x^6+\frac{1}{5}\left(3\,Cab^2+Ab^3\right)x^5+\frac{3}{4}\left(Da^2b+Bab^2\right)x^4+Ba^3\log(x)+\left(Ca^2b+Aab^2\right)x^3-\frac{Aa^3}{x}+\frac{1}{2}\left(Da^3+3\,Ba^2b\right)x^2+\left(Ca^3+3\,Aa^2b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5
*(3*C*a*b^2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*lo
g(x) + (C*a^2*b + A*a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b
)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

Fricas [A] time = 0.2274, size = 198, normalized size = 1.6

$$\frac{105\,Db^3x^9+120\,Cb^3x^8+140\left(3\,Dab^2+Bb^3\right)x^7+168\left(3\,Cab^2+Ab^3\right)x^6+630\left(Da^2b+Bab^2\right)x^5+840\,Ba^3x\log(x)+840\,(C}{840\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^
7 + 168*(3*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 8
40*B*a^3*x*log(x) + 840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420
*(D*a^3 + 3*B*a^2*b)*x^3 + 840*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A] time = 0.903227, size = 150, normalized size = 1.21

$$-\frac{Aa^3}{x}+Ba^3\log(x)+\frac{Cb^3x^7}{7}+\frac{Db^3x^8}{8}+x^6\left(\frac{Bb^3}{6}+\frac{Dab^2}{2}\right)+x^5\left(\frac{Ab^3}{5}+\frac{3Cab^2}{5}\right)+x^4\left(\frac{3Bab^2}{4}+\frac{3Da^2b}{4}\right)+x^3\left(Aab^2+Ca^2b\right)+x^2\left(\frac{3Ba^2b}{2}+\frac{Da^3}{2}\right)+x\left(3Aa^2b+Ca^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*
(B*b**3/6 + D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(
3*B*a*b**2/4 + 3*D*a**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*
(3*B*a**2*b/2 + D*a**3/2) + x*(3*A*a**2*b + C*a**3)
```

GIAC/XCAS [A] time = 0.222207, size = 196, normalized size = 1.58

$$\frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{2}Dab^2x^6 + \frac{1}{6}Bb^3x^6 + \frac{3}{5}Cab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{4}Da^2bx^4 + \frac{3}{4}Bab^2x^4 + Ca^2bx^3 + Aab^2x^3 + \frac{1}{2}Da^3x^2 + \frac{3}{2}Ba^2bx^2 + Ca^3x + 3Aa^2bx + Ba^3\ln(|x|) - \frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 +
 3/5*C*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*
x^4 + C*a^2*b*x^3 + A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2
 + C*a^3*x + 3*A*a^2*b*x + B*a^3*ln(abs(x)) - A*a^3/x
```

3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b
+ a*C)*x^2)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 +
 (b^2*(b*B + 3*a*D)*x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*
(3*A*b + a*C)*Log[x]
```

Rubi [A] time = 0.284122, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b
+ a*C)*x^2)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 +
 (b^2*(b*B + 3*a*D)*x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*
(3*A*b + a*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^3}{2x^2}-\frac{Ba^3}{x}+\frac{Cb^3x^6}{6}+\frac{Db^3x^7}{7}+a^2(3Ab+Ca)\log(x)+abx^3(Bb+Da)+3ab(Ab+Ca)\int x\,dx+\frac{b^2x^5(Bb+3Da)}{5}+\frac{b^2x^4(Ab+3Ca)}{4}+\frac{a^2(3Bb+Da)\int D\,dx}{D}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) - B*a**3/x + C*b**3*x**6/6 + D*b**3*x**7/7 + a**
2*(3*A*b + C*a)*log(x) + a*b*x**3*(B*b + D*a) + 3*a*b*(A*b + C*a)
*Integral(x, x) + b**2*x**5*(B*b + 3*D*a)/5 + b**2*x**4*(A*b + 3*
C*a)/4 + a**2*(3*B*b + D*a)*Integral(D, x)/D
```

Mathematica [A] time = 0.142639, size = 124, normalized size = 0.92

$$-\frac{a^3(A+2Bx-2Dx^3)}{2x^2}+a^2\log(x)(aC+3Ab)+\frac{1}{2}a^2bx(6B+x(3C+2Dx))+\frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx)))+\frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (a^2*b*x*(6*B + x*(3*C + 2
*D*x)))/2 + (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20
+ (b^3*x^4*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3
*A*b + a*C)*Log[x]
```

Maple [A] time = 0.011, size = 144, normalized size = 1.1

$$\frac{b^3Dx^7}{7}+\frac{b^3Cx^6}{6}+\frac{Bx^5b^3}{5}+\frac{3\,Dx^5ab^2}{5}+\frac{Ax^4b^3}{4}+\frac{3\,Cx^4ab^2}{4}+Bx^3ab^2+Dx^3a^2b+\frac{3\,Ax^2ab^2}{2}+\frac{3\,Cx^2a^2b}{2}+3\,Bxa^2b+Dxa^3+3\,A\ln(x)\,a^2b+C\ln(x)\,a^3-\frac{Aa^3}{2\,x^2}-\frac{Ba^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*B*x^5*b^3+3/5*D*x^5*a*b^2+1/4*A*x
^4*b^3+3/4*C*x^4*a*b^2+B*x^3*a*b^2+D*x^3*a^2*b+3/2*A*x^2*a*b^2+3/
2*C*x^2*a^2*b+3*B*x*a^2*b+D*x*a^3+3*A*ln(x)*a^2*b+C*ln(x)*a^3-1/2
*a^3*A/x^2-a^3*B/x
```

Maxima [A] time = 1.34474, size = 188, normalized size = 1.39

$$\frac{1}{7}\,Db^3x^7+\frac{1}{6}\,Cb^3x^6+\frac{1}{5}\left(3\,Dab^2+Bb^3\right)x^5+\frac{1}{4}\left(3\,Cab^2+Ab^3\right)x^4+\left(Da^2b+Bab^2\right)x^3+\frac{3}{2}\left(Ca^2b+Aab^2\right)x^2+\left(Da^3+3\,Ba^2b\right)x+\left(Ca^3+3\,Aa^2b\right)\log(x)-\frac{2\,Ba^3x+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4
*(3*C*a*b^2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b
 + A*a*b^2)*x^2 + (D*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log
(x) - 1/2*(2*B*a^3*x + A*a^3)/x^2
```

Fricas [A] time = 0.223617, size = 198, normalized size = 1.47

$$\frac{60\,Db^3x^9+70\,Cb^3x^8+84\left(3\,Dab^2+Bb^3\right)x^7+105\left(3\,Cab^2+Ab^3\right)x^6+420\left(Da^2b+Bab^2\right)x^5-420\,Ba^3x+630\,(Ca^2b+Aab}{420\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 +
 105*(3*C*a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*
B*a^3*x + 630*(C*a^2*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 +
3*B*a^2*b)*x^3 + 420*(C*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A] time = 1.19946, size = 150, normalized size = 1.11

$$\frac{Cb^3x^6}{6}+\frac{Db^3x^7}{7}+a^2\left(3Ab+Ca\right)\log(x)+x^5\left(\frac{Bb^3}{5}+\frac{3Dab^2}{5}\right)+x^4\left(\frac{Ab^3}{4}+\frac{3Cab^2}{4}\right)+x^3\left(Bab^2+Da^2b\right)+x^2\left(\frac{3Aab^2}{2}+\frac{3Ca^2b}{2}\right)+x\left(3Ba^2b+Da^3\right)-\frac{Aa^3+2Ba^3x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*
(B*b**3/5 + 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3
*(B*a*b**2 + D*a**2*b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(
3*B*a**2*b + D*a**3) - (A*a**3 + 2*B*a**3*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.220849, size = 194, normalized size = 1.44

$$\frac{1}{7}Db^3x^7 + \frac{1}{6}Cb^3x^6 + \frac{3}{5}Dab^2x^5 + \frac{1}{5}Bb^3x^5 + \frac{3}{4}Cab^2x^4 + \frac{1}{4}Ab^3x^4 + Da^2bx^3 + Bab^2x^3 + \frac{3}{2}Ca^2bx^2 + \frac{3}{2}Aab^2x^2 + Da^3x + 3\,Ba^2bx + \left(Ca^3 + 3\,Aa^2b\right)\ln\left(|x|\right) - \frac{2\,Ba^3x + Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 +
 3/4*C*a*b^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/
2*C*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3
+ 3*A*a^2*b)*ln(abs(x)) - 1/2*(2*B*a^3*x + A*a^3)/x^2
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*
b*(A*b + a*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*
x^3)/3 + (b^2*(b*B + 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/
6 + a^2*(3*b*B + a*D)*Log[x]
```

Rubi [A] time = 0.284305, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*
b*(A*b + a*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*
x^3)/3 + (b^2*(b*B + 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/
6 + a^2*(3*b*B + a*D)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^3}{3x^3}-\frac{Ba^3}{2x^2}+\frac{Cb^3x^5}{5}+\frac{Db^3x^6}{6}+a^2\left(3Bb+Da\right)\log\left(x\right)-\frac{a^2\left(3Ab+Ca\right)}{x}+3abx\left(Ab+Ca\right)+3ab\left(Bb+Da\right)\int x\,dx+\frac{b^2x^4\left(Bb+3Da\right)}{4}+\frac{b^2x^3\left(Ab+3Ca\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] -A*a**3/(3*x**3) - B*a**3/(2*x**2) + C*b**3*x**5/5 + D*b**3*x**6/
6 + a**2*(3*B*b + D*a)*log(x) - a**2*(3*A*b + C*a)/x + 3*a*b*x*(A
*b + C*a) + 3*a*b*(B*b + D*a)*Integral(x, x) + b**2*x**4*(B*b + 3
*D*a)/4 + b**2*x**3*(A*b + 3*C*a)/3
```

Mathematica [A] time = 0.138119, size = 124, normalized size = 0.89

$$-\frac{a^3(2A+3x(B+2Cx))}{6x^3}+\frac{3a^2b\left(x^2(2C+Dx)-2A\right)}{2x}+a^2\log(x)(aD+3bB)+\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) + (3*a^2*b*(-2*A + x^2*(2*
C + D*x)))/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4
 + (b^3*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*
B + a*D)*Log[x]
```

Maple [A] time = 0.011, size = 146, normalized size = 1.1

$$\frac{b^3Dx^6}{6}+\frac{b^3Cx^5}{5}+\frac{Bx^4b^3}{4}+\frac{3\,Dx^4ab^2}{4}+\frac{Ax^3b^3}{3}+Cx^3ab^2+\frac{3\,Bx^2ab^2}{2}+\frac{3\,Dx^2a^2b}{2}+3\,Axab^2+3\,Cxa^2b+3\,B\ln(x)\,a^2b+D\ln(x)\,a^3-\frac{Aa^3}{3\,x^3}-\frac{Ba^3}{2\,x^2}-3\,\frac{Aa^2b}{x}-\frac{a^3C}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*x^4*b^3+3/4*D*x^4*a*b^2+1/3*A*x
^3*b^3+C*x^3*a*b^2+3/2*B*x^2*a*b^2+3/2*D*x^2*a^2*b+3*A*x*a*b^2+3*
C*x*a^2*b+3*B*ln(x)*a^2*b+D*ln(x)*a^3-1/3*a^3*A/x^3-1/2*a^3*B/x^2
-3*a^2/x*A*b-a^3/x*C
```

Maxima [A] time = 1.36271, size = 192, normalized size = 1.38

$$\frac{1}{6}\,Db^3x^6+\frac{1}{5}\,Cb^3x^5+\frac{1}{4}\left(3\,Dab^2+Bb^3\right)x^4+\frac{1}{3}\left(3\,Cab^2+Ab^3\right)x^3+\frac{3}{2}\left(Da^2b+Bab^2\right)x^2+3\left(Ca^2b+Aab^2\right)x+\left(Da^3+3\,Ba^2b\right)\log(x)-\frac{3\,Ba^3x+2\,Aa^3+6\left(Ca^3+3\,Aa^2b\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3
*(3*C*a*b^2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2
*b + A*a*b^2)*x + (D*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2
*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)*x^2)/x^3
```

Fricas [A] time = 0.228882, size = 198, normalized size = 1.42

$$\frac{10\,Db^3x^9+12\,Cb^3x^8+15\left(3\,Dab^2+Bb^3\right)x^7+20\left(3\,Cab^2+Ab^3\right)x^6+90\left(Da^2b+Bab^2\right)x^5-30\,Ba^3x+180\left(Ca^2b+Aab^2\right)x}{60\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 +
20*(3*C*a*b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^
3*x + 180*(C*a^2*b + A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*lo
g(x) - 20*A*a^3 - 60*(C*a^3 + 3*A*a^2*b)*x^2)/x^3
```

Sympy [A] time = 2.15736, size = 153, normalized size = 1.1

$$\frac{Cb^3x^5}{5}+\frac{Db^3x^6}{6}+a^2\left(3Bb+Da\right)\log(x)+x^4\left(\frac{Bb^3}{4}+\frac{3Dab^2}{4}\right)+x^3\left(\frac{Ab^3}{3}+Cab^2\right)+x^2\left(\frac{3Bab^2}{2}+\frac{3Da^2b}{2}\right)+x\left(3Aab^2+3Ca^2b\right)-\frac{2Aa^3+3Ba^3x+x^2\left(18Aa^2b+6Ca^3\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*
(B*b**3/4 + 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*
B*a*b**2/2 + 3*D*a**2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) - (2*A*a
**3 + 3*B*a**3*x + x**2*(18*A*a**2*b + 6*C*a**3))/(6*x**3)
```

GIAC/XCAS [A] time = 0.223223, size = 197, normalized size = 1.42

$$\frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{3}{4}\,Dab^2x^4 + \frac{1}{4}\,Bb^3x^4 + Cab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Da^2bx^2 + \frac{3}{2}\,Bab^2x^2 + 3\,Ca^2bx + 3\,Aab^2x + \left(Da^3 + 3\,Ba^2b\right)\ln\left(|x|\right) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\,\left(Ca^3 + 3\,Aa^2b\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 +
 C*a*b^2*x^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2
+ 3*C*a^2*b*x + 3*A*a*b^2*x + (D*a^3 + 3*B*a^2*b)*ln(abs(x)) - 1/
6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)*x^2)/x^3
```

3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=151

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}$$

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b -
a*C)*x^3)/(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (
D*x^6)/(6*b) + (a^(3/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(7/2) + (a^2*(b*B - a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.324638, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b -
a*C)*x^3)/(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (
D*x^6)/(6*b) + (a^(3/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(7/2) + (a^2*(b*B - a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{a^{\frac{3}{2}}\left(Ab-Ca\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{7}{2}}}+\frac{a^2\left(Bb-Da\right)\log\left(a+bx^2\right)}{2b^4}-\frac{a\left(Bb-Da\right)\int x\,dx}{b^3}+\frac{x^4\left(Bb-Da\right)}{4b^2}+\frac{x^3\left(Ab-Ca\right)}{3b^2}-\frac{\left(Ab-Ca\right)\int a\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + a**(3/2)*(A*b - C*a)*atan(sqrt(b)*x
/sqrt(a))/b**(7/2) + a**2*(B*b - D*a)*log(a + b*x**2)/(2*b**4) -
a*(B*b - D*a)*Integral(x, x)/b**3 + x**4*(B*b - D*a)/(4*b**2) + x
**3*(A*b - C*a)/(3*b**2) - (A*b - C*a)*Integral(a, x)/b**3
```

Mathematica [A] time = 0.156339, size = 130, normalized size = 0.86

$$\frac{-60a^{3/2}\sqrt{b}(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+bx\left(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5\right.}{60b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x)
)) + b^2*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*
B) + a*D)*Log[a + b*x^2])/(60*b^4)
```

Maple [A] time = 0.008, size = 176, normalized size = 1.2

$$\frac{Dx^6}{6b}+\frac{Cx^5}{5b}+\frac{Bx^4}{4b}-\frac{Dx^4a}{4b^2}+\frac{Ax^3}{3b}-\frac{Cx^3a}{3b^2}-\frac{Bax^2}{2b^2}+\frac{Dx^2a^2}{2b^3}-\frac{aAx}{b^2}+\frac{Cxa^2}{b^3}+\frac{a^2\ln\left(bx^2+a\right)B}{2b^3}-\frac{a^3\ln\left(bx^2+a\right)D}{2b^4}+\frac{Aa^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3C}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/6*D*x^6/b+1/5*C*x^5/b+1/4/b*B*x^4-1/4/b^2*D*x^4*a+1/3/b*A*x^3-1
/3/b^2*C*x^3*a-1/2/b^2*B*x^2*a+1/2/b^3*D*x^2*a^2-1/b^2*A*x*a+1/b^
3*C*x*a^2+1/2*a^2/b^3*ln(b*x^2+a)*B-1/2*a^3/b^4*ln(b*x^2+a)*D+a^2
/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-a^3/b^3/(a*b)^(1/2)*ar
ctan(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262755, size = 1, normalized size = 0.01

$$\left[\frac{10\,Db^3x^6+12\,Cb^3x^5-15\left(Dab^2-Bb^3\right)x^4-20\left(Cab^2-Ab^3\right)x^3+30\left(Da^2b-Bab^2\right)x^2-30\left(Ca^2b-Aab^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 \cdots}{\cdots}\right.\cdots}{60\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 2
0*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*
b - A*a*b^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
 + a)) + 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^
2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*
b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2
- 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 60*(C*a^
2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4]
```

Sympy [A] time = 2.07887, size = 308, normalized size = 2.04

$$\frac{Cx^5}{5b} + \frac{Dx^6}{6b} + \left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right) + \left(-\frac{a^2(-Bb+Da)}{2b^4} + \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} + \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right) - \frac{x^4(-Bb+Da)}{4b^2} - \frac{x^3(-Ab+Ca)}{3b^2} + \frac{x^2(-Bab+Da^2)}{2b^3} + \frac{x(-Aab+Ca^2)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt
(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (B*a**2*b - D*a**3 -
2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*
a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) + (-a**2*(-B*b + D*a)/(2*b*
*4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (B*a**2*b -
 D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*
(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) - x**4*(-B*b + D*
a)/(4*b**2) - x**3*(-A*b + C*a)/(3*b**2) + x**2*(-B*a*b + D*a**2)
/(2*b**3) + x*(-A*a*b + C*a**2)/b**3
```

GIAC/XCAS [A] time = 0.221778, size = 217, normalized size = 1.44

$$-\frac{(Ca^3 - Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{(Da^3 - Ba^2b)\ln(bx^2+a)}{2\,b^4} + \frac{10\,Db^5x^6 + 12\,Cb^5x^5 - 15\,Dab^4x^4 + 15\,Bb^5x^4 - 20\,Cab^4x^3 + 20\,Ab^5x^3 + 30\,Da^2b^3x^2 - 30\,Bab^4x^2 + 60\,Ca^2b^3x - 60\,Aab^4x}{60\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D
*a^3 - B*a^2*b)*ln(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5
*x^5 - 15*D*a*b^4*x^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*
x^3 + 30*D*a^2*b^3*x^2 - 30*B*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a
*b^4*x)/b^6
```

3.87 $\int \frac{x^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2}\, dx$

Optimal. Leaf size=130

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*
D)*x^3)/(3*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B -
 a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a
 + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.27825, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*
D)*x^3)/(3*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B -
 a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a
 + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{\frac{3}{2}}\left(Bb-Da\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{7}{2}}}-\frac{a\left(Ab-Ca\right)\log\left(a+bx^2\right)}{2b^3}+\frac{x^3\left(Bb-Da\right)}{3b^2}+\frac{\left(Ab-Ca\right)\int x\,dx}{b^2}-\frac{\left(Bb-Da\right)\int a\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + a**(3/2)*(B*b - D*a)*atan(sqrt(b)*x
/sqrt(a))/b**(7/2) - a*(A*b - C*a)*log(a + b*x**2)/(2*b**3) + x**
3*(B*b - D*a)/(3*b**2) + (A*b - C*a)*Integral(x, x)/b**2 - (B*b -
 D*a)*Integral(a, x)/b**3
```

Mathematica [A] time = 0.209906, size = 114, normalized size = 0.88

$$\frac{x\left(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx)))\right)+30a(aC-Ab)\log\left(a+bx^2\right)}{60b^3}-\frac{a^{3/2}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) +
 (x*(60*a^2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*
(20*B + 3*x*(5*C + 4*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2]
)/(60*b^3)
```

Maple [A] time = 0.008, size = 152, normalized size = 1.2

$$\frac{Dx^5}{5\,b}+\frac{Cx^4}{4\,b}+\frac{Bx^3}{3\,b}-\frac{Dx^3a}{3\,b^2}+\frac{Ax^2}{2\,b}-\frac{Cx^2a}{2\,b^2}-\frac{Bxa}{b^2}+\frac{Dxa^2}{b^3}-\frac{a\ln\left(bx^2+a\right)A}{2\,b^2}+\frac{a^2\ln\left(bx^2+a\right)C}{2\,b^3}+\frac{a^2B}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3D}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/5*D*x^5/b+1/4*C*x^4/b+1/3*B/b*x^3-1/3/b^2*D*x^3*a+1/2/b*A*x^2-1
/2/b^2*C*x^2*a-1/b^2*B*x*a+1/b^3*D*x*a^2-1/2*a/b^2*ln(b*x^2+a)*A+
1/2*a^2/b^3*ln(b*x^2+a)*C+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*B-a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257928, size = 1, normalized size = 0.01

$$\left[\frac{12\,Db^2x^5+15\,Cb^2x^4-20\left(Dab-Bb^2\right)x^3-30\left(Cab-Ab^2\right)x^2-30\left(Da^2-Bab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+60\left(Da^2-\right.}{60\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*
(C*a*b - A*b^2)*x^2 - 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 +
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C
*a^2 - A*a*b)*log(b*x^2 + a))/b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*
x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 - 60*(D*a^2
 - B*a*b)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 60*(D*a^2 - B*a*b)*x +
30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [A] time = 2.04661, size = 269, normalized size = 2.07

$$\frac{Cx^4}{4b} + \frac{Dx^5}{5b} + \left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x + \frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}-\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right) + \left(\frac{a(-Ab+Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x + \frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}+\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right) - \frac{x^3(-Bb+Da)}{3b^2} - \frac{x^2(-Ab+Ca)}{2b^2} + \frac{x(-Bab+Da^2)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + (a*(-A*b + C*a)/(2*b**3) - sqrt(-a*
*3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3
*(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7
)))/(-B*a*b + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b*
*7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(
-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(
-B*a*b + D*a**2)) - x**3*(-B*b + D*a)/(3*b**2) - x**2*(-A*b + C*a
)/(2*b**2) + x*(-B*a*b + D*a**2)/b**3
```

GIAC/XCAS [A] time = 0.223358, size = 185, normalized size = 1.42

$$\frac{(Ca^2 - Aab)\ln(bx^2+a)}{2\,b^3} - \frac{(Da^3 - Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4x^5 + 15\,Cb^4x^4 - 20\,Dab^3x^3 + 20\,Bb^4x^3 - 30\,Cab^3x^2 + 30\,Ab^4x^2 + 60\,Da^2b^2x - 60\,Bab^3x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*ln(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^
4 - 20*D*a*b^3*x^3 + 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2
 + 60*D*a^2*b^2*x - 60*B*a*b^3*x)/b^5
```

3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) +
 (D*x^4)/(4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/b^(5/2) - (a*(b*B - a*D)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.248523, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) +
 (D*x^4)/(4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/b^(5/2) - (a*(b*B - a*D)*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}\,(Ab-Ca)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}-\frac{a\,(Bb-Da)\log\left(a+bx^2\right)}{2b^3}+(Ab-Ca)\int\frac{1}{b^2}\,dx+\frac{(Bb-Da)\int x\,dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) - sqrt(a)*(A*b - C*a)*atan(sqrt(b)*x/
sqrt(a))/b**(5/2) - a*(B*b - D*a)*log(a + b*x**2)/(2*b**3) + (A*b
 - C*a)*Integral(b**(-2), x) + (B*b - D*a)*Integral(x, x)/b**2
```

Mathematica [A] time = 0.101558, size = 95, normalized size = 0.86

$$\frac{bx\left(-6a(2C+Dx)+12Ab+bx\left(6B+4Cx+3Dx^2\right)\right)+12\sqrt{a}\sqrt{b}(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6a(aD-bB)\log\left(a+bx^2\right)}{12b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 1
2*Sqrt[a]*Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*
a*(-(b*B) + a*D)*Log[a + b*x^2])/(12*b^3)
```

Maple [A] time = 0.007, size = 128, normalized size = 1.2

$$\frac{Dx^4}{4b}+\frac{Cx^3}{3b}+\frac{Bx^2}{2b}-\frac{Dx^2a}{2b^2}+\frac{Ax}{b}-\frac{Cxa}{b^2}-\frac{a\ln\left(bx^2+a\right)B}{2b^2}+\frac{a^2\ln\left(bx^2+a\right)D}{2b^3}-\frac{aA}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2C}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/4*D*x^4/b+1/3*C*x^3/b+1/2*B/b*x^2-1/2/b^2*D*x^2*a+1/b*A*x-1/b^2
*C*x*a-1/2*a/b^2*ln(b*x^2+a)*B+1/2*a^2/b^3*ln(b*x^2+a)*D-a/b/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+a^2/b^2/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271887, size = 1, normalized size = 0.01

$$\left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\left(Dab-Bb^2\right)x^2-6\left(Cab-Ab^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\left(Cab-Ab^2\right)x+6\left(Da^2-Bab\right)\log}{12\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a
*b - A*b^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/
b^3, 1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12
*(C*a*b - A*b^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 12*(C*a*b - A*b^
2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [A] time = 1.90719, size = 243, normalized size = 2.19

$$\frac{Cx^3}{3b}+\frac{Dx^4}{4b}+\left(\frac{a\left(-Bb+Da\right)}{2b^3}-\frac{\sqrt{-ab^7}\left(-Ab+Ca\right)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)+\left(\frac{a\left(-Bb+Da\right)}{2b^3}+\frac{\sqrt{-ab^7}\left(-Ab+Ca\right)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)-\frac{x^2\left(-Bb+Da\right)}{2b^2}-\frac{x\left(-Ab+Ca\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*
b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*
(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A
*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b
+ C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)
/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*
b)) - x**2*(-B*b + D*a)/(2*b**2) - x*(-A*b + C*a)/b**2
```

GIAC/XCAS [A] time = 0.220411, size = 151, normalized size = 1.36

$$\frac{\left(Ca^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{\left(Da^2 - Bab\right)\ln\left(bx^2 + a\right)}{2\,b^3} + \frac{3\,Db^3x^4 + 4\,Cb^3x^3 - 6\,Dab^2x^2 + 6\,Bb^3x^2 - 12\,Cab^2x + 12\,Ab^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^
2 - B*a*b)*ln(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 -
6*D*a*b^2*x^2 + 6*B*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

Optimal. Leaf size=92

$$\frac{(Ab-aC)\log(a+bx^2)}{2b^2}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log
[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.210285, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(Ab-aC)\log(a+bx^2)}{2b^2}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log
[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{C\int x\,dx}{b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}\,(Bb-Da)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+(Bb-Da)\int\frac{1}{b^2}\,dx+\frac{(Ab-Ca)\log(a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*Integral(x, x)/b + D*x**3/(3*b) - sqrt(a)*(B*b - D*a)*atan(sqrt
(b)*x/sqrt(a))/b**(5/2) + (B*b - D*a)*Integral(b**(-2), x) + (A*b
 - C*a)*log(a + b*x**2)/(2*b**2)
```

Mathematica [A] time = 0.123848, size = 81, normalized size = 0.88

$$\frac{3(Ab-aC)\log(a+bx^2)+x(-6aD+6bB+bx(3C+2Dx))}{6b^2}+\frac{\sqrt{a}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x
*(6*b*B - 6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^
2])/(6*b^2)
```

Maple [A] time = 0.006, size = 106, normalized size = 1.2

$$\frac{Dx^3}{3\,b}+\frac{Cx^2}{2\,b}+\frac{Bx}{b}-\frac{Dxa}{b^2}+\frac{\ln\left(bx^2+a\right)A}{2\,b}-\frac{\ln\left(bx^2+a\right)aC}{2\,b^2}-\frac{Ba}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2D}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/3*D*x^3/b+1/2*C*x^2/b+B*x/b-1/b^2*D*x*a+1/2/b*ln(b*x^2+a)*A-1/2
/b^2*ln(b*x^2+a)*a*C-1/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*B+
1/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239165, size = 1, normalized size = 0.01

$$\left[\frac{2\,Dbx^3+3\,Cbx^2-3\,(Da-Bb)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2},\ \frac{2\,Dbx^3+3\,Cbx^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*D*b*x^3 + 3*C*b*x^2 - 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a
- A*b)*log(b*x^2 + a))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a -
 B*b)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 6*(D*a - B*b)*x - 3*(C*a -
A*b)*log(b*x^2 + a))/b^2]
```

Sympy [A] time = 1.80254, size = 211, normalized size = 2.29

$$\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)+\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)-\frac{x\,(-Bb+Da)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + (-(-A*b + C*a)/(2*b**2) - sqrt(-a*b
**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b
+ C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D
*a)) + (-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b*
*5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(
-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) - x*(-B*b + D*a)/b
**2
```

GIAC/XCAS [A] time = 0.237712, size = 119, normalized size = 1.29

$$-\frac{(Ca - Ab)\ln\left(bx^2 + a\right)}{2\,b^2} + \frac{\left(Da^2 - Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2x^3 + 3\,Cb^2x^2 - 6\,Dabx + 6\,Bb^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*ln(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*
a*b*x + 6*B*b^2*x)/b^3
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$

Optimal. Leaf size=73

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(Sqrt[a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.160775, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(Sqrt[a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{D\int x\,dx}{b}+\frac{\int C\,dx}{b}+\frac{(Bb-Da)\log\left(a+bx^2\right)}{2b^2}+\frac{(Ab-Ca)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] D*Integral(x, x)/b + Integral(C, x)/b + (B*b - D*a)*log(a + b*x**
2)/(2*b**2) + (A*b - C*a)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/
2))
```

Mathematica [A] time = 0.0754814, size = 68, normalized size = 0.93

$$\frac{\frac{2\sqrt{b}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)+bx(2C+Dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/Sqrt[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.005, size = 83, normalized size = 1.1

$$\frac{Dx^2}{2\,b}+\frac{Cx}{b}+\frac{\ln\left(bx^2+a\right)B}{2\,b}-\frac{\ln\left(bx^2+a\right)aD}{2\,b^2}+A\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{aC}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/2*D*x^2/b+C*x/b+1/2/b*ln(b*x^2+a)*B-1/2/b^2*ln(b*x^2+a)*a*D+1/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-1/b/(a*b)^(1/2)*arctan(x*b/(
a*b)^(1/2))*a*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233294, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Cab-Ab^{2}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)-\left(Dbx^{2}+2\,Cbx-\left(Da-Bb\right)\log\left(bx^{2}+a\right)\right)\sqrt{-ab}}{2\,\sqrt{-ab}b^{2}},\right.$$
$$\left.-\frac{2\left(Cab-Ab^{2}\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(Dbx^{2}+2\,Cbx-\left(Da-Bb\right)\log\left(bx^{2}+a\right)\right)\sqrt{ab}}{2\,\sqrt{ab}b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/2*((C*a*b - A*b^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*
x^2 + a)) - (D*b*x^2 + 2*C*b*x - (D*a - B*b)*log(b*x^2 + a))*sqrt
(-a*b))/(sqrt(-a*b)*b^2), -1/2*(2*(C*a*b - A*b^2)*arctan(sqrt(a*b
)*x/a) - (D*b*x^2 + 2*C*b*x - (D*a - B*b)*log(b*x^2 + a))*sqrt(a*
b))/(sqrt(a*b)*b^2)]
```

Sympy [A] time = 1.79122, size = 219, normalized size = 3.

$$\frac{Cx}{b}+\frac{Dx^{2}}{2b}$$
$$+\left(-\frac{-Bb+Da}{2b^{2}}-\frac{\sqrt{-ab^{5}}\left(-Ab+Ca\right)}{2ab^{4}}\right)\log\left(x+\frac{Bab-Da^{2}-2ab^{2}\left(-\frac{-Bb+Da}{2b^{2}}-\frac{\sqrt{-ab^{5}}(-Ab+Ca)}{2ab^{4}}\right)}{-Ab^{2}+Cab}\right)$$
$$+\left(-\frac{-Bb+Da}{2b^{2}}+\frac{\sqrt{-ab^{5}}\left(-Ab+Ca\right)}{2ab^{4}}\right)\log\left(x+\frac{Bab-Da^{2}-2ab^{2}\left(-\frac{-Bb+Da}{2b^{2}}+\frac{\sqrt{-ab^{5}}(-Ab+Ca)}{2ab^{4}}\right)}{-Ab^{2}+Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-
A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b
```

```
 + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b*
*2 + C*a*b)) + (-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*
a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a
*b))
```

GIAC/XCAS [A] time = 0.236983, size = 89, normalized size = 1.22

$$-\frac{(Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da - Bb)\ln\left(bx^2 + a\right)}{2\,b^2} + \frac{Dbx^2 + 2\,Cbx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b
)*ln(b*x^2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2)) + (A*Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.209655, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2)) + (A*Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{A\log{(x)}}{a}+\frac{\int D\,dx}{b}-\frac{(Ab-Ca)\log{\left(a+bx^2\right)}}{2ab}+\frac{(Bb-Da)\operatorname{atan}{\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] A*log(x)/a + Integral(D, x)/b - (A*b - C*a)*log(a + b*x**2)/(2*a*
b) + (B*b - D*a)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/2))
```

Mathematica [A] time = 0.0994818, size = 73, normalized size = 1.01

$$\frac{(aC-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}-\frac{(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b
^(3/2)) + (A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.009, size = 80, normalized size = 1.1

$$\frac{Dx}{b}+\frac{A\ln{(x)}}{a}-\frac{\ln{\left(bx^2+a\right)}A}{2a}+\frac{\ln{\left(bx^2+a\right)}C}{2b}+B\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{aD}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x)
```

```
[Out] D*x/b+A*ln(x)/a-1/2/a*ln(b*x^2+a)*A+1/2/b*ln(b*x^2+a)*C+1/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*B-a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270926, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Da^2 - Bab\right)\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right) - \left(2\,Dax + 2\,Ab\log\left(x\right) + \left(Ca - Ab\right)\log\left(bx^2 + a\right)\right)\sqrt{-ab}}{2\,\sqrt{-ab}ab},\right.$$
$$\left.-\frac{2\left(Da^2 - Bab\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right) - \left(2\,Dax + 2\,Ab\log\left(x\right) + \left(Ca - Ab\right)\log\left(bx^2 + a\right)\right)\sqrt{ab}}{2\,\sqrt{ab}ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a^2 - B*a*b)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*
x^2 + a)) - (2*D*a*x + 2*A*b*log(x) + (C*a - A*b)*log(b*x^2 + a))
*sqrt(-a*b))/(sqrt(-a*b)*a*b), -1/2*(2*(D*a^2 - B*a*b)*arctan(sqr
t(a*b)*x/a) - (2*D*a*x + 2*A*b*log(x) + (C*a - A*b)*log(b*x^2 + a
))*sqrt(a*b))/(sqrt(a*b)*a*b)]
```

Sympy [A] time = 39.1185, size = 1268, normalized size = 17.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] A*log(x)/a + D*x/b + ((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B
*b + D*a)/(2*a**2*b**3))*log(x + (-6*A**3*b**4 + 8*A**2*C*a*b**3
- 6*A**2*a*b**4*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b +
D*a)/(2*a**2*b**3)) + 2*A*B**2*a*b**3 - 4*A*B*D*a**2*b**2 - 2*A*C
**2*a**2*b**2 - 4*A*C*a**2*b**3*((-A*b + C*a)/(2*a*b) - sqrt(-a**
3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A*D**2*a**3*b + 12*A*a**2
*b**4*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a*
*2*b**3))**2 - 2*B**2*a**2*b**3*((-A*b + C*a)/(2*a*b) - sqrt(-a**
3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 4*B*D*a**3*b**2*((-A*b + C*
a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*C**
2*a**3*b**2*((-A*b + C*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)
```

```
/(2*a**2*b**3)) - 4*C*a**3*b**3*((-A*b + C*a)/(2*a*b) - sqrt(-a**
3*b**3)*(-B*b + D*a)/(2*a**2*b**3))**2 - 2*D**2*a**4*b*((-A*b + C
*a)/(2*a*b) - sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)))/(-9*A
**2*B*b**4 + 9*A**2*D*a*b**3 + 6*A*B*C*a*b**3 - 6*A*C*D*a**2*b**2
 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - B*C**2*a**2*b**2 - 3*B*D**2
*a**3*b + C**2*D*a**3*b + D**3*a**4)) + ((-A*b + C*a)/(2*a*b) + s
qrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3))*log(x + (-6*A**3*b**4
 + 8*A**2*C*a*b**3 - 6*A**2*a*b**4*((-A*b + C*a)/(2*a*b) + sqrt(-
a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A*B**2*a*b**3 - 4*A*B*
D*a**2*b**2 - 2*A*C**2*a**2*b**2 - 4*A*C*a**2*b**3*((-A*b + C*a)/
(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 2*A*D**2
*a**3*b + 12*A*a**2*b**4*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)
*(-B*b + D*a)/(2*a**2*b**3))**2 - 2*B**2*a**2*b**3*((-A*b + C*a)/
(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3)) + 4*B*D*a*
*3*b**2*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*
a**2*b**3)) + 2*C**2*a**3*b**2*((-A*b + C*a)/(2*a*b) + sqrt(-a**3
*b**3)*(-B*b + D*a)/(2*a**2*b**3)) - 4*C*a**3*b**3*((-A*b + C*a)/
(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2*a**2*b**3))**2 - 2*D**
2*a**4*b*((-A*b + C*a)/(2*a*b) + sqrt(-a**3*b**3)*(-B*b + D*a)/(2
*a**2*b**3)))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 6*A*B*C*a*b**3
- 6*A*C*D*a**2*b**2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - B*C**2*a
**2*b**2 - 3*B*D**2*a**3*b + C**2*D*a**3*b + D**3*a**4))
```

GIAC/XCAS [A] time = 0.224664, size = 89, normalized size = 1.24

$$\frac{Dx}{b} + \frac{A\ln\left(|x|\right)}{a} - \frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{(Ca - Ab)\ln\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] D*x/b + A*ln(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b) + 1/2*(C*a - A*b)*ln(b*x^2 + a)/(a*b)
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=76

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b]) + (B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.20578, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b]) + (B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rubi in Sympy [A] time = 36.9897, size = 61, normalized size = 0.8

$$-\frac{A}{ax}+\frac{B\log(x)}{a}-\frac{(Bb-Da)\log\left(a+bx^2\right)}{2ab}-\frac{(Ab-Ca)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) + B*log(x)/a - (B*b - D*a)*log(a + b*x**2)/(2*a*b) - (A*
b - C*a)*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*sqrt(b))
```

Mathematica [A] time = 0.0955344, size = 75, normalized size = 0.99

$$\frac{(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}+\frac{(aD-bB)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2
)*Sqrt[b]) + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*
b)
```

Maple [A] time = 0.012, size = 83, normalized size = 1.1

$$-\frac{A}{ax}+\frac{B\ln(x)}{a}-\frac{\ln\left(bx^2+a\right)B}{2a}+\frac{\ln\left(bx^2+a\right)D}{2b}-\frac{Ab}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+C\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x)
```

```
[Out] -A/a/x+B*ln(x)/a-1/2/a*ln(b*x^2+a)*B+1/2/b*ln(b*x^2+a)*D-1/a/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*A*b+1/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265203, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Cab-Ab^2\right)x\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-\left(2\,Bbx\log\left(x\right)+\left(Da-Bb\right)x\log\left(bx^2+a\right)-2\,Ab\right)\sqrt{-ab}}{2\,\sqrt{-ab}abx},\ \frac{2\left(Cab-Ab^2\right)x\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((C*a*b - A*b^2)*x*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/
(b*x^2 + a)) - (2*B*b*x*log(x) + (D*a - B*b)*x*log(b*x^2 + a) - 2
*A*b)*sqrt(-a*b))/(sqrt(-a*b)*a*b*x), 1/2*(2*(C*a*b - A*b^2)*x*ar
ctan(sqrt(a*b)*x/a) + (2*B*b*x*log(x) + (D*a - B*b)*x*log(b*x^2 +
 a) - 2*A*b)*sqrt(a*b))/(sqrt(a*b)*a*b*x)]
```

Sympy [A] time = 36.1482, size = 1258, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) + B*log(x)/a + ((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*
(-A*b + C*a)/(2*a**3*b**2))*log(x + (-2*A**2*B*a*b**3 + 2*A**2*a*
*2*b**3*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*
a**3*b**2)) + 4*A*B*C*a**2*b**2 - 4*A*C*a**3*b**2*((-B*b + D*a)/(
2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 6*B**3*a*
*2*b**2 - 8*B**2*D*a**3*b + 6*B**2*a**3*b**2*((-B*b + D*a)/(2*a*b
) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*B*C**2*a**3*
b + 2*B*D**2*a**4 + 4*B*D*a**4*b*((-B*b + D*a)/(2*a*b) - sqrt(-a*
*3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 12*B*a**4*b**2*((-B*b + D*
a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2 + 2*
C**2*a**4*b*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*b**3)*(-A*b + C*a)
/(2*a**3*b**2)) - 2*D**2*a**5*((-B*b + D*a)/(2*a*b) - sqrt(-a**3*
b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*D*a**5*b*((-B*b + D*a)/(2*a
*b) - sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2)/(-A**3*b**
4 + 3*A**2*C*a*b**3 - 9*A*B**2*a*b**3 + 6*A*B*D*a**2*b**2 - 3*A*C
**2*a**2*b**2 - A*D**2*a**3*b + 9*B**2*C*a**2*b**2 - 6*B*C*D*a**3
```

```
*b + C**3*a**3*b + C*D**2*a**4)) + ((-B*b + D*a)/(2*a*b) + sqrt(-
a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))*log(x + (-2*A**2*B*a*b**3
+ 2*A**2*a**2*b**3*((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b
 + C*a)/(2*a**3*b**2)) + 4*A*B*C*a**2*b**2 - 4*A*C*a**3*b**2*((-B
*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))
+ 6*B**3*a**2*b**2 - 8*B**2*D*a**3*b + 6*B**2*a**3*b**2*((-B*b +
D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 2*B
*C**2*a**3*b + 2*B*D**2*a**4 + 4*B*D*a**4*b*((-B*b + D*a)/(2*a*b)
 + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) - 12*B*a**4*b**2*
((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**
2))**2 + 2*C**2*a**4*b*((-B*b + D*a)/(2*a*b) + sqrt(-a**3*b**3)*(
-A*b + C*a)/(2*a**3*b**2)) - 2*D**2*a**5*((-B*b + D*a)/(2*a*b) +
sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2)) + 4*D*a**5*b*((-B*b
+ D*a)/(2*a*b) + sqrt(-a**3*b**3)*(-A*b + C*a)/(2*a**3*b**2))**2)
/(-A**3*b**4 + 3*A**2*C*a*b**3 - 9*A*B**2*a*b**3 + 6*A*B*D*a**2*b
**2 - 3*A*C**2*a**2*b**2 - A*D**2*a**3*b + 9*B**2*C*a**2*b**2 - 6
*B*C*D*a**3*b + C**3*a**3*b + C*D**2*a**4))
```

GIAC/XCAS [A] time = 0.241668, size = 92, normalized size = 1.21

$$\frac{B\ln(|x|)}{a}+\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{(Da-Bb)\ln\left(bx^2+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] B*ln(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
+ 1/2*(D*a - B*b)*ln(b*x^2 + a)/(a*b) - A/(a*x)
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=92

$$-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(3/2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log
[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.237514, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(3/2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log
[a + b*x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 43.2924, size = 76, normalized size = 0.83

$$-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-Ca)\log(x)}{a^2}+\frac{(Ab-Ca)\log\left(a+bx^2\right)}{2a^2}-\frac{(Bb-Da)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) - B/(a*x) - (A*b - C*a)*log(x)/a**2 + (A*b - C*a)*l
og(a + b*x**2)/(2*a**2) - (B*b - D*a)*atan(sqrt(b)*x/sqrt(a))/(a*
*(3/2)*sqrt(b))
```

Mathematica [A] time = 0.14513, size = 84, normalized size = 0.91

$$\frac{(Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-Ab)-\frac{aA}{x^2}+\frac{2\sqrt{a}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)
*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.011, size = 102, normalized size = 1.1

$$-\frac{A}{2\,a x^{2}}-\frac{B}{a x}-\frac{A\ln\left(x\right) b}{a^{2}}+\frac{\ln\left(x\right) C}{a}+\frac{b\ln\left(b x^{2}+a\right) A}{2\,a^{2}}-\frac{\ln\left(b x^{2}+a\right) C}{2\,a}-\frac{B b}{a}\arctan\left(b x\frac{1}{\sqrt{a b}}\right)\frac{1}{\sqrt{a b}}+D\arctan\left(b x\frac{1}{\sqrt{a b}}\right)\frac{1}{\sqrt{a b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-B/a/x-1/a^2*ln(x)*A*b+1/a*ln(x)*C+1/2/a^2*b*ln(b*x^2
+a)*A-1/2/a*ln(b*x^2+a)*C-1/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
*b*B+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282118, size = 1, normalized size = 0.01

$$\left[-\frac{\left(D a^{2}-B a b\right) x^{2} \log\left(-\frac{2\,a b x-\left(b x^{2}-a\right)\sqrt{-a b}}{b x^{2}+a}\right)+\left(\left(C a-A b\right) x^{2}\log\left(b x^{2}+a\right)-2\left(C a-A b\right) x^{2}\log\left(x\right)+2\,B a x+A a\right)\sqrt{-a b}}{2\sqrt{-a b}\,a^{2} x^{2}},\ 2\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a^2 - B*a*b)*x^2*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b)
)/(b*x^2 + a)) + ((C*a - A*b)*x^2*log(b*x^2 + a) - 2*(C*a - A*b)*
x^2*log(x) + 2*B*a*x + A*a)*sqrt(-a*b))/(sqrt(-a*b)*a^2*x^2), 1/2
*(2*(D*a^2 - B*a*b)*x^2*arctan(sqrt(a*b)*x/a) - ((C*a - A*b)*x^2*
log(b*x^2 + a) - 2*(C*a - A*b)*x^2*log(x) + 2*B*a*x + A*a)*sqrt(a
*b))/(sqrt(a*b)*a^2*x^2)]
```

Sympy [A] time = 39.1408, size = 1686, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] (-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))*
log(x + (-6*A**3*b**4 + 18*A**2*C*a*b**3 + 6*A**2*a**2*b**3*(-(-A
*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 2*A
*B**2*a*b**3 - 4*A*B*D*a**2*b**2 - 18*A*C**2*a**2*b**2 - 12*A*C*a
**3*b**2*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*
a**4*b)) + 2*A*D**2*a**3*b + 12*A*a**4*b**2*(-(-A*b + C*a)/(2*a**
```

```
2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 - 2*B**2*C*a**2*b*
*2 + 2*B**2*a**3*b**2*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B
*b + D*a)/(2*a**4*b)) + 4*B*C*D*a**3*b - 4*B*D*a**4*b*(-(-A*b + C
*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 6*C**3*a*
*3*b + 6*C**2*a**4*b*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*
b + D*a)/(2*a**4*b)) - 2*C*D**2*a**4 - 12*C*a**5*b*(-(-A*b + C*a)
/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 + 2*D**2*a*
*5*(-(-A*b + C*a)/(2*a**2) - sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b
)))/(-9*A**2*B*b**4 + 9*A**2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A*C*
D*a**2*b**2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - 9*B*C**2*a**2*b*
*2 - 3*B*D**2*a**3*b + 9*C**2*D*a**3*b + D**3*a**4)) + (-(-A*b +
C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))*log(x + (-
6*A**3*b**4 + 18*A**2*C*a*b**3 + 6*A**2*a**2*b**3*(-(-A*b + C*a)/
(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 2*A*B**2*a*b*
*3 - 4*A*B*D*a**2*b**2 - 18*A*C**2*a**2*b**2 - 12*A*C*a**3*b**2*(
-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) +
 2*A*D**2*a**3*b + 12*A*a**4*b**2*(-(-A*b + C*a)/(2*a**2) + sqrt(
-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 - 2*B**2*C*a**2*b**2 + 2*B**
2*a**3*b**2*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/
(2*a**4*b)) + 4*B*C*D*a**3*b - 4*B*D*a**4*b*(-(-A*b + C*a)/(2*a**
2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)) + 6*C**3*a**3*b + 6*C
**2*a**4*b*(-(-A*b + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(
2*a**4*b)) - 2*C*D**2*a**4 - 12*C*a**5*b*(-(-A*b + C*a)/(2*a**2)
+ sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b))**2 + 2*D**2*a**5*(-(-A*b
 + C*a)/(2*a**2) + sqrt(-a**5*b)*(-B*b + D*a)/(2*a**4*b)))/(-9*A*
*2*B*b**4 + 9*A**2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A*C*D*a**2*b**
2 - B**3*a*b**3 + 3*B**2*D*a**2*b**2 - 9*B*C**2*a**2*b**2 - 3*B*D
**2*a**3*b + 9*C**2*D*a**3*b + D**3*a**4)) - (A + 2*B*x)/(2*a*x**
2) + (-A*b + C*a)*log(x + (-6*A**3*b**4 + 18*A**2*C*a*b**3 + 6*A*
*2*b**3*(-A*b + C*a) + 2*A*B**2*a*b**3 - 4*A*B*D*a**2*b**2 - 18*A
*C**2*a**2*b**2 - 12*A*C*a*b**2*(-A*b + C*a) + 2*A*D**2*a**3*b +
12*A*b**2*(-A*b + C*a)**2 - 2*B**2*C*a**2*b**2 + 2*B**2*a*b**2*(-
A*b + C*a) + 4*B*C*D*a**3*b - 4*B*D*a**2*b*(-A*b + C*a) + 6*C**3*
a**3*b + 6*C**2*a**2*b*(-A*b + C*a) - 2*C*D**2*a**4 - 12*C*a*b*(-
A*b + C*a)**2 + 2*D**2*a**3*(-A*b + C*a))/(-9*A**2*B*b**4 + 9*A**
2*D*a*b**3 + 18*A*B*C*a*b**3 - 18*A*C*D*a**2*b**2 - B**3*a*b**3 +
 3*B**2*D*a**2*b**2 - 9*B*C**2*a**2*b**2 - 3*B*D**2*a**3*b + 9*C*
*2*D*a**3*b + D**3*a**4))/a**2
```

GIAC/XCAS [A] time = 0.226842, size = 108, normalized size = 1.17

$$\frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca - Ab)\ln\left(bx^2 + a\right)}{2\,a^2} + \frac{(Ca - Ab)\ln\left(|x|\right)}{a^2} - \frac{2\,Bax + Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)
*ln(b*x^2 + a)/a^2 + (C*a - A*b)*ln(abs(x))/a^2 - 1/2*(2*B*a*x +
A*a)/(a^2*x^2)
```

3.94 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{a}(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3Ab-5aC)}{2b^3}-\frac{x^3(3Ab-5aC)}{6ab^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(2bB-3aD)}{2b^3}+\frac{Dx^4}{4b^2}$$

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3
*A*b - 5*a*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*
D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5
*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*
D)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.552634, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{\sqrt{a}(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3Ab-5aC)}{2b^3}-\frac{x^3(3Ab-5aC)}{6ab^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(2bB-3aD)}{2b^3}+\frac{Dx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3
*A*b - 5*a*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*
D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5
*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*
D)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}-\frac{\sqrt{a}\,(3Ab-5Ca)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{7}{2}}}+\frac{ax\,(Ab-Ca+x\,(Bb-Da))}{2b^3\,(a+bx^2)}-\frac{a\,(2Bb-3Da)\log\left(a+bx^2\right)}{2b^4}+\frac{x\,(Ab-2Ca)}{b^3}+\frac{(Bb-2Da)\int x\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) - sqrt(a)*(3*A*b - 5*C*a)*atan(
sqrt(b)*x/sqrt(a))/(2*b**(7/2)) + a*x*(A*b - C*a + x*(B*b - D*a))
/(2*b**3*(a + b*x**2)) - a*(2*B*b - 3*D*a)*log(a + b*x**2)/(2*b**
4) + x*(A*b - 2*C*a)/b**3 + (B*b - 2*D*a)*Integral(x, x)/b**3
```

Mathematica [A] time = 0.240112, size = 139, normalized size = 0.79

$$\frac{\frac{6a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{a+bx^2}+12bx(Ab-2aC)+6\sqrt{a}\sqrt{b}(5aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6bx^2(bB-2aD)+6a(3aD-2bB)\log\left(a+b\right.}{12b^4}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b
^2*D*x^4 + (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) +
6*Sqrt[a]*Sqrt[b]*(-3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] +
6*a*(-2*b*B + 3*a*D)*Log[a + b*x^2])/(12*b^4)
```

Maple [A]   time = 0.013, size = 201, normalized size = 1.1

$$\frac{Dx^4}{4\,b^2}+\frac{Cx^3}{3\,b^2}+\frac{Bx^2}{2\,b^2}-\frac{Dx^2a}{b^3}+\frac{Ax}{b^2}-2\,\frac{Cxa}{b^3}+\frac{aAx}{2\,b^2\,(bx^2+a)}-\frac{Cxa^2}{2\,b^3\,(bx^2+a)}$$
$$-\frac{a^2B}{2\,b^3\,(bx^2+a)}+\frac{a^3D}{2\,b^4\,(bx^2+a)}-\frac{\ln\left(bx^2+a\right)Ba}{b^3}+\frac{3\,a^2\ln\left(bx^2+a\right)D}{2\,b^4}$$
$$-\frac{3\,Aa}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,a^2C}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*D*x^4/b^2+1/3/b^2*C*x^3+1/2*B*x^2/b^2-1/b^3*D*x^2*a+1/b^2*A*x
-2/b^3*C*x*a+1/2*a/b^2/(b*x^2+a)*A*x-1/2*a^2/b^3/(b*x^2+a)*C*x-1/
2/b^3*a^2/(b*x^2+a)*B+1/2*a^3/b^4/(b*x^2+a)*D-1/b^3*ln(b*x^2+a)*B
*a+3/2*a^2/b^4*ln(b*x^2+a)*D-3/2*a/b^2/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*A+5/2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*C
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.233795, size = 1, normalized size = 0.01

$$\Bigg[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\left(3\,Dab^2-2\,Bb^3\right)x^4+6\,Da^3-6\,Ba^2b-4\left(5\,Cab^2-3\,Ab^3\right)x^3-6\left(2\,Da^2b-Bab^2\right)x^2-3\left(5\,Ca^2b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 +
6*D*a^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b
- B*a*b^2)*x^2 - 3*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)
*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
- 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^2*b + (3*D*a^2
*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4), 1/12*(3*D
*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)
*x^2 + 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt
(a/b)*arctan(x/sqrt(a/b)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*
```

```
a^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b
^5*x^2 + a*b^4)]
```

Sympy [A] time = 7.29329, size = 333, normalized size = 1.89

$$\frac{Cx^3}{3b^2} + \frac{Dx^4}{4b^2} + \left(\frac{a\left(-2Bb + 3Da\right)}{2b^4} - \frac{\sqrt{-ab^9}\left(-3Ab + 5Ca\right)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} - \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2 + 5Cab}\right) + \left(\frac{a\left(-2Bb + 3Da\right)}{2b^4} + \frac{\sqrt{-ab^9}\left(-3Ab + 5Ca\right)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} + \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2 + 5Cab}\right) - \frac{Ba^2b - Da^3 + x\left(-Aab^2 + Ca^2b\right)}{2ab^4 + 2b^5x^2} - \frac{x^2\left(-Bb + 2Da\right)}{2b^3} - \frac{x\left(-Ab + 2Ca\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + (a*(-2*B*b + 3*D*a)/(2*b**4)
- sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D
*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A
*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (a*(-2*B*b + 3*D*
a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log(x + (4
*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a
*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) - (B*a*
*2*b - D*a**3 + x*(-A*a*b**2 + C*a**2*b))/(2*a*b**4 + 2*b**5*x**2
) - x**2*(-B*b + 2*D*a)/(2*b**3) - x*(-A*b + 2*C*a)/b**3
```

GIAC/XCAS [A] time = 0.224767, size = 215, normalized size = 1.22

$$\frac{\left(5\,Ca^2 - 3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{\left(3\,Da^2 - 2\,Bab\right)\ln\left(bx^2 + a\right)}{2\,b^4} + \frac{Da^3 - Ba^2b - \left(Ca^2b - Aab^2\right)x}{2\,(bx^2 + a)b^4} + \frac{3\,Db^6x^4 + 4\,Cb^6x^3 - 12\,Dab^5x^2 + 6\,Bb^6x^2 - 24\,Cab^5x + 12\,Ab^6x}{12\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1
/2*(3*D*a^2 - 2*B*a*b)*ln(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b -
 (C*a^2*b - A*a*b^2)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4
*C*b^6*x^3 - 12*D*a*b^5*x^2 + 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b
^6*x)/b^8
```

3.95 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=154

$$\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(Ab-2aC)}{2ab^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3bB-5aD)}{2b^3}+\frac{Dx^3}{3b^2}$$

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*
x^3)/(3*b^2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a
+ b*x^2)) - (Sqrt[a]*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.50113, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(Ab-2aC)}{2ab^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3bB-5aD)}{2b^3}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*
x^3)/(3*b^2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a
+ b*x^2)) - (Sqrt[a]*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{C\int x\,dx}{b^2}+\frac{Dx^3}{3b^2}-\frac{\sqrt{a}\,(3Bb-5Da)\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{7}{2}}}+\frac{x\,(Bb-2Da)}{b^3}+\frac{x\,(a\,(Bb-Da)-bx\,(Ab-Ca))}{2b^3\,(a+bx^2)}+\frac{(Ab-2Ca)\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*Integral(x, x)/b**2 + D*x**3/(3*b**2) - sqrt(a)*(3*B*b - 5*D*a)
*atan(sqrt(b)*x/sqrt(a))/(2*b**(7/2)) + x*(B*b - 2*D*a)/b**3 + x*
(a*(B*b - D*a) - b*x*(A*b - C*a))/(2*b**3*(a + b*x**2)) + (A*b -
2*C*a)*log(a + b*x**2)/(2*b**3)
```

Mathematica [A] time = 0.143791, size = 128, normalized size = 0.83

$$\frac{a(-a(C+Dx)+Ab+bBx)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}+\frac{\sqrt{a}(5aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(bB-2aD)}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A
*b + b*B*x - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B
 + 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*
C)*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.013, size = 177, normalized size = 1.2

$$\frac{Dx^3}{3\,b^2}+\frac{Cx^2}{2\,b^2}+\frac{Bx}{b^2}-2\,\frac{Dxa}{b^3}+\frac{Bxa}{2\,b^2\,(bx^2+a)}-\frac{Dxa^2}{2\,b^3\,(bx^2+a)}+\frac{aA}{2\,b^2\,(bx^2+a)}-\frac{a^2C}{2\,b^3\,(bx^2+a)}+\frac{\ln\left(bx^2+a\right)A}{2\,b^2}-\frac{\ln\left(bx^2+a\right)aC}{b^3}-\frac{3\,Ba}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,a^2D}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*D*x^3/b^2+1/2/b^2*C*x^2+B*x/b^2-2/b^3*D*x*a+1/2/b^2/(b*x^2+a)
*B*x*a-1/2/b^3/(b*x^2+a)*D*x*a^2+1/2/b^2*a/(b*x^2+a)*A-1/2/b^3/(b
*x^2+a)*a^2*C+1/2/b^2*ln(b*x^2+a)*A-1/b^3*ln(b*x^2+a)*a*C-3/2/b^2
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*B+5/2/b^3/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*a^2*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238777, size = 1, normalized size = 0.01

$$\left[\frac{4\,Db^2x^5+6\,Cb^2x^4+6\,Cabx^2-4\left(5\,Dab-3\,Bb^2\right)x^3-6\,Ca^2+6\,Aab-3\left(5\,Da^2-3\,Bab+\left(5\,Dab-3\,Bb^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\Bigg(\ldots}{12\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B
*b^2)*x^3 - 6*C*a^2 + 6*A*a*b - 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b -
 3*B*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)) - 6*(5*D*a^2 - 3*B*a*b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*
b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/6*(2*D*b^2*x
^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b - 3*B*b^2)*x^3 - 3*C*
a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*s
qrt(a/b)*arctan(x/sqrt(a/b)) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*(2*C*a
^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*
```

```
b^3)]
```

Sympy [A] time = 6.90362, size = 287, normalized size = 1.86

$$\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)-\frac{-Aab+Ca^2+x\left(-Bab+Da^2\right)}{2ab^3+2b^4x^2}-\frac{x\,(-Bb+2Da)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + (-(-A*b + 2*C*a)/(2*b**3) - s
qrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a +
 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a
)/(4*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt
(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*
b**3*(-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(
4*b**7)))/(-3*B*b + 5*D*a)) - (-A*a*b + C*a**2 + x*(-B*a*b + D*a*
*2))/(2*a*b**3 + 2*b**4*x**2) - x*(-B*b + 2*D*a)/b**3
```

GIAC/XCAS [A] time = 0.223106, size = 177, normalized size = 1.15

$$-\frac{(2\,Ca-Ab)\ln\left(bx^2+a\right)}{2\,b^3}+\frac{\left(5\,Da^2-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\,(bx^2+a)b^3}+\frac{2\,Db^4x^3+3\,Cb^4x^2-12\,Dab^3x+6\,Bb^4x}{6\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*ln(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2
 - B*a*b)*x)/((b*x^2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 -
 12*D*a*b^3*x + 6*B*b^4*x)/b^6
```

3.96 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=134

$$\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*
D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log
[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.478134, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*
D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log
[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx}{b^2}+\frac{D\int x\,dx}{b^2}-\frac{x\left(Ab-Ca+x\left(Bb-Da\right)\right)}{2b^2\left(a+bx^2\right)}+\frac{(Bb-2Da)\log\left(a+bx^2\right)}{2b^3}+\frac{(Ab-3Ca)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*Integral(x, x)/b**2 - x*(A*b - C*a + x*(B*b - D*a))/
(2*b**2*(a + b*x**2)) + (B*b - 2*D*a)*log(a + b*x**2)/(2*b**3) +
(A*b - 3*C*a)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.147992, size = 100, normalized size = 0.75

$$\frac{\frac{a^2(-D)+ab(B+Cx)-Ab^2x}{a+bx^2}+\frac{\sqrt{b}(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log\left(a+bx^2\right)+2bCx+bDx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*
x^2) + (Sqrt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a
] + (b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.013, size = 154, normalized size = 1.2

$$\frac{Dx^2}{2\,b^2}+\frac{Cx}{b^2}-\frac{Ax}{2\,b\,(bx^2+a)}+\frac{Cxa}{2\,b^2\,(bx^2+a)}+\frac{Ba}{2\,b^2\,(bx^2+a)}-\frac{a^2D}{2\,b^3\,(bx^2+a)}+\frac{B\ln\left(bx^2+a\right)}{2\,b^2}-\frac{\ln\left(bx^2+a\right)\,aD}{b^3}+\frac{A}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,aC}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*D*x^2/b^2+1/b^2*C*x-1/2/b/(b*x^2+a)*A*x+1/2/b^2/(b*x^2+a)*C*x
*a+1/2/b^2/(b*x^2+a)*B*a-1/2/b^3/(b*x^2+a)*a^2*D+1/2*B/b^2*ln(b*x
^2+a)-1/b^3*ln(b*x^2+a)*a*D+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*A-3/2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239986, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,Ca^2b-Aab^2+\left(3\,Cab^2-Ab^3\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(Db^2x^4+2\,Cb^2x^3+Dabx^2-Da^2+Bab+\left(3\,Cab-A\right.\right.}{4\left(b^4x^2+ab^3\right)\sqrt{-ab}}\right.$$

$$-\frac{\left(3\,Ca^2b-Aab^2+\left(3\,Cab^2-Ab^3\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(Db^2x^4+2\,Cb^2x^3+Dabx^2-Da^2+Bab+\left(3\,Cab-Ab^2\right)x-\left(2\,D\right.\right.}{2\left(b^4x^2+ab^3\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*C*a^2*b - A*a*b^2 + (3*C*a*b^2 - A*b^3)*x^2)*log((2*a*b
*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(D*b^2*x^4 + 2*C*b^
2*x^3 + D*a*b*x^2 - D*a^2 + B*a*b + (3*C*a*b - A*b^2)*x - (2*D*a^
2 - B*a*b + (2*D*a*b - B*b^2)*x^2)*log(b*x^2 + a))*sqrt(-a*b))/((
b^4*x^2 + a*b^3)*sqrt(-a*b)), -1/2*((3*C*a^2*b - A*a*b^2 + (3*C*a
*b^2 - A*b^3)*x^2)*arctan(sqrt(a*b)*x/a) - (D*b^2*x^4 + 2*C*b^2*x
^3 + D*a*b*x^2 - D*a^2 + B*a*b + (3*C*a*b - A*b^2)*x - (2*D*a^2 -
 B*a*b + (2*D*a*b - B*b^2)*x^2)*log(b*x^2 + a))*sqrt(a*b))/((b^4*
x^2 + a*b^3)*sqrt(a*b))]
```

Sympy [A] time = 6.5623, size = 284, normalized size = 2.12

$$\frac{Cx}{b^2} + \frac{Dx^2}{2b^2} + \left(-\frac{-Bb + 2Da}{2b^3} - \frac{\sqrt{-ab^7}\,(-Ab + 3Ca)}{4ab^6}\right)\log\left(x + \frac{2Bab - 4Da^2 - 4ab^3\left(-\frac{-Bb+2Da}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2 + 3Cab}\right) + \left(-\frac{-Bb + 2Da}{2b^3} + \frac{\sqrt{-ab^7}\,(-Ab + 3Ca)}{4ab^6}\right)\log\left(x + \frac{2Bab - 4Da^2 - 4ab^3\left(-\frac{-Bb+2Da}{2b^3} + \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2 + 3Cab}\right) + \frac{Bab - Da^2 + x\left(-Ab^2 + Cab\right)}{2ab^3 + 2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*
b**7)*(-A*b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*
a*b**3*(-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(
4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (-(-B*b + 2*D*a)/(2*b**3) + sq
rt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**
2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*
C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x*(-A*
b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

GIAC/XCAS [A] time = 0.223688, size = 150, normalized size = 1.12

$$-\frac{(3\,Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{(2\,Da - Bb)\ln\left(bx^2 + a\right)}{2\,b^3} + \frac{Db^2x^2 + 2\,Cb^2x}{2\,b^4} - \frac{Da^2 - Bab - \left(Cab - Ab^2\right)x}{2\,(bx^2 + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2
*D*a - B*b)*ln(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 -
 1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x
^2)) + ((b*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(
5/2)) + (C*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.254629, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x
^2)) + ((b*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(
5/2)) + (C*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 32.4923, size = 83, normalized size = 0.82

$$\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{3Dx}{2b^2}-\frac{A+Bx+Cx^2+Dx^3}{2b\left(a+bx^2\right)}+\frac{(Bb-3Da)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*log(a + b*x**2)/(2*b**2) + 3*D*x/(2*b**2) - (A + B*x + C*x**2 +
 D*x**3)/(2*b*(a + b*x**2)) + (B*b - 3*D*a)*atan(sqrt(b)*x/sqrt(a
))/(2*sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.0998942, size = 92, normalized size = 0.91

$$\frac{aC+aDx-Ab-bBx}{2b^2\left(a+bx^2\right)}-\frac{(3aD-bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) -
((-(b*B) + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)
) + (C*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.013, size = 127, normalized size = 1.3

$$\frac{Dx}{b^2}-\frac{Bx}{2\,b\,(bx^2+a)}+\frac{Dxa}{2\,b^2\,(bx^2+a)}-\frac{A}{2\,b\,(bx^2+a)}+\frac{aC}{2\,b^2\,(bx^2+a)}+\frac{C\ln\left(bx^2+a\right)}{2\,b^2}+\frac{B}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,aD}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] D*x/b^2-1/2/b/(b*x^2+a)*x*B+1/2/b^2/(b*x^2+a)*D*x*a-1/2/b/(b*x^2+
a)*A+1/2/b^2/(b*x^2+a)*a*C+1/2*C*ln(b*x^2+a)/b^2+1/2/b/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*B-3/2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*a*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240564, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,Da^2-Bab+\left(3\,Dab-Bb^2\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(2\,Dbx^3+Ca-Ab+\left(3\,Da-Bb\right)x+\left(Cbx^2+Ca\right)\log\left(bx^2+a\right)\right)\sqrt{-ab}}{4\left(b^3x^2+ab^2\right)\sqrt{-ab}},\ -\frac{\left(3\,Da^2-Bab+\left(3\,Dab-Bb^2\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(2\,Dbx^3+Ca-Ab+\left(3\,Da-Bb\right)x+\left(Cbx^2+Ca\right)\log\left(bx^2+a\right)\right)\sqrt{ab}}{2\left(b^3x^2+ab^2\right)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*D*a^2 - B*a*b + (3*D*a*b - B*b^2)*x^2)*log((2*a*b*x + (
b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(2*D*b*x^3 + C*a - A*b +
(3*D*a - B*b)*x + (C*b*x^2 + C*a)*log(b*x^2 + a))*sqrt(-a*b))/((b
^3*x^2 + a*b^2)*sqrt(-a*b)), -1/2*((3*D*a^2 - B*a*b + (3*D*a*b -
B*b^2)*x^2)*arctan(sqrt(a*b)*x/a) - (2*D*b*x^3 + C*a - A*b + (3*D
*a - B*b)*x + (C*b*x^2 + C*a)*log(b*x^2 + a))*sqrt(a*b))/((b^3*x^
2 + a*b^2)*sqrt(a*b))]
```

Sympy [A] time = 5.39716, size = 212, normalized size = 2.1

$$\frac{Dx}{b^2}+\left(\frac{C}{2b^2}-\frac{\sqrt{-ab^5}\left(-Bb+3Da\right)}{4ab^5}\right)\log\left(x+\frac{2Ca-4ab^2\left(\frac{C}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right)+\left(\frac{C}{2b^2}+\frac{\sqrt{-ab^5}\left(-Bb+3Da\right)}{4ab^5}\right)\log\left(x+\frac{2Ca-4ab^2\left(\frac{C}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right)+\frac{-Ab+Ca+x\left(-Bb+Da\right)}{2ab^2+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))
*log(x + (2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*
D*a)/(4*a*b**5)))/(-B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(
-B*b + 3*D*a)/(4*a*b**5))*log(x + (2*C*a - 4*a*b**2*(C/(2*b**2) +
 sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-B*b + 3*D*a)) + (-A*
b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

GIAC/XCAS [A] time = 0.22131, size = 109, normalized size = 1.08

$$\frac{Dx}{b^2} + \frac{C\ln\left(bx^2 + a\right)}{2\,b^2} - \frac{(3\,Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Ca - Ab + (Da - Bb)x}{2\,(bx^2 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*ln(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*
x^2 + a)*b^2)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=93

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b +
a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a
+ b*x^2])/(2*b^2)
```

Rubi [A] time = 0.145849, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b +
a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a
+ b*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 32.4821, size = 75, normalized size = 0.81

$$\frac{D\log\left(a+bx^2\right)}{2b^2}+\frac{x\left(Ab-Ca+x\left(Bb-Da\right)\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab+Ca)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*log(a + b*x**2)/(2*b**2) + x*(A*b - C*a + x*(B*b - D*a))/(2*a*b
*(a + b*x**2)) + (A*b + C*a)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*
b**(3/2))
```

Mathematica [A] time = 0.164582, size = 83, normalized size = 0.89

$$\frac{\frac{\sqrt{b}(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{a^2D-ab(B+Cx)+Ab^2x}{a(a+bx^2)}+D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*
b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])
/(2*b^2)
```

Maple [A] time = 0.017, size = 100, normalized size = 1.1

$$\frac{1}{bx^2+a}\left(\frac{(Ab-aC)x}{2ab}-\frac{Bb-aD}{2b^2}\right)+\frac{D\ln\left(ab\left(bx^2+a\right)\right)}{2b^2}+\frac{A}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{C}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2*D/b^2*ln(a*
b*(b*x^2+a))+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+1/2/b/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235445, size = 1, normalized size = 0.01

$$\left[\frac{\left(Ca^2b+Aab^2+\left(Cab^2+Ab^3\right)x^2\right)\log\left(\frac{2abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(Da^2-Bab-\left(Cab-Ab^2\right)x+\left(Dabx^2+Da^2\right)\log\left(bx^2+a\right.\right.}{4\left(ab^3x^2+a^2b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((C*a^2*b + A*a*b^2 + (C*a*b^2 + A*b^3)*x^2)*log((2*a*b*x +
(b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(D*a^2 - B*a*b - (C*a*b
- A*b^2)*x + (D*a*b*x^2 + D*a^2)*log(b*x^2 + a))*sqrt(-a*b))/((a*
b^3*x^2 + a^2*b^2)*sqrt(-a*b)), 1/2*((C*a^2*b + A*a*b^2 + (C*a*b^
2 + A*b^3)*x^2)*arctan(sqrt(a*b)*x/a) + (D*a^2 - B*a*b - (C*a*b -
 A*b^2)*x + (D*a*b*x^2 + D*a^2)*log(b*x^2 + a))*sqrt(a*b))/((a*b^
3*x^2 + a^2*b^2)*sqrt(a*b))]
```

Sympy [A] time = 4.37742, size = 233, normalized size = 2.51

$$\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)+\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)-\frac{Bab-Da^2+x\left(-Ab^2+Cab\right)}{2a^2b^2+2ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x +
 (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*
b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**2 + 4*a**2*b**2
*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/(A*b*
*2 + C*a*b)) - (B*a*b - D*a**2 + x*(-A*b**2 + C*a*b))/(2*a**2*b**
2 + 2*a*b**3*x**2)
```

GIAC/XCAS [A] time = 0.239666, size = 119, normalized size = 1.28

$$\frac{D\ln\left(bx^2+a\right)}{2\,b^2}+\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*ln(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2
+ a)*a*b)
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2} \, dx$

Optimal. Leaf size=95

$$\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 -
 (A*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.268661, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 -
 (A*Log[a + b*x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 43.913, size = 85, normalized size = 0.89

$$\frac{C\log(x)}{ab}-\frac{C\log\left(a+bx^2\right)}{2ab}+\frac{x\left(\frac{Ab}{x}+Bb-\frac{Ca}{x}-Da\right)}{2ab\left(a+bx^2\right)}+\frac{(Bb+Da)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] C*log(x)/(a*b) - C*log(a + b*x**2)/(2*a*b) + x*(A*b/x + B*b - C*a
/x - D*a)/(2*a*b*(a + b*x**2)) + (B*b + D*a)*atan(sqrt(b)*x/sqrt(
a))/(2*a**(3/2)*b**(3/2))
```

Mathematica [A] time = 0.144014, size = 85, normalized size = 0.89

$$\frac{\frac{a(-a(C+Dx)+Ab+bBx)}{b(a+bx^2)}-A\log\left(a+bx^2\right)+\frac{\sqrt{a}(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B
+ a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[
a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.019, size = 127, normalized size = 1.3

$$\frac{A\ln(x)}{a^2}+\frac{Bx}{2a(bx^2+a)}-\frac{xD}{(2bx^2+2a)b}+\frac{A}{2a(bx^2+a)}-\frac{C}{(2bx^2+2a)b}-\frac{A\ln\left(b\left(bx^2+a\right)\right)}{2a^2}+\frac{B}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{D}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2+1/2/a/(b*x^2+a)*B*x-1/2/(b*x^2+a)*x/b*D+1/2/a/(b*x^2+
a)*A-1/2/(b*x^2+a)/b*C-1/2/a^2*A*ln(b*(b*x^2+a))+1/2/a/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*B+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26292, size = 1, normalized size = 0.01

$$\left[\frac{\left(Da^3+Ba^2b+\left(Da^2b+Bab^2\right)x^2\right)\log\left(\frac{2abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(Ca^2-Aab+\left(Da^2-Bab\right)x+\left(Ab^2x^2+Aab\right)\log\left(bx^2+a\right.\right.}{4\left(a^2b^2x^2+a^3b\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/4*((D*a^3 + B*a^2*b + (D*a^2*b + B*a*b^2)*x^2)*log((2*a*b*x +
(b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(C*a^2 - A*a*b + (D*a^2
- B*a*b)*x + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b^2*x^2 +
A*a*b)*log(x))*sqrt(-a*b))/((a^2*b^2*x^2 + a^3*b)*sqrt(-a*b)), 1/
2*((D*a^3 + B*a^2*b + (D*a^2*b + B*a*b^2)*x^2)*arctan(sqrt(a*b)*x
/a) - (C*a^2 - A*a*b + (D*a^2 - B*a*b)*x + (A*b^2*x^2 + A*a*b)*lo
g(b*x^2 + a) - 2*(A*b^2*x^2 + A*a*b)*log(x))*sqrt(a*b))/((a^2*b^2
*x^2 + a^3*b)*sqrt(a*b))]
```

Sympy [A] time = 14.5026, size = 797, normalized size = 8.39

$$\frac{A\log(x)}{a^2} + \left(-\frac{A}{2a^2} - \frac{\sqrt{-a^5b^3}\,(Bb+Da)}{4a^4b^3}\right)\log\left(x + \frac{48A^3b^4 + 48A^2a^2b^4\left(-\frac{A}{2a^2} - \frac{\sqrt{-a^5b^3}(Bb+Da)}{4a^4b^3}\right) - 4AB^2ab^3 - 8ABDa^2b^2 - 4AD^2a^3b - 96Aa^4b^4}{36A^2Bb^4 +}\right.$$

$$+ \left(-\frac{A}{2a^2} + \frac{\sqrt{-a^5b^3}\,(Bb+Da)}{4a^4b^3}\right)\log\left(x + \frac{48A^3b^4 + 48A^2a^2b^4\left(-\frac{A}{2a^2} + \frac{\sqrt{-a^5b^3}(Bb+Da)}{4a^4b^3}\right) - 4AB^2ab^3 - 8ABDa^2b^2 - 4AD^2a^3b - 96Aa^4b^4}{36A^2Bb^4 + 3}\right.$$

$$- \frac{-Ab + Ca + x(-Bb + Da)}{2a^2b + 2ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 + (-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a*
*4*b**3))*log(x + (48*A**3*b**4 + 48*A**2*a**2*b**4*(-A/(2*a**2)
- sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) - 4*A*B**2*a*b**3 -
 8*A*B*D*a**2*b**2 - 4*A*D**2*a**3*b - 96*A*a**4*b**4*(-A/(2*a**2
) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3))**2 + 4*B**2*a**3*
b**3*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) +
 8*B*D*a**4*b**2*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b + D*a)/(4*a
**4*b**3)) + 4*D**2*a**5*b*(-A/(2*a**2) - sqrt(-a**5*b**3)*(B*b +
 D*a)/(4*a**4*b**3)))/(36*A**2*B*b**4 + 36*A**2*D*a*b**3 + B**3*a
*b**3 + 3*B**2*D*a**2*b**2 + 3*B*D**2*a**3*b + D**3*a**4)) + (-A/
(2*a**2) + sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3))*log(x + (4
8*A**3*b**4 + 48*A**2*a**2*b**4*(-A/(2*a**2) + sqrt(-a**5*b**3)*(
B*b + D*a)/(4*a**4*b**3)) - 4*A*B**2*a*b**3 - 8*A*B*D*a**2*b**2 -
 4*A*D**2*a**3*b - 96*A*a**4*b**4*(-A/(2*a**2) + sqrt(-a**5*b**3)
*(B*b + D*a)/(4*a**4*b**3))**2 + 4*B**2*a**3*b**3*(-A/(2*a**2) +
sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) + 8*B*D*a**4*b**2*(-A
/(2*a**2) + sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3)) + 4*D**2*
a**5*b*(-A/(2*a**2) + sqrt(-a**5*b**3)*(B*b + D*a)/(4*a**4*b**3))
)/(36*A**2*B*b**4 + 36*A**2*D*a*b**3 + B**3*a*b**3 + 3*B**2*D*a**
2*b**2 + 3*B*D**2*a**3*b + D**3*a**4)) - (-A*b + C*a + x*(-B*b +
D*a))/(2*a**2*b + 2*a*b**2*x**2)
```

GIAC/XCAS [A] time = 0.23593, size = 126, normalized size = 1.33

$$-\frac{A\ln\left(bx^2+a\right)}{2\,a^2} + \frac{A\ln\left(|x|\right)}{a^2} + \frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{Ca^2 - Aab + \left(Da^2 - Bab\right)x}{2\,(bx^2+a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] -1/2*A*ln(b*x^2 + a)/a^2 + A*ln(abs(x))/a^2 + 1/2*(D*a + B*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2
- B*a*b)*x)/((b*x^2 + a)*a^2*b)
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)
) - ((3*A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b
]) + (B*Log[x])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.282796, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)
) - ((3*A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b
]) + (B*Log[x])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 44.2476, size = 92, normalized size = 0.84

$$-\frac{C}{abx}-\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}+\frac{D\log\left(x\right)}{ab}-\frac{D\log\left(a+bx^2\right)}{2ab}+\frac{x\left(\frac{Ab}{x^2}+\frac{Bb}{x}-\frac{Ca}{x^2}-\frac{Da}{x}\right)}{2ab\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -C/(a*b*x) - C*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*sqrt(b)) + D*log
(x)/(a*b) - D*log(a + b*x**2)/(2*a*b) + x*(A*b/x**2 + B*b/x - C*a
/x**2 - D*a/x)/(2*a*b*(a + b*x**2))
```

Mathematica [A] time = 0.134197, size = 110, normalized size = 1.

$$\frac{(aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{a^2(-D)+abB+abCx-Ab^2x}{2a^2b\left(a+bx^2\right)}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a +
b*x^2)) + ((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)
*Sqrt[b]) + (B*Log[x])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.018, size = 136, normalized size = 1.2

$$-\frac{A}{xa^2}+\frac{\ln(x)B}{a^2}-\frac{Axb}{2a^2(bx^2+a)}+\frac{Cx}{2a(bx^2+a)}+\frac{B}{2a(bx^2+a)}-\frac{D}{(2bx^2+2a)b}-\frac{\ln\left(bx^2+a\right)B}{2a^2}-\frac{3Ab}{2a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{C}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -A/x/a^2+1/a^2*ln(x)*B-1/2/a^2/(b*x^2+a)*A*x*b+1/2/a/(b*x^2+a)*C*
x+1/2/a/(b*x^2+a)*B-1/2/(b*x^2+a)/b*D-1/2/a^2*ln(b*x^2+a)*B-3/2/a
^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A*b+1/2/a/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264915, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(Cab^2-3Ab^3\right)x^3+\left(Ca^2b-3Aab^2\right)x\right)\log\left(-\frac{2abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(2Aab-\left(Cab-3Ab^2\right)x^2+\left(Da^2-Bab\right)x+\left(B\right.\right.}{4\left(a^2b^2x^3+a^3bx\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((C*a*b^2 - 3*A*b^3)*x^3 + (C*a^2*b - 3*A*a*b^2)*x)*log(-(
2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*A*a*b - (C*
a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x + (B*b^2*x^3 + B*a*b*x)*lo
g(b*x^2 + a) - 2*(B*b^2*x^3 + B*a*b*x)*log(x))*sqrt(-a*b))/((a^2*
b^2*x^3 + a^3*b*x)*sqrt(-a*b)), 1/2*(((C*a*b^2 - 3*A*b^3)*x^3 + (
C*a^2*b - 3*A*a*b^2)*x)*arctan(sqrt(a*b)*x/a) - (2*A*a*b - (C*a*b
 - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x + (B*b^2*x^3 + B*a*b*x)*log(b
*x^2 + a) - 2*(B*b^2*x^3 + B*a*b*x)*log(x))*sqrt(a*b))/((a^2*b^2*
x^3 + a^3*b*x)*sqrt(a*b))]
```

Sympy [A] time = 15.8858, size = 782, normalized size = 7.11

$$\frac{B\log(x)}{a^2} + \left(-\frac{B}{2a^2} - \frac{\sqrt{-a^5b}\,(-3Ab+Ca)}{4a^5b}\right)\log\left(x + \frac{-36A^2Bab^2 + 36A^2a^3b^2\left(-\frac{B}{2a^2} - \frac{\sqrt{-a^5b}(-3Ab+Ca)}{4a^5b}\right) + 24ABCa^2b - 24ACa^4b\left(-\frac{B}{2a^2} - \frac{\sqrt{-a^5b}(\cdot}{4\cdot}\right.}{-27A^3b^3\cdots}\right.$$

$$+\left(-\frac{B}{2a^2} + \frac{\sqrt{-a^5b}\,(-3Ab+Ca)}{4a^5b}\right)\log\left(x + \frac{-36A^2Bab^2 + 36A^2a^3b^2\left(-\frac{B}{2a^2} + \frac{\sqrt{-a^5b}(-3Ab+Ca)}{4a^5b}\right) + 24ABCa^2b - 24ACa^4b\left(-\frac{B}{2a^2} + \frac{\sqrt{-a^5b}(-\cdot}{4a\cdot}\right.}{-27A^3b^3+\cdots}\right.$$

$$+\frac{-2Aab + x^2\left(-3Ab^2 + Cab\right) + x\left(Bab - Da^2\right)}{2a^3bx + 2a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] B*log(x)/a**2 + (-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a*
*5*b))*log(x + (-36*A**2*B*a*b**2 + 36*A**2*a**3*b**2*(-B/(2*a**2
) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) + 24*A*B*C*a**2*b -
24*A*C*a**4*b*(-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5
*b)) + 48*B**3*a**2*b + 48*B**2*a**4*b*(-B/(2*a**2) - sqrt(-a**5*
b)*(-3*A*b + C*a)/(4*a**5*b)) - 4*B*C**2*a**3 - 96*B*a**6*b*(-B/(
2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b))**2 + 4*C**2*a*
*5*(-B/(2*a**2) - sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)))/(-27*
A**3*b**3 + 27*A**2*C*a*b**2 - 108*A*B**2*a*b**2 - 9*A*C**2*a**2*
b + 36*B**2*C*a**2*b + C**3*a**3)) + (-B/(2*a**2) + sqrt(-a**5*b)
*(-3*A*b + C*a)/(4*a**5*b))*log(x + (-36*A**2*B*a*b**2 + 36*A**2*
a**3*b**2*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b))
 + 24*A*B*C*a**2*b - 24*A*C*a**4*b*(-B/(2*a**2) + sqrt(-a**5*b)*(
-3*A*b + C*a)/(4*a**5*b)) + 48*B**3*a**2*b + 48*B**2*a**4*b*(-B/(
2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a**5*b)) - 4*B*C**2*a**
3 - 96*B*a**6*b*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*a)/(4*a*
*5*b))**2 + 4*C**2*a**5*(-B/(2*a**2) + sqrt(-a**5*b)*(-3*A*b + C*
a)/(4*a**5*b)))/(-27*A**3*b**3 + 27*A**2*C*a*b**2 - 108*A*B**2*a*
b**2 - 9*A*C**2*a**2*b + 36*B**2*C*a**2*b + C**3*a**3)) + (-2*A*a
*b + x**2*(-3*A*b**2 + C*a*b) + x*(B*a*b - D*a**2))/(2*a**3*b*x +
 2*a**2*b**2*x**3)
```

GIAC/XCAS [A] time = 0.240638, size = 139, normalized size = 1.26

$$-\frac{B\ln\left(bx^2+a\right)}{2\,a^2} + \frac{B\ln\left(|x|\right)}{a^2} + \frac{(Ca - 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} + \frac{Cabx^2 - 3\,Ab^2x^2 - Da^2x + Babx - 2\,Aab}{2\,(bx^3+ax)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*B*ln(b*x^2 + a)/a^2 + B*ln(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x
^2 - D*a^2*x + B*a*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a
*(a + b*x^2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^
(5/2)*Sqrt[b]) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[
a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.412557, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a
*(a + b*x^2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^
(5/2)*Sqrt[b]) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[
a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 51.6954, size = 105, normalized size = 0.78

$$-\frac{C}{2abx^2}-\frac{C\log(x)}{a^2}+\frac{C\log\left(a+bx^2\right)}{2a^2}-\frac{D}{abx}-\frac{D\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}+\frac{x\left(\frac{Ab}{x^3}+\frac{Bb}{x^2}-\frac{Ca}{x^3}-\frac{Da}{x^2}\right)}{2ab\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] -C/(2*a*b*x**2) - C*log(x)/a**2 + C*log(a + b*x**2)/(2*a**2) - D/
(a*b*x) - D*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*sqrt(b)) + x*(A*b/x
**3 + B*b/x**2 - C*a/x**3 - D*a/x**2)/(2*a*b*(a + b*x**2))
```

Mathematica [A] time = 0.215616, size = 112, normalized size = 0.83

$$\frac{\frac{a(a(C+Dx)-Ab-bBx)}{a+bx^2}+(2Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-2Ab)-\frac{aA}{x^2}+\frac{\sqrt{a}(aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a
 + b*x^2) + (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
Sqrt[b] + 2*(-2*A*b + a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])
/(2*a^3)
```

Maple [A] time = 0.02, size = 169, normalized size = 1.3

$$-\frac{A}{2\,a^2x^2}-\frac{B}{xa^2}-2\,\frac{A\ln(x)\,b}{a^3}+\frac{\ln(x)\,C}{a^2}-\frac{bBx}{2\,a^2\,(bx^2+a)}+\frac{Dx}{2\,a\,(bx^2+a)}-\frac{Ab}{2\,a^2\,(bx^2+a)}+\frac{C}{2\,a\,(bx^2+a)}+\frac{b\ln\left(bx^2+a\right)A}{a^3}-\frac{\ln\left(bx^2+a\right)C}{2\,a^2}-\frac{3\,Bb}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{D}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-B/x/a^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*C-1/2/a^2/(b*x
^2+a)*B*x*b+1/2/a/(b*x^2+a)*D*x-1/2/a^2*b/(b*x^2+a)*A+1/2/a/(b*x^
2+a)*C+1/a^3*b*ln(b*x^2+a)*A-1/2/a^2*ln(b*x^2+a)*C-3/2/a^2/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*b*B+1/2/a/(a*b)^(1/2)*arctan(x*b/(a
*b)^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287234, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(Da^2b-3\,Bab^2\right)x^4+\left(Da^3-3\,Ba^2b\right)x^2\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,Ba^2x-\left(Da^2-3\,Bab\right)x^3+Aa^2-\left(Ca^2-2\,A\right.\right.}{4\left(a^3bx^4+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(((D*a^2*b - 3*B*a*b^2)*x^4 + (D*a^3 - 3*B*a^2*b)*x^2)*log(
-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*B*a^2*x -
 (D*a^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2 + ((C*a*b
- 2*A*b^2)*x^4 + (C*a^2 - 2*A*a*b)*x^2)*log(b*x^2 + a) - 2*((C*a*
b - 2*A*b^2)*x^4 + (C*a^2 - 2*A*a*b)*x^2)*log(x))*sqrt(-a*b))/((a
^3*b*x^4 + a^4*x^2)*sqrt(-a*b)), 1/2*(((D*a^2*b - 3*B*a*b^2)*x^4
+ (D*a^3 - 3*B*a^2*b)*x^2)*arctan(sqrt(a*b)*x/a) - (2*B*a^2*x - (
D*a^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2 + ((C*a*b -
2*A*b^2)*x^4 + (C*a^2 - 2*A*a*b)*x^2)*log(b*x^2 + a) - 2*((C*a*b
- 2*A*b^2)*x^4 + (C*a^2 - 2*A*a*b)*x^2)*log(x))*sqrt(a*b))/((a^3*
b*x^4 + a^4*x^2)*sqrt(a*b))]
```

Sympy [A] time = 60.9099, size = 1807, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*
b))*log(x + (-384*A**3*b**4 + 576*A**2*C*a*b**3 + 192*A**2*a**3*b
**3*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a
**6*b)) + 72*A*B**2*a*b**3 - 48*A*B*D*a**2*b**2 - 288*A*C**2*a**2
*b**2 - 192*A*C*a**4*b**2*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*
b)*(-3*B*b + D*a)/(4*a**6*b)) + 8*A*D**2*a**3*b + 192*A*a**6*b**2
*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6
*b))**2 - 36*B**2*C*a**2*b**2 + 36*B**2*a**4*b**2*(-(-2*A*b + C*a
)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 24*B*C*D*
a**3*b - 24*B*D*a**5*b*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*
(-3*B*b + D*a)/(4*a**6*b)) + 48*C**3*a**3*b + 48*C**2*a**5*b*(-(-
2*A*b + C*a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))
- 4*C*D**2*a**4 - 96*C*a**7*b*(-(-2*A*b + C*a)/(2*a**3) - sqrt(-a
**7*b)*(-3*B*b + D*a)/(4*a**6*b))**2 + 4*D**2*a**6*(-(-2*A*b + C*
a)/(2*a**3) - sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)))/(-432*A**
2*B*b**4 + 144*A**2*D*a*b**3 + 432*A*B*C*a*b**3 - 144*A*C*D*a**2*
b**2 - 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2 - 108*B*C**2*a**2*b**
2 - 9*B*D**2*a**3*b + 36*C**2*D*a**3*b + D**3*a**4)) + (-(-2*A*b
+ C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))*log(x
+ (-384*A**3*b**4 + 576*A**2*C*a*b**3 + 192*A**2*a**3*b**3*(-(-2*
A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) +
72*A*B**2*a*b**3 - 48*A*B*D*a**2*b**2 - 288*A*C**2*a**2*b**2 - 19
2*A*C*a**4*b**2*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b
 + D*a)/(4*a**6*b)) + 8*A*D**2*a**3*b + 192*A*a**6*b**2*(-(-2*A*b
 + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b))**2 -
36*B**2*C*a**2*b**2 + 36*B**2*a**4*b**2*(-(-2*A*b + C*a)/(2*a**3)
 + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) + 24*B*C*D*a**3*b - 2
4*B*D*a**5*b*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b +
D*a)/(4*a**6*b)) + 48*C**3*a**3*b + 48*C**2*a**5*b*(-(-2*A*b + C*
a)/(2*a**3) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)) - 4*C*D**2
*a**4 - 96*C*a**7*b*(-(-2*A*b + C*a)/(2*a**3) + sqrt(-a**7*b)*(-3
*B*b + D*a)/(4*a**6*b))**2 + 4*D**2*a**6*(-(-2*A*b + C*a)/(2*a**3
) + sqrt(-a**7*b)*(-3*B*b + D*a)/(4*a**6*b)))/(-432*A**2*B*b**4 +
 144*A**2*D*a*b**3 + 432*A*B*C*a*b**3 - 144*A*C*D*a**2*b**2 - 27*
B**3*a*b**3 + 27*B**2*D*a**2*b**2 - 108*B*C**2*a**2*b**2 - 9*B*D*
*2*a**3*b + 36*C**2*D*a**3*b + D**3*a**4)) + (-A*a - 2*B*a*x + x*
*3*(-3*B*b + D*a) + x**2*(-2*A*b + C*a))/(2*a**3*x**2 + 2*a**2*b*
x**4) + (-2*A*b + C*a)*log(x + (-384*A**3*b**4 + 576*A**2*C*a*b**
3 + 192*A**2*b**3*(-2*A*b + C*a) + 72*A*B**2*a*b**3 - 48*A*B*D*a*
*2*b**2 - 288*A*C**2*a**2*b**2 - 192*A*C*a*b**2*(-2*A*b + C*a) +
8*A*D**2*a**3*b + 192*A*b**2*(-2*A*b + C*a)**2 - 36*B**2*C*a**2*b
**2 + 36*B**2*a*b**2*(-2*A*b + C*a) + 24*B*C*D*a**3*b - 24*B*D*a*
*2*b*(-2*A*b + C*a) + 48*C**3*a**3*b + 48*C**2*a**2*b*(-2*A*b + C
*a) - 4*C*D**2*a**4 - 96*C*a*b*(-2*A*b + C*a)**2 + 4*D**2*a**3*(-
2*A*b + C*a))/(-432*A**2*B*b**4 + 144*A**2*D*a*b**3 + 432*A*B*C*a
*b**3 - 144*A*C*D*a**2*b**2 - 27*B**3*a*b**3 + 27*B**2*D*a**2*b**
2 - 108*B*C**2*a**2*b**2 - 9*B*D**2*a**3*b + 36*C**2*D*a**3*b + D
**3*a**4))/a**3
```

GIAC/XCAS [A] time = 0.227448, size = 170, normalized size = 1.26

$$\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{(Ca-2\,Ab)\ln\left(bx^2+a\right)}{2\,a^3}+\frac{(Ca-2\,Ab)\ln\left(|x|\right)}{a^3}-\frac{2\,Ba^2x-\left(Da^2-3\,Bab\right)x^3+Aa^2-\left(Ca^2-2\,Aab\right)x^2}{2\,(bx^2+a)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*
a - 2*A*b)*ln(b*x^2 + a)/a^3 + (C*a - 2*A*b)*ln(abs(x))/a^3 - 1/2
*(2*B*a^2*x - (D*a^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x
```

```
^2)/((b*x^2 + a)*a^3*x^2)
```

3.102 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=185

$$\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{(bB-3aD)\log\left(a+bx^2\right)}{2b^4}-\frac{x^2(bB-3aD)}{2ab^3}$$

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) -
(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (
x^3*(A*b - 5*a*C + 4*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3
*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) +
 ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.713244, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{(bB-3aD)\log\left(a+bx^2\right)}{2b^4}-\frac{x^2(bB-3aD)}{2ab^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) -
(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (
x^3*(A*b - 5*a*C + 4*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3
*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) +
 ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.235367, size = 139, normalized size = 0.75

$$\frac{\frac{2a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{\left(a+bx^2\right)^2}+\frac{-12a^2D+8abB+9abCx-5Ab^2x}{a+bx^2}+\frac{3\sqrt{b}(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)\log\left(a+bx^2\right)+8bCx+4bDx^2}{8b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*
x)/(a + b*x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x
```

```
^2)^2 + (3*Sqrt[b]*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[a] + 4*(b*B - 3*a*D)*Log[a + b*x^2])/(8*b^4)
```

Maple [A] time = 0.017, size = 235, normalized size = 1.3

$$\frac{Dx^2}{2\,b^3}+\frac{Cx}{b^3}-\frac{5\,Ax^3}{8\,b\,(bx^2+a)^2}+\frac{9\,Cx^3a}{8\,b^2\,(bx^2+a)^2}+\frac{Bax^2}{b^2\,(bx^2+a)^2}-\frac{3\,Dx^2a^2}{2\,b^3\,(bx^2+a)^2}-\frac{3\,aAx}{8\,b^2\,(bx^2+a)^2}+\frac{7\,Cxa^2}{8\,b^3\,(bx^2+a)^2}+\frac{3\,a^2B}{4\,b^3\,(bx^2+a)^2}-\frac{5\,a^3D}{4\,b^4\,(bx^2+a)^2}+\frac{B\ln\left(bx^2+a\right)}{2\,b^3}-\frac{3\,\ln\left(bx^2+a\right)aD}{2\,b^4}+\frac{3\,A}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,aC}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2/b^3*D*x^2+1/b^3*C*x-5/8/b/(b*x^2+a)^2*A*x^3+9/8/b^2/(b*x^2+a)
^2*C*x^3*a+1/b^2/(b*x^2+a)^2*B*x^2*a-3/2/b^3/(b*x^2+a)^2*D*x^2*a^
2-3/8/b^2/(b*x^2+a)^2*A*x*a+7/8/b^3/(b*x^2+a)^2*C*x*a^2+3/4*a^2/b
^3/(b*x^2+a)^2*B-5/4/b^4/(b*x^2+a)^2*a^3*D+1/2*B*ln(b*x^2+a)/b^3-
3/2/b^4*ln(b*x^2+a)*a*D+3/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))*A-15/8/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236375, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,Ca^3b-Aa^2b^2+\left(5\,Cab^3-Ab^4\right)x^4+2\left(5\,Ca^2b^2-Aab^3\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(4\,Db^3x^6+8\,Cb^3x^5+8\,Dal\cdots\right.}{\cdots}\right.$$

$$-\frac{3\left(5\,Ca^3b-Aa^2b^2+\left(5\,Cab^3-Ab^4\right)x^4+2\left(5\,Ca^2b^2-Aab^3\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(4\,Db^3x^6+8\,Cb^3x^5+8\,Dab^2x^4-10\,Da\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*C*a^3*b - A*a^2*b^2 + (5*C*a*b^3 - A*b^4)*x^4 + 2*(5
*C*a^2*b^2 - A*a*b^3)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)) - 2*(4*D*b^3*x^6 + 8*C*b^3*x^5 + 8*D*a*b^2*x^4 - 10
*D*a^3 + 6*B*a^2*b + 5*(5*C*a*b^2 - A*b^3)*x^3 - 8*(D*a^2*b - B*a
*b^2)*x^2 + 3*(5*C*a^2*b - A*a*b^2)*x - 4*((3*D*a*b^2 - B*b^3)*x^
4 + 3*D*a^3 - B*a^2*b + 2*(3*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 +
a))*sqrt(-a*b))/((b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*sqrt(-a*b)), -
1/8*(3*(5*C*a^3*b - A*a^2*b^2 + (5*C*a*b^3 - A*b^4)*x^4 + 2*(5*C*
a^2*b^2 - A*a*b^3)*x^2)*arctan(sqrt(a*b)*x/a) - (4*D*b^3*x^6 + 8*
C*b^3*x^5 + 8*D*a*b^2*x^4 - 10*D*a^3 + 6*B*a^2*b + 5*(5*C*a*b^2 -
```

```
 A*b^3)*x^3 - 8*(D*a^2*b - B*a*b^2)*x^2 + 3*(5*C*a^2*b - A*a*b^2)
*x - 4*((3*D*a*b^2 - B*b^3)*x^4 + 3*D*a^3 - B*a^2*b + 2*(3*D*a^2*
b - B*a*b^2)*x^2)*log(b*x^2 + a))*sqrt(a*b))/((b^6*x^4 + 2*a*b^5*
x^2 + a^2*b^4)*sqrt(a*b))]
```

Sympy [A] time = 41.516, size = 357, normalized size = 1.93

$$\frac{Cx}{b^3} + \frac{Dx^2}{2b^3} + \left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}\left(-Ab+5Ca\right)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}\left(-Ab+5Ca\right)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \frac{6Ba^2b - 10Da^3 + x^3\left(-5Ab^3 + 9Cab^2\right) + x^2\left(8Bab^2 - 12Da^2b\right) + x\left(-3Aab^2 + 7Ca^2b\right)}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-
a*b**9)*(-A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2
- 16*a*b**4*(-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5
*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2
*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B
*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-
a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b)) + (6
*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B
*a*b**2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b*
*4 + 16*a*b**5*x**2 + 8*b**6*x**4)
```

GIAC/XCAS [A] time = 0.247485, size = 212, normalized size = 1.15

$$-\frac{3\left(5\,Ca - Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} - \frac{\left(3\,Da - Bb\right)\ln\left(bx^2 + a\right)}{2\,b^4} + \frac{Db^3x^2 + 2\,Cb^3x}{2\,b^6} - \frac{10\,Da^3 - 6\,Ba^2b - \left(9\,Cab^2 - 5\,Ab^3\right)x^3 + 4\left(3\,Da^2b - 2\,Bab^2\right)x^2 - \left(7\,Ca^2b - 3\,Aab^2\right)x}{8\left(bx^2 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^4/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3
*D*a - B*b)*ln(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 -
 1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a
^2*b - 2*B*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2
*b^4)
```

3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{3x(bB-5aD)}{8ab^3}+\frac{C\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}$$

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a
*C)*x))/(4*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))
/(8*a*b^2*(a + b*x^2)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.494439, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{3x(bB-5aD)}{8ab^3}+\frac{C\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a
*C)*x))/(4*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))
/(8*a*b^2*(a + b*x^2)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [A] time = 133.624, size = 131, normalized size = 0.85

$$\frac{C\log\left(a+bx^2\right)}{2b^3}+\frac{Dx}{b^3}+\frac{x\left(a\left(Bb-Da\right)-bx\left(Ab-Ca\right)\right)}{4b^3\left(a+bx^2\right)^2}-\frac{x\left(a\left(5Bb-9Da\right)-2bx\left(Ab-3Ca\right)\right)}{8ab^3\left(a+bx^2\right)}+\frac{3\left(Bb-5Da\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*log(a + b*x**2)/(2*b**3) + D*x/b**3 + x*(a*(B*b - D*a) - b*x*(A
*b - C*a))/(4*b**3*(a + b*x**2)**2) - x*(a*(5*B*b - 9*D*a) - 2*b*
x*(A*b - 3*C*a))/(8*a*b**3*(a + b*x**2)) + 3*(B*b - 5*D*a)*atan(s
qrt(b)*x/sqrt(a))/(8*sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.150641, size = 126, normalized size = 0.81

$$\frac{a(-a(C+Dx)+Ab+bBx)}{4b^3\left(a+bx^2\right)^2}+\frac{8aC+9aDx-4Ab-5bBx}{8b^3\left(a+bx^2\right)}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}+\frac{Dx}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^
2)) + (a*(A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*
(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) +
(C*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.015, size = 206, normalized size = 1.3

$$\frac{Dx}{b^3} - \frac{5\,Bx^3}{8\,b\,(bx^2+a)^2} + \frac{9\,Dx^3a}{8\,b^2\,(bx^2+a)^2} - \frac{Ax^2}{2\,b\,(bx^2+a)^2} + \frac{Cx^2a}{b^2\,(bx^2+a)^2} - \frac{3\,Bxa}{8\,b^2\,(bx^2+a)^2} + \frac{7\,Dxa^2}{8\,b^3\,(bx^2+a)^2} - \frac{Aa}{4\,b^2\,(bx^2+a)^2} + \frac{3\,a^2C}{4\,b^3\,(bx^2+a)^2} + \frac{C\ln\left(bx^2+a\right)}{2\,b^3} + \frac{3\,B}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{15\,aD}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] D/b^3*x-5/8/b/(b*x^2+a)^2*B*x^3+9/8/b^2/(b*x^2+a)^2*D*x^3*a-1/2/b
/(b*x^2+a)^2*A*x^2+1/b^2/(b*x^2+a)^2*C*x^2*a-3/8/b^2/(b*x^2+a)^2*
B*x*a+7/8/b^3/(b*x^2+a)^2*D*x*a^2-1/4*a/b^2/(b*x^2+a)^2*A+3/4/b^3
/(b*x^2+a)^2*a^2*C+1/2*C*ln(b*x^2+a)/b^3+3/8/b^2/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*B-15/8/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*a*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242437, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(5\,Dab^2 - Bb^3\right)x^4 + 5\,Da^3 - Ba^2b + 2\left(5\,Da^2b - Bab^2\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right) - 2\left(8\,Db^2x^5 + 5\left(5\,Dab - Bb^2\right)x^3 \cdots\right.}{16\left(b^5x^4 + 2\,ab^4x^2 + a \cdots\right.}\right.$$

$$-\frac{3\left(\left(5\,Dab^2 - Bb^3\right)x^4 + 5\,Da^3 - Ba^2b + 2\left(5\,Da^2b - Bab^2\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right) - \left(8\,Db^2x^5 + 5\left(5\,Dab - Bb^2\right)x^3 + 6\,Ca^2 - 2 \cdots\right.}{8\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)\sqrt{a \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D*a^3 - B*a^2*b + 2*(5*D*a
^2*b - B*a*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^
2 + a)) - 2*(8*D*b^2*x^5 + 5*(5*D*a*b - B*b^2)*x^3 + 6*C*a^2 - 2*
```

```
A*a*b + 4*(2*C*a*b - A*b^2)*x^2 + 3*(5*D*a^2 - B*a*b)*x + 4*(C*b^
2*x^4 + 2*C*a*b*x^2 + C*a^2)*log(b*x^2 + a))*sqrt(-a*b))/((b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3)*sqrt(-a*b)), -1/8*(3*((5*D*a*b^2 - B*b
^3)*x^4 + 5*D*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*arctan
(sqrt(a*b)*x/a) - (8*D*b^2*x^5 + 5*(5*D*a*b - B*b^2)*x^3 + 6*C*a^
2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2 + 3*(5*D*a^2 - B*a*b)*x + 4
*(C*b^2*x^4 + 2*C*a*b*x^2 + C*a^2)*log(b*x^2 + a))*sqrt(a*b))/((b
^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*sqrt(a*b))]
```

Sympy [A] time = 38.2683, size = 282, normalized size = 1.82

$$\frac{Dx}{b^3}+\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\left(-Bb+5Da\right)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}\left(-Bb+5Da\right)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\frac{-2Aab+6Ca^2+x^3\left(-5Bb^2+9Dab\right)+x^2\left(-4Ab^2+8Cab\right)+x\left(-3Bab+7Da^2\right)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*
b + 5*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqr
t(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x + (8*C*a - 16*a*b**3
*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7)))/(-3*B
*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*b**2 + 9*D*a*b)
 + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*a**2*
b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

GIAC/XCAS [A] time = 0.227834, size = 165, normalized size = 1.06

$$\frac{Dx}{b^3}+\frac{C\ln\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+\left(7\,Da^2-3\,Bab\right)x}{8\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^3/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] D*x/b^3 + 1/2*C*ln(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a
^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/(
(b*x^2 + a)^2*b^3)
```

3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=136

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) -
(x*(A*b + 3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A
*b + 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D
*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.356606, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) -
(x*(A*b + 3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A
*b + 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D
*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [A] time = 100.162, size = 114, normalized size = 0.84

$$\frac{D\log\left(a+bx^2\right)}{2b^3}-\frac{x\left(Ab-Ca+x\left(Bb-Da\right)\right)}{4b^2\left(a+bx^2\right)^2}+\frac{x\left(Ab-5Ca+x\left(2Bb-6Da\right)\right)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3Ca)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*log(a + b*x**2)/(2*b**3) - x*(A*b - C*a + x*(B*b - D*a))/(4*b**
2*(a + b*x**2)**2) + x*(A*b - 5*C*a + x*(2*B*b - 6*D*a))/(8*a*b**
2*(a + b*x**2)) + (A*b + 3*C*a)*atan(sqrt(b)*x/sqrt(a))/(8*a**(3/
2)*b**(5/2))
```

Mathematica [A] time = 0.204529, size = 122, normalized size = 0.9

$$\frac{\frac{\sqrt{b}(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{-2a^2D+2ab(B+Cx)-2Ab^2x}{(a+bx^2)^2}+\frac{8a^2D-ab(4B+5Cx)+Ab^2x}{a(a+bx^2)}+4D\log\left(a+bx^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*
D + A*b^2*x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b
+ 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2
```

```
])/(8*b^3)
```

Maple [A] time = 0.019, size = 138, normalized size = 1.

$$\frac{1}{(bx^2+a)^2}\left(\frac{(Ab-5\,aC)\,x^3}{8\,ab}-\frac{(Bb-2\,aD)\,x^2}{2\,b^2}-\frac{(Ab+3\,aC)\,x}{8\,b^2}-\frac{a\,(Bb-3\,aD)}{4\,b^3}\right)+\frac{D\ln\left(ab^2\left(bx^2+a\right)\right)}{2\,b^3}+\frac{A}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,C}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/
b^2*x-1/4*a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/2*D/b^3*ln(a*b^2*(b*x^
2+a))+1/8/a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+3/8/b^2/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237941, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,Ca^3b+Aa^2b^2+\left(3\,Cab^3+Ab^4\right)x^4+2\left(3\,Ca^2b^2+Aab^3\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(6\,Da^3-2\,Ba^2b-\left(5\,Cab^2-A\right.\right.}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*a^3*b + A*a^2*b^2 + (3*C*a*b^3 + A*b^4)*x^4 + 2*(3*C*
a^2*b^2 + A*a*b^3)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b
*x^2 + a)) + 2*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4
*(2*D*a^2*b - B*a*b^2)*x^2 - (3*C*a^2*b + A*a*b^2)*x + 4*(D*a*b^2
*x^4 + 2*D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))*sqrt(-a*b))/((a*b^5
*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)*sqrt(-a*b)), 1/8*((3*C*a^3*b + A*
a^2*b^2 + (3*C*a*b^3 + A*b^4)*x^4 + 2*(3*C*a^2*b^2 + A*a*b^3)*x^2
)*arctan(sqrt(a*b)*x/a) + (6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b
^3)*x^3 + 4*(2*D*a^2*b - B*a*b^2)*x^2 - (3*C*a^2*b + A*a*b^2)*x +
 4*(D*a*b^2*x^4 + 2*D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))*sqrt(a*b
))/((a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)*sqrt(a*b))]
```

Sympy [A] time = 33.8903, size = 303, normalized size = 2.23

$$\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2 + 16a^2b^3\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right) + \left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2 + 16a^2b^3\left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right) - \frac{2Ba^2b - 6Da^3 + x^3\left(-Ab^3+5Cab^2\right) + x^2\left(4Bab^2 - 8Da^2b\right) + x\left(Aab^2 + 3Ca^2b\right)}{8a^3b^3 + 16a^2b^4x^2 + 8ab^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(
x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b
 + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sq
rt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*a**2 +
 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a*
*3*b**6)))/(A*b**2 + 3*C*a*b)) - (2*B*a**2*b - 6*D*a**3 + x**3*(-
A*b**3 + 5*C*a*b**2) + x**2*(4*B*a*b**2 - 8*D*a**2*b) + x*(A*a*b*
*2 + 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**
4)
```

GIAC/XCAS [A] time = 0.226742, size = 173, normalized size = 1.27

$$\frac{D\ln\left(bx^2+a\right)}{2\,b^3} + \frac{(3\,Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{\left(5\,Cab - Ab^2\right)x^3 - 4\left(2\,Da^2 - Bab\right)x^2 + \left(3\,Ca^2 + Aab\right)x - \frac{2\,(3\,Da^3 - Ba^2b)}{b}}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)*x^2/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*ln(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*
a*b)*x^2 + (3*C*a^2 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2
 + a)^2*a*b^2)
```

3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=119

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2(a+bx^2)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}$$

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2
*(A*b + a*C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3
*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.249769, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2(a+bx^2)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2
*(A*b + a*C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3
*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rubi in Sympy [A] time = 31.224, size = 88, normalized size = 0.74

$$-\frac{A+Bx+Cx^2+Dx^3}{4b(a+bx^2)^2}-\frac{2Ca-x(Bb-3Da)}{8ab^2(a+bx^2)}+\frac{(Bb+3Da)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -(A + B*x + C*x**2 + D*x**3)/(4*b*(a + b*x**2)**2) - (2*C*a - x*(
B*b - 3*D*a))/(8*a*b**2*(a + b*x**2)) + (B*b + 3*D*a)*atan(sqrt(b
)*x/sqrt(a))/(8*a**(3/2)*b**(5/2))
```

Mathematica [A] time = 0.204017, size = 99, normalized size = 0.83

$$\frac{\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}\left(-a^2(2C+3Dx)-ab\left(2A+x\left(B+4Cx+5Dx^2\right)\right)+b^2Bx^3\right)}{a(a+bx^2)^2}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*
x + 5*D*x^2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/a^(3/2))/(8*b^(5/2))
```

Maple [A] time = 0.013, size = 110, normalized size = 0.9

$$\frac{1}{(bx^2+a)^2}\left(\frac{(Bb-5\,aD)\,x^3}{8\,ab}-\frac{Cx^2}{2\,b}-\frac{(Bb+3\,aD)\,x}{8\,b^2}-\frac{Ab+aC}{4\,b^2}\right)+\frac{B}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,D}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A
*b+C*a)/b^2)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*B+3/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238685, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(3\,Dab^2+Bb^3\right)x^4+3\,Da^3+Ba^2b+2\left(3\,Da^2b+Bab^2\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(4\,Cabx^2+\left(5\,Dab-Bb^2\right)x^3+2\,C\right.}{16\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*
b + B*a*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) - 2*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b
 + (3*D*a^2 + B*a*b)*x)*sqrt(-a*b))/((a*b^4*x^4 + 2*a^2*b^3*x^2 +
 a^3*b^2)*sqrt(-a*b)), 1/8*(((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 +
B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)*arctan(sqrt(a*b)*x/a) - (4
*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2
 + B*a*b)*x)*sqrt(a*b))/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*sq
rt(a*b))]
```

Sympy [A] time = 25.8248, size = 177, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}-\frac{2Aab+2Ca^2+4Cabx^2+x^3\left(-Bb^2+5Dab\right)+x\left(Bab+3Da^2\right)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*
b**5)) + x)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2
*sqrt(-1/(a**3*b**5)) + x)/16 - (2*A*a*b + 2*C*a**2 + 4*C*a*b*x**
2 + x**3*(-B*b**2 + 5*D*a*b) + x*(B*a*b + 3*D*a**2))/(8*a**3*b**2
 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

GIAC/XCAS [A] time = 0.223047, size = 131, normalized size = 1.1

$$\frac{(3\,Da + Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Dabx^3 - Bb^2x^3 + 4\,Cabx^2 + 3\,Da^2x + Babx + 2\,Ca^2 + 2\,Aab}{8\,(bx^2 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)*x/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(
5*D*a*b*x^3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C
*a^2 + 2*A*a*b)/((b*x^2 + a)^2*a*b^2)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=116

$$\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2(a+bx^2)}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab(a+bx^2)^2}$$

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2
*D - b*(3*A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.173001, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2(a+bx^2)}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2
*D - b*(3*A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 34.5027, size = 100, normalized size = 0.86

$$\frac{x(Ab-Ca+x(Bb-Da))}{4ab(a+bx^2)^2}-\frac{2a(Bb+Da)-bx(3Ab+Ca)}{8a^2b^2(a+bx^2)}+\frac{(3Ab+Ca)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] x*(A*b - C*a + x*(B*b - D*a))/(4*a*b*(a + b*x**2)**2) - (2*a*(B*b
 + D*a) - b*x*(3*A*b + C*a))/(8*a**2*b**2*(a + b*x**2)) + (3*A*b
+ C*a)*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*b**(3/2))
```

Mathematica [A] time = 0.200031, size = 104, normalized size = 0.9

$$\frac{\frac{\sqrt{a}(-2a^3D-a^2b(2B+x(C+4Dx))+ab^2x(5A+Cx^2)+3Ab^3x^3)}{(a+bx^2)^2}+\sqrt{b}(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b
*(2*B + x*(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^2)
```

Maple [A] time = 0.012, size = 111, normalized size = 1.

$$\frac{1}{(bx^2+a)^2}\left(\frac{(3Ab+aC)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-aC)x}{8ab}-\frac{Bb+aD}{4b^2}\right)+\frac{3A}{8a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{C}{8ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B
*b+D*a)/b^2)/(b*x^2+a)^2+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*A+1/8/a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237723, size = 1, normalized size = 0.01

$$\left[\frac{\left(Ca^3b+3Aa^2b^2+\left(Cab^3+3Ab^4\right)x^4+2\left(Ca^2b^2+3Aab^3\right)x^2\right)\log\left(\frac{2abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(4Da^2bx^2+2Da^3+2Ba^2b-\right.}{16\left(a^2b^4x^4+2a^3b^3x^2+a^4b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((C*a^3*b + 3*A*a^2*b^2 + (C*a*b^3 + 3*A*b^4)*x^4 + 2*(C*a^
2*b^2 + 3*A*a*b^3)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b
*x^2 + a)) - 2*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 +
3*A*b^3)*x^3 + (C*a^2*b - 5*A*a*b^2)*x)*sqrt(-a*b))/((a^2*b^4*x^4
 + 2*a^3*b^3*x^2 + a^4*b^2)*sqrt(-a*b)), 1/8*((C*a^3*b + 3*A*a^2*
b^2 + (C*a*b^3 + 3*A*b^4)*x^4 + 2*(C*a^2*b^2 + 3*A*a*b^3)*x^2)*ar
ctan(sqrt(a*b)*x/a) - (4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a
*b^2 + 3*A*b^3)*x^3 + (C*a^2*b - 5*A*a*b^2)*x)*sqrt(a*b))/((a^2*b
^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sqrt(a*b))]
```

Sympy [A] time = 16.4478, size = 184, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\left(3Ab^3+Cab^2\right)+x\left(5Aab^2-Ca^2b\right)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**
3)) + x)/16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(
-1/(a**5*b**3)) + x)/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x*
*2 + x**3*(3*A*b**3 + C*a*b**2) + x*(5*A*a*b**2 - C*a**2*b))/(8*a
**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

GIAC/XCAS [A] time = 0.219677, size = 143, normalized size = 1.23

$$\frac{(Ca + 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3 + 3\,Ab^3x^3 - 4\,Da^2bx^2 - Ca^2bx + 5\,Aab^2x - 2\,Da^3 - 2\,Ba^2b}{8\,(bx^2 + a)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(
C*a*b^2*x^3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2
*x - 2*D*a^3 - 2*B*a^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3} dx$

Optimal. Leaf size=130

$$\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b
*B + a*D)*x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a +
 b*x^2])/(2*a^3)
```

Rubi [A] time = 0.341028, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b
*B + a*D)*x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a +
 b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 63.8864, size = 121, normalized size = 0.93

$$\frac{C\log(x)}{a^2b}-\frac{C\log\left(a+bx^2\right)}{2a^2b}+\frac{x\left(\frac{Ab}{x}+Bb-\frac{Ca}{x}-Da\right)}{4ab\left(a+bx^2\right)^2}+\frac{4Ca+x\left(3Bb+Da\right)}{8a^2b\left(a+bx^2\right)}+\frac{(3Bb+Da)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] C*log(x)/(a**2*b) - C*log(a + b*x**2)/(2*a**2*b) + x*(A*b/x + B*b
 - C*a/x - D*a)/(4*a*b*(a + b*x**2)**2) + (4*C*a + x*(3*B*b + D*a
))/(8*a**2*b*(a + b*x**2)) + (3*B*b + D*a)*atan(sqrt(b)*x/sqrt(a)
)/(8*a**(5/2)*b**(3/2))
```

Mathematica [A] time = 0.211728, size = 117, normalized size = 0.9

$$\frac{\frac{2a^2(-a(C+Dx)+Ab+bBx)}{b(a+bx^2)^2}+\frac{a(aDx+4Ab+3bBx)}{b(a+bx^2)}-4A\log\left(a+bx^2\right)+\frac{\sqrt{a}(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*
B*x - a*(C + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x
```

```
^2])/(8*a^3)
```

Maple [A] time = 0.019, size = 186, normalized size = 1.4

$$\frac{A\ln(x)}{a^3}+\frac{3\,bBx^3}{8\,a^2\,(bx^2+a)^2}+\frac{Dx^3}{8\,a\,(bx^2+a)^2}+\frac{Ax^2b}{2\,a^2\,(bx^2+a)^2}+\frac{5\,Bx}{8\,a\,(bx^2+a)^2}-\frac{xD}{8\,(bx^2+a)^2\,b}+\frac{3\,A}{4\,a\,(bx^2+a)^2}-\frac{C}{4\,(bx^2+a)^2\,b}-\frac{A\ln\left(b\,(bx^2+a)\right)}{2\,a^3}+\frac{3\,B}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{D}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x)
```

```
[Out] A*ln(x)/a^3+3/8/a^2/(b*x^2+a)^2*B*x^3*b+1/8/a/(b*x^2+a)^2*D*x^3+1
/2/a^2/(b*x^2+a)^2*A*x^2*b+5/8/a/(b*x^2+a)^2*B*x-1/8/(b*x^2+a)^2/
b*x*D+3/4/a/(b*x^2+a)^2*A-1/4/(b*x^2+a)^2/b*C-1/2/a^3*A*ln(b*(b*x
^2+a))+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B+1/8/a/b/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26664, size = 1, normalized size = 0.01

$$\left[\frac{\left(Da^4+3\,Ba^3b+\left(Da^2b^2+3\,Bab^3\right)x^4+2\,\left(Da^3b+3\,Ba^2b^2\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\,\left(4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+\left(\right.\right.}{16\,(a^3b^3x^4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x),x, algorithm="fricas")
```

```
[Out] [1/16*((D*a^4 + 3*B*a^3*b + (D*a^2*b^2 + 3*B*a*b^3)*x^4 + 2*(D*a^
3*b + 3*B*a^2*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b
*x^2 + a)) + 2*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b + (D*a^2*b +
3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x - 4*(A*b^3*x^4 + 2*A*a*b^2
*x^2 + A*a^2*b)*log(b*x^2 + a) + 8*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A
*a^2*b)*log(x))*sqrt(-a*b))/((a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b
)*sqrt(-a*b)), 1/8*((D*a^4 + 3*B*a^3*b + (D*a^2*b^2 + 3*B*a*b^3)*
x^4 + 2*(D*a^3*b + 3*B*a^2*b^2)*x^2)*arctan(sqrt(a*b)*x/a) + (4*A
*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b + (D*a^2*b + 3*B*a*b^2)*x^3 - (D
*a^3 - 5*B*a^2*b)*x - 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log
(b*x^2 + a) + 8*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(x))*sqr
t(a*b))/((a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*sqrt(a*b))]
```

Sympy [A] time = 30.4944, size = 872, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 + (-A/(2*a**3) - sqrt(-a**7*b**3)*(3*B*b + D*a)/(16
*a**6*b**3))*log(x + (3072*A**3*b**4 + 3072*A**2*a**3*b**4*(-A/(2
*a**3) - sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) - 144*A*B
**2*a*b**3 - 96*A*B*D*a**2*b**2 - 16*A*D**2*a**3*b - 6144*A*a**6*
b**4*(-A/(2*a**3) - sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)
)**2 + 144*B**2*a**4*b**3*(-A/(2*a**3) - sqrt(-a**7*b**3)*(3*B*b
+ D*a)/(16*a**6*b**3)) + 96*B*D*a**5*b**2*(-A/(2*a**3) - sqrt(-a*
*7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 16*D**2*a**6*b*(-A/(2*a*
*3) - sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)))/(1728*A**2*
B*b**4 + 576*A**2*D*a*b**3 + 27*B**3*a*b**3 + 27*B**2*D*a**2*b**2
 + 9*B*D**2*a**3*b + D**3*a**4)) + (-A/(2*a**3) + sqrt(-a**7*b**3
)*(3*B*b + D*a)/(16*a**6*b**3))*log(x + (3072*A**3*b**4 + 3072*A*
*2*a**3*b**4*(-A/(2*a**3) + sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a*
*6*b**3)) - 144*A*B**2*a*b**3 - 96*A*B*D*a**2*b**2 - 16*A*D**2*a*
*3*b - 6144*A*a**6*b**4*(-A/(2*a**3) + sqrt(-a**7*b**3)*(3*B*b +
D*a)/(16*a**6*b**3))**2 + 144*B**2*a**4*b**3*(-A/(2*a**3) + sqrt(
-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 96*B*D*a**5*b**2*(-A/
(2*a**3) + sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*b**3)) + 16*D*
*2*a**6*b*(-A/(2*a**3) + sqrt(-a**7*b**3)*(3*B*b + D*a)/(16*a**6*
b**3)))/(1728*A**2*B*b**4 + 576*A**2*D*a*b**3 + 27*B**3*a*b**3 +
27*B**2*D*a**2*b**2 + 9*B*D**2*a**3*b + D**3*a**4)) + (6*A*a*b +
4*A*b**2*x**2 - 2*C*a**2 + x**3*(3*B*b**2 + D*a*b) + x*(5*B*a*b -
 D*a**2))/(8*a**4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

GIAC/XCAS [A] time = 0.228001, size = 173, normalized size = 1.33

$$-\frac{A\ln\left(bx^2+a\right)}{2\,a^3}+\frac{A\ln\left(|x|\right)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}+\frac{4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+\left(Da^2b+3\,Bab^2\right)x^3-\left(Da^3-5\,Ba^2b\right)x}{8\,(bx^2+a)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*A*ln(b*x^2 + a)/a^3 + A*ln(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C
*a^3 + 6*A*a^2*b + (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b
)*x)/((b*x^2 + a)^2*a^3*b)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3} dx$

Optimal. Leaf size=144

$$-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)
^2) + (4*B - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b
 - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log
[x])/a^3 - (B*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.446339, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)
^2) + (4*B - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b
 - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log
[x])/a^3 - (B*Log[a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 71.7661, size = 124, normalized size = 0.86

$$-\frac{3C}{2a^2bx}-\frac{3C\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\sqrt{b}}+\frac{D\log(x)}{a^2b}-\frac{D\log\left(a+bx^2\right)}{2a^2b}+\frac{x\left(\frac{Ab}{x^2}+\frac{Bb}{x}-\frac{Ca}{x^2}-\frac{Da}{x}\right)}{4ab\left(a+bx^2\right)^2}+\frac{C+Dx}{2abx\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -3*C/(2*a**2*b*x) - 3*C*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/2)*sqrt(
b)) + D*log(x)/(a**2*b) - D*log(a + b*x**2)/(2*a**2*b) + x*(A*b/x
**2 + B*b/x - C*a/x**2 - D*a/x)/(4*a*b*(a + b*x**2)**2) + (C + D*
x)/(2*a*b*x*(a + b*x**2))
```

Mathematica [A] time = 0.182523, size = 141, normalized size = 0.98

$$\frac{3(aC-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{4aB+3aCx-7Abx}{8a^3\left(a+bx^2\right)}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{a^2(-D)+abB+abCx-Ab^2x}{4a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a +
b*x^2)^2) + (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*
(-5*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) +
 (B*Log[x])/a^3 - (B*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.023, size = 195, normalized size = 1.4

$$-\frac{A}{a^3x}+\frac{\ln(x)B}{a^3}-\frac{7Ax^3b^2}{8a^3(bx^2+a)^2}+\frac{3bCx^3}{8a^2(bx^2+a)^2}+\frac{Bbx^2}{2a^2(bx^2+a)^2}-\frac{9Axb}{8a^2(bx^2+a)^2}+\frac{5Cx}{8a(bx^2+a)^2}+\frac{3B}{4a(bx^2+a)^2}-\frac{D}{4(bx^2+a)^2b}-\frac{\ln(bx^2+a)B}{2a^3}-\frac{15Ab}{8a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3C}{8a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -A/a^3/x+1/a^3*ln(x)*B-7/8/a^3/(b*x^2+a)^2*A*x^3*b^2+3/8/a^2/(b*x
^2+a)^2*C*x^3*b+1/2/a^2/(b*x^2+a)^2*B*x^2*b-9/8/a^2/(b*x^2+a)^2*A
*x*b+5/8/a/(b*x^2+a)^2*C*x+3/4/a/(b*x^2+a)^2*B-1/4/(b*x^2+a)^2/b*
D-1/2/a^3*ln(b*x^2+a)*B-15/8/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*A*b+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250744, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(Cab^3-5Ab^4\right)x^5+2\left(Ca^2b^2-5Aab^3\right)x^3+\left(Ca^3b-5Aa^2b^2\right)x\right)\log\left(-\frac{2abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(4Bab^2x^3+3\left(Cab^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((C*a*b^3 - 5*A*b^4)*x^5 + 2*(C*a^2*b^2 - 5*A*a*b^3)*x^
3 + (C*a^3*b - 5*A*a^2*b^2)*x)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-
a*b))/(b*x^2 + a)) - 2*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4
 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b
)*x - 4*(B*b^3*x^5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) +
8*(B*b^3*x^5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*log(x))*sqrt(-a*b))/((a
^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5*b*x)*sqrt(-a*b)), 1/8*(3*((C*a*b
^3 - 5*A*b^4)*x^5 + 2*(C*a^2*b^2 - 5*A*a*b^3)*x^3 + (C*a^3*b - 5*
A*a^2*b^2)*x)*arctan(sqrt(a*b)*x/a) + (4*B*a*b^2*x^3 + 3*(C*a*b^2
 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*
a^3 - 3*B*a^2*b)*x - 4*(B*b^3*x^5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*lo
```

```
g(b*x^2 + a) + 8*(B*b^3*x^5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*log(x))*
sqrt(a*b))/((a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5*b*x)*sqrt(a*b))]
```

Sympy [A] time = 37.9281, size = 860, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] B*log(x)/a**3 + (-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16
*a**7*b))*log(x + (-1200*A**2*B*a*b**2 + 1200*A**2*a**4*b**2*(-B/
(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) + 480*A*B*
C*a**2*b - 480*A*C*a**5*b*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b
+ C*a)/(16*a**7*b)) + 1024*B**3*a**2*b + 1024*B**2*a**5*b*(-B/(2*
a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) - 48*B*C**2*a
**3 - 2048*B*a**8*b*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-5*A*b + C*a)
/(16*a**7*b))**2 + 48*C**2*a**6*(-B/(2*a**3) - 3*sqrt(-a**7*b)*(-
5*A*b + C*a)/(16*a**7*b)))/(-1125*A**3*b**3 + 675*A**2*C*a*b**2 -
 2880*A*B**2*a*b**2 - 135*A*C**2*a**2*b + 576*B**2*C*a**2*b + 9*C
**3*a**3)) + (-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a*
*7*b))*log(x + (-1200*A**2*B*a*b**2 + 1200*A**2*a**4*b**2*(-B/(2*
a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) + 480*A*B*C*a
**2*b - 480*A*C*a**5*b*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C
*a)/(16*a**7*b)) + 1024*B**3*a**2*b + 1024*B**2*a**5*b*(-B/(2*a**
3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(16*a**7*b)) - 48*B*C**2*a**3
 - 2048*B*a**8*b*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A*b + C*a)/(1
6*a**7*b))**2 + 48*C**2*a**6*(-B/(2*a**3) + 3*sqrt(-a**7*b)*(-5*A
*b + C*a)/(16*a**7*b)))/(-1125*A**3*b**3 + 675*A**2*C*a*b**2 - 28
80*A*B**2*a*b**2 - 135*A*C**2*a**2*b + 576*B**2*C*a**2*b + 9*C**3
*a**3)) + (-8*A*a**2*b + 4*B*a*b**2*x**3 + x**4*(-15*A*b**3 + 3*C
*a*b**2) + x**2*(-25*A*a*b**2 + 5*C*a**2*b) + x*(6*B*a**2*b - 2*D
*a**3))/(8*a**5*b*x + 16*a**4*b**2*x**3 + 8*a**3*b**3*x**5)
```

GIAC/XCAS [A] time = 0.226215, size = 190, normalized size = 1.32

$$-\frac{B\ln\left(bx^2+a\right)}{2\,a^3}+\frac{B\ln\left(|x|\right)}{a^3}+\frac{3\left(Ca-5\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}+\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-5\,Aab^2\right)x^2-2\left(Da^3-3\,Ba^2b\right)x}{8\left(bx^2+a\right)^2a^3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] -1/2*B*ln(b*x^2 + a)/a^3 + B*ln(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*
a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 -
2*(D*a^3 - 3*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b*x)
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a
*(a + b*x^2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a
 + b*x^2)) - (3*(5*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
7/2)*Sqrt[b]) - ((3*A*b - a*C)*Log[x])/a^4 + ((3*A*b - a*C)*Log[a
 + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.628237, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a
*(a + b*x^2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a
 + b*x^2)) - (3*(5*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
7/2)*Sqrt[b]) - ((3*A*b - a*C)*Log[x])/a^4 + ((3*A*b - a*C)*Log[a
 + b*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 74.1163, size = 136, normalized size = 0.78

$$-\frac{C}{a^2bx^2}-\frac{2C\log(x)}{a^3}+\frac{C\log\left(a+bx^2\right)}{a^3}-\frac{3D}{2a^2bx}-\frac{3D\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\sqrt{b}}+\frac{x\left(\frac{Ab}{x^3}+\frac{Bb}{x^2}-\frac{Ca}{x^3}-\frac{Da}{x^2}\right)}{4ab\left(a+bx^2\right)^2}+\frac{C+Dx}{2abx^2\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] -C/(a**2*b*x**2) - 2*C*log(x)/a**3 + C*log(a + b*x**2)/a**3 - 3*D
/(2*a**2*b*x) - 3*D*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/2)*sqrt(b))
+ x*(A*b/x**3 + B*b/x**2 - C*a/x**3 - D*a/x**2)/(4*a*b*(a + b*x**
2)**2) + (C + D*x)/(2*a*b*x**2*(a + b*x**2))
```

Mathematica [A] time = 0.321675, size = 147, normalized size = 0.84

$$\frac{\frac{2a^2(a(C+Dx)-Ab-bBx)}{(a+bx^2)^2}+\frac{a(4aC+3aDx-8Ab-7bBx)}{a+bx^2}+4(3Ab-aC)\log\left(a+bx^2\right)+8\log(x)(aC-3Ab)-\frac{4aA}{x^2}+\frac{3\sqrt{a}(aD-5bB)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{\sqrt{b}}}{8a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*
x))/(a + b*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x
^2)^2 + (3*Sqrt[a]*(-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sq
rt[b] + 8*(-3*A*b + a*C)*Log[x] + 4*(3*A*b - a*C)*Log[a + b*x^2])
/(8*a^4)
```

Maple [A] time = 0.024, size = 250, normalized size = 1.4

$$-\frac{A}{2\,a^3x^2}-\frac{B}{a^3x}-3\,\frac{A\ln(x)\,b}{a^4}+\frac{\ln(x)\,C}{a^3}-\frac{7\,Bb^2x^3}{8\,a^3\,(bx^2+a)^2}+\frac{3\,bDx^3}{8\,a^2\,(bx^2+a)^2}-\frac{Ax^2b^2}{a^3\,(bx^2+a)^2}+\frac{bCx^2}{2\,a^2\,(bx^2+a)^2}-\frac{9\,bBx}{8\,a^2\,(bx^2+a)^2}+\frac{5\,Dx}{8\,a\,(bx^2+a)^2}-\frac{5\,Ab}{4\,a^2\,(bx^2+a)^2}+\frac{3\,C}{4\,a\,(bx^2+a)^2}+\frac{3\,b\ln\left(bx^2+a\right)A}{2\,a^4}-\frac{\ln\left(bx^2+a\right)C}{2\,a^3}-\frac{15\,Bb}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,D}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*A/a^3/x^2-B/a^3/x-3/a^4*ln(x)*A*b+1/a^3*ln(x)*C-7/8/a^3/(b*x
^2+a)^2*B*x^3*b^2+3/8/a^2/(b*x^2+a)^2*D*x^3*b-1/a^3/(b*x^2+a)^2*A
*x^2*b^2+1/2/a^2/(b*x^2+a)^2*C*x^2*b-9/8/a^2/(b*x^2+a)^2*B*x*b+5/
8/a/(b*x^2+a)^2*D*x-5/4/a^2*b/(b*x^2+a)^2*A+3/4/a/(b*x^2+a)^2*C+3
/2/a^4*b*ln(b*x^2+a)*A-1/2/a^3*ln(b*x^2+a)*C-15/8/a^3/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*b*B+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*D
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299975, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((D*a^2*b^2 - 5*B*a*b^3)*x^6 + 2*(D*a^3*b - 5*B*a^2*b^2
)*x^4 + (D*a^4 - 5*B*a^3*b)*x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt
(-a*b))/(b*x^2 + a)) - 2*(3*(D*a^2*b - 5*B*a*b^2)*x^5 - 8*B*a^3*x
 + 4*(C*a^2*b - 3*A*a*b^2)*x^4 - 4*A*a^3 + 5*(D*a^3 - 5*B*a^2*b)*
x^3 + 6*(C*a^3 - 3*A*a^2*b)*x^2 - 4*((C*a*b^2 - 3*A*b^3)*x^6 + 2*
(C*a^2*b - 3*A*a*b^2)*x^4 + (C*a^3 - 3*A*a^2*b)*x^2)*log(b*x^2 +
a) + 8*((C*a*b^2 - 3*A*b^3)*x^6 + 2*(C*a^2*b - 3*A*a*b^2)*x^4 + (
C*a^3 - 3*A*a^2*b)*x^2)*log(x))*sqrt(-a*b))/((a^4*b^2*x^6 + 2*a^5
*b*x^4 + a^6*x^2)*sqrt(-a*b)), 1/8*(3*((D*a^2*b^2 - 5*B*a*b^3)*x^
```

```
6 + 2*(D*a^3*b - 5*B*a^2*b^2)*x^4 + (D*a^4 - 5*B*a^3*b)*x^2)*arct
an(sqrt(a*b)*x/a) + (3*(D*a^2*b - 5*B*a*b^2)*x^5 - 8*B*a^3*x + 4*
(C*a^2*b - 3*A*a*b^2)*x^4 - 4*A*a^3 + 5*(D*a^3 - 5*B*a^2*b)*x^3 +
 6*(C*a^3 - 3*A*a^2*b)*x^2 - 4*((C*a*b^2 - 3*A*b^3)*x^6 + 2*(C*a^
2*b - 3*A*a*b^2)*x^4 + (C*a^3 - 3*A*a^2*b)*x^2)*log(b*x^2 + a) +
8*((C*a*b^2 - 3*A*b^3)*x^6 + 2*(C*a^2*b - 3*A*a*b^2)*x^4 + (C*a^3
 - 3*A*a^2*b)*x^2)*log(x))*sqrt(a*b))/((a^4*b^2*x^6 + 2*a^5*b*x^4
 + a^6*x^2)*sqrt(a*b))]
```

Sympy [A] time = 96.7322, size = 1904, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a*
*8*b))*log(x + (-27648*A**3*b**4 + 27648*A**2*C*a*b**3 + 9216*A**
2*a**4*b**3*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b +
 D*a)/(16*a**8*b)) + 3600*A*B**2*a*b**3 - 1440*A*B*D*a**2*b**2 -
9216*A*C**2*a**2*b**2 - 6144*A*C*a**5*b**2*(-(-3*A*b + C*a)/(2*a*
*4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 144*A*D**2*a*
*3*b + 6144*A*a**8*b**2*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*
b)*(-5*B*b + D*a)/(16*a**8*b))**2 - 1200*B**2*C*a**2*b**2 + 1200*
B**2*a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*
b + D*a)/(16*a**8*b)) + 480*B*C*D*a**3*b - 480*B*D*a**6*b*(-(-3*A
*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b))
+ 1024*C**3*a**3*b + 1024*C**2*a**6*b*(-(-3*A*b + C*a)/(2*a**4) -
 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) - 48*C*D**2*a**4 - 2
048*C*a**9*b*(-(-3*A*b + C*a)/(2*a**4) - 3*sqrt(-a**9*b)*(-5*B*b
+ D*a)/(16*a**8*b))**2 + 48*D**2*a**7*(-(-3*A*b + C*a)/(2*a**4) -
 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)))/(-25920*A**2*B*b**4
 + 5184*A**2*D*a*b**3 + 17280*A*B*C*a*b**3 - 3456*A*C*D*a**2*b**2
 - 1125*B**3*a*b**3 + 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**
2 - 135*B*D**2*a**3*b + 576*C**2*D*a**3*b + 9*D**3*a**4)) + (-(-3
*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)
)*log(x + (-27648*A**3*b**4 + 27648*A**2*C*a*b**3 + 9216*A**2*a**
4*b**3*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)
/(16*a**8*b)) + 3600*A*B**2*a*b**3 - 1440*A*B*D*a**2*b**2 - 9216*
A*C**2*a**2*b**2 - 6144*A*C*a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) +
 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 144*A*D**2*a**3*b
+ 6144*A*a**8*b**2*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-
5*B*b + D*a)/(16*a**8*b))**2 - 1200*B**2*C*a**2*b**2 + 1200*B**2*
a**5*b**2*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D
*a)/(16*a**8*b)) + 480*B*C*D*a**3*b - 480*B*D*a**6*b*(-(-3*A*b +
C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) + 102
4*C**3*a**3*b + 1024*C**2*a**6*b*(-(-3*A*b + C*a)/(2*a**4) + 3*sq
rt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)) - 48*C*D**2*a**4 - 2048*C
*a**9*b*(-(-3*A*b + C*a)/(2*a**4) + 3*sqrt(-a**9*b)*(-5*B*b + D*a
)/(16*a**8*b))**2 + 48*D**2*a**7*(-(-3*A*b + C*a)/(2*a**4) + 3*sq
rt(-a**9*b)*(-5*B*b + D*a)/(16*a**8*b)))/(-25920*A**2*B*b**4 + 51
84*A**2*D*a*b**3 + 17280*A*B*C*a*b**3 - 3456*A*C*D*a**2*b**2 - 11
25*B**3*a*b**3 + 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**2 - 1
35*B*D**2*a**3*b + 576*C**2*D*a**3*b + 9*D**3*a**4)) + (-4*A*a**2
 - 8*B*a**2*x + x**5*(-15*B*b**2 + 3*D*a*b) + x**4*(-12*A*b**2 +
4*C*a*b) + x**3*(-25*B*a*b + 5*D*a**2) + x**2*(-18*A*a*b + 6*C*a*
*2))/(8*a**5*x**2 + 16*a**4*b*x**4 + 8*a**3*b**2*x**6) + (-3*A*b
+ C*a)*log(x + (-27648*A**3*b**4 + 27648*A**2*C*a*b**3 + 9216*A**
2*b**3*(-3*A*b + C*a) + 3600*A*B**2*a*b**3 - 1440*A*B*D*a**2*b**2
 - 9216*A*C**2*a**2*b**2 - 6144*A*C*a*b**2*(-3*A*b + C*a) + 144*A
*D**2*a**3*b + 6144*A*b**2*(-3*A*b + C*a)**2 - 1200*B**2*C*a**2*b
**2 + 1200*B**2*a*b**2*(-3*A*b + C*a) + 480*B*C*D*a**3*b - 480*B*
D*a**2*b*(-3*A*b + C*a) + 1024*C**3*a**3*b + 1024*C**2*a**2*b*(-3
*A*b + C*a) - 48*C*D**2*a**4 - 2048*C*a*b*(-3*A*b + C*a)**2 + 48*
D**2*a**3*(-3*A*b + C*a))/(-25920*A**2*B*b**4 + 5184*A**2*D*a*b**
3 + 17280*A*B*C*a*b**3 - 3456*A*C*D*a**2*b**2 - 1125*B**3*a*b**3
+ 675*B**2*D*a**2*b**2 - 2880*B*C**2*a**2*b**2 - 135*B*D**2*a**3*
b + 576*C**2*D*a**3*b + 9*D**3*a**4))/a**4
```

GIAC/XCAS [A] time = 0.225452, size = 219, normalized size = 1.26

$$\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\ln\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\ln\left(|x|\right)}{a^4}+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-4\,Aa^2}{8\,(bx^3+ax)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/((b*x^2 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*
a - 3*A*b)*ln(b*x^2 + a)/a^4 + (C*a - 3*A*b)*ln(abs(x))/a^4 + 1/8
*(3*D*a*b*x^5 - 15*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a
^2*x^3 - 25*B*a*b*x^3 + 6*C*a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x -
4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

3.110 $\int \frac{-x+4x^3}{(5+x^2)^2} \, dx$

Optimal. Leaf size=20

$$\frac{21}{2(x^2+5)} + 2\log\left(x^2+5\right)$$

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rubi [A] time = 0.049741, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{21}{2(x^2+5)} + 2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + 4*x^3)/(5 + x^2)^2,x]

[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rubi in Sympy [A] time = 9.17442, size = 15, normalized size = 0.75

$$2\log\left(x^2+5\right) + \frac{21}{2(x^2+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**3-x)/(x**2+5)**2,x)

[Out] 2*log(x**2 + 5) + 21/(2*(x**2 + 5))
```

Mathematica [A] time = 0.0126905, size = 20, normalized size = 1.

$$\frac{21}{2(x^2+5)} + 2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]

[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Maple [A] time = 0.014, size = 19, normalized size = 1.

$$\frac{21}{2x^2+10} + 2\ln\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^3-x)/(x^2+5)^2,x)

[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Maxima [A] time = 1.34696, size = 24, normalized size = 1.2

$$\frac{21}{2\,(x^2+5)} + 2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x)/(x^2 + 5)^2,x, algorithm="maxima")

[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

Fricas [A] time = 0.223175, size = 32, normalized size = 1.6

$$\frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\,(x^2+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x)/(x^2 + 5)^2,x, algorithm="fricas")

[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

Sympy [A] time = 0.097345, size = 15, normalized size = 0.75

$$2\log\left(x^2+5\right) + \frac{21}{2x^2+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**3-x)/(x**2+5)**2,x)

[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

GIAC/XCAS [A] time = 0.219991, size = 34, normalized size = 1.7

$$-\frac{4\,x^2-1}{2\,(x^2+5)} + 2\ln\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - x)/(x^2 + 5)^2,x, algorithm="giac")

[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*ln(x^2 + 5)
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}} \, dx$

Optimal. Leaf size=23

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0526903, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 7.20262, size = 17, normalized size = 0.74

$$\frac{\left(x^2-2\right)^{\frac{3}{2}}}{3}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x)/(x**2-2)**(1/2),x)
```

```
[Out] (x**2 - 2)**(3/2)/3 + sqrt(x**2 - 2)
```

Mathematica [A] time = 0.0121626, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{x^2-2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

Maple [A] time = 0.008, size = 15, normalized size = 0.7

$$\frac{x^2+1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x)/(x^2-2)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)*(x^2-2)^(1/2)
```

Maxima [A] time = 1.32909, size = 30, normalized size = 1.3

$$\frac{1}{3}\sqrt{x^2-2}x^2+\frac{1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/sqrt(x^2 - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

Fricas [A] time = 0.23735, size = 93, normalized size = 4.04

$$-\frac{2\,x^6-3\,x^4-3\,x^2-\left(2\,x^5-x^3-3\,x\right)\sqrt{x^2-2}+2}{3\left(2\,x^3-(2\,x^2-1)\sqrt{x^2-2}-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/sqrt(x^2 - 2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^6 - 3*x^4 - 3*x^2 - (2*x^5 - x^3 - 3*x)*sqrt(x^2 - 2) +
 2)/(2*x^3 - (2*x^2 - 1)*sqrt(x^2 - 2) - 3*x)
```

Sympy [A] time = 0.480152, size = 22, normalized size = 0.96

$$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x)/(x**2-2)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

GIAC/XCAS [A] time = 0.214884, size = 23, normalized size = 1.

$$\frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/sqrt(x^2 - 2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2} \, dx$

Optimal. Leaf size=25

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0591201, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]

[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 11.6984, size = 20, normalized size = 0.8

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\,\text{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4-x**2)/(2*x**2+1),x)

[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

Mathematica [A] time = 0.0185529, size = 25, normalized size = 1.

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]

[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.005, size = 21, normalized size = 0.8

$$-x + \frac{x^3}{3} + \frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4-x^2)/(2*x^2+1),x)
```

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.50371, size = 27, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 - x^2)/(2*x^2 + 1),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Fricas [A] time = 0.219631, size = 34, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(\sqrt{2}\left(x^3 - 3x\right) + 3\arctan\left(\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 - x^2)/(2*x^2 + 1),x, algorithm="fricas")

[Out] 1/6*sqrt(2)*(sqrt(2)*(x^3 - 3*x) + 3*arctan(sqrt(2)*x))
```

Sympy [A] time = 0.086394, size = 20, normalized size = 0.8

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4-x**2)/(2*x**2+1),x)

[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

GIAC/XCAS [A] time = 0.221173, size = 27, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 - x^2)/(2*x^2 + 1),x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

3.113 $\int \frac{x^3+x^4}{1+x^2}\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0595498, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}-x-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**3)/(x**2+1),x)
```

```
[Out] x**3/3 - x - log(x**2 + 1)/2 + atan(x) + Integral(x, x)
```

Mathematica [A] time = 0.0094795, size = 30, normalized size = 1.

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^3)/(x^2+1),x)
```

```
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.49592, size = 32, normalized size = 1.07

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.22549, size = 32, normalized size = 1.07

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.090556, size = 22, normalized size = 0.73

$$\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**3)/(x**2+1),x)
```

```
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.221957, size = 32, normalized size = 1.07

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*ln(x^2 + 1)
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{13/2}}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) +
 ((b*e - a*f)*x^9)/(9*b^2) + (f*x^11)/(11*b) - (a^(5/2)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

Rubi [A] time = 0.34738, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{13/2}}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) +
 ((b*e - a*f)*x^9)/(9*b^2) + (f*x^11)/(11*b) - (a^(5/2)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{\frac{5}{2}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{13}{2}}}+\frac{ax^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^5}+\frac{fx^{11}}{11b}-\frac{x^9\left(af-be\right)}{9b^2}+\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{x^5\left(a^3f-a^2be+ab^2d-b^3c\right)}{5b^4}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int a^2\,dx}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a**(5/2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/s
qrt(a))/b**(13/2) + a*x**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c
)/(3*b**5) + f*x**11/(11*b) - x**9*(a*f - b*e)/(9*b**2) + x**7*(a
**2*f - a*b*e + b**2*d)/(7*b**3) - x**5*(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c)/(5*b**4) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*Integral(a**2, x)/b**6
```

Mathematica [A] time = 0.292217, size = 210, normalized size = 1.

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^6}+\frac{ax^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{13/2}}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/
(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^11)/(11*b) + (a^(5/2)*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/b^(13/2)
```

Maple [A] time = 0.006, size = 278, normalized size = 1.3

$$\frac{fx^{11}}{11\,b}-\frac{x^9af}{9\,b^2}+\frac{x^9e}{9\,b}+\frac{x^7a^2f}{7\,b^3}-\frac{x^7ae}{7\,b^2}+\frac{x^7d}{7\,b}-\frac{x^5a^3f}{5\,b^4}+\frac{x^5a^2e}{5\,b^3}-\frac{x^5ad}{5\,b^2}+\frac{x^5c}{5\,b}+\frac{x^3a^4f}{3\,b^5}-\frac{x^3a^3e}{3\,b^4}+\frac{x^3a^2d}{3\,b^3}-\frac{ax^3c}{3\,b^2}-\frac{a^5fx}{b^6}+\frac{a^4ex}{b^5}-\frac{a^3dx}{b^4}+\frac{a^2cx}{b^3}+\frac{a^6f}{b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^5e}{b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^4d}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3c}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/11*f*x^11/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/7/b^3*x^7*a^2*f-1/7/b
^2*x^7*a*e+1/7/b*x^7*d-1/5/b^4*x^5*a^3*f+1/5/b^3*x^5*a^2*e-1/5/b^
2*x^5*a*d+1/5/b*x^5*c+1/3/b^5*x^3*a^4*f-1/3/b^4*x^3*a^3*e+1/3/b^3
*x^3*a^2*d-1/3/b^2*x^3*a*c-1/b^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*
d*x+1/b^3*a^2*c*x+a^6/b^6/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-a
^5/b^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e+a^4/b^4/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*d-a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230557, size = 1, normalized size = 0.

$$\left[\frac{630\,b^5fx^{11}+770\left(b^5e-ab^4f\right)x^9+990\left(b^5d-ab^4e+a^2b^3f\right)x^7+1386\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^5-2310\left(ab^4c-a\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d
- a*b^4*e + a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e -
a^3*b^2*f)*x^5 - 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)
*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*
log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6930*(a^2*b^3*c
 - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/3465*(315*b^5*f*x^11 +
385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2*b^3*f)*x^7
 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*b^
4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 3465*(
a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6]
```

Sympy [A] time = 1.77579, size = 366, normalized size = 1.74

$$-\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{fx^{11}}{11b}-\frac{x^9\left(af-be\right)}{9b^2}+\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{x^5\left(a^3f-a^2be+ab^2d-b^3c\right)}{5b^4}+\frac{x^3\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{3b^5}-\frac{x\left(a^5f-a^4be+a^3b^2d-a^2b^3c\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b
**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + sqrt(-a**5/
b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**6*sqrt(-a**
5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b
*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*b) - x**9*(a
*f - b*e)/(9*b**2) + x**7*(a**2*f - a*b*e + b**2*d)/(7*b**3) - x*
*5*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(5*b**4) + x**3*(a**4*
f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*b**5) - x*(a**5*f - a**
4*b*e + a**3*b**2*d - a**2*b**3*c)/b**6
```

GIAC/XCAS [A] time = 0.22397, size = 338, normalized size = 1.61

$$-\frac{\left(a^3b^3c-a^4b^2d-a^6f+a^5be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6}+\frac{315\,b^{10}fx^{11}-385\,ab^9fx^9+385\,b^{10}x^9e+495\,b^{10}dx^7+495\,a^2b^8fx^7-495\,ab^9x^7e+693\,b^{10}cx^5-693\,ab^9dx^5-693\,a^3b^7fx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^6) + 1/3465*(315*b^10*f*x^11 - 385*a*b^9*f*x^9 + 385
*b^10*x^9*e + 495*b^10*d*x^7 + 495*a^2*b^8*f*x^7 - 495*a*b^9*x^7*
e + 693*b^10*c*x^5 - 693*a*b^9*d*x^5 - 693*a^3*b^7*f*x^5 + 693*a^
2*b^8*x^5*e - 1155*a*b^9*c*x^3 + 1155*a^2*b^8*d*x^3 + 1155*a^4*b^
```

```
6*f*x^3 - 1155*a^3*b^7*x^3*e + 3465*a^2*b^8*c*x - 3465*a^3*b^7*d*
x - 3465*a^5*b^5*f*x + 3465*a^4*b^6*x*e)/b^11
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=172

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{11/2}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^9}{9b}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^
5)/(5*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/b^(11/2)
```

Rubi [A] time = 0.262295, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{11/2}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^
5)/(5*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/b^(11/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{3}{2}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{11}{2}}}+\frac{fx^9}{9b}-\frac{x^7\left(af-be\right)}{7b^2}+\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{x^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^4}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\int a\,dx}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**(3/2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/
sqrt(a))/b**(11/2) + f*x**9/(9*b) - x**7*(a*f - b*e)/(7*b**2) + x
**5*(a**2*f - a*b*e + b**2*d)/(5*b**3) - x**3*(a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)/(3*b**4) + (a**3*f - a**2*b*e + a*b**2*d - b
**3*c)*Integral(a, x)/b**5
```

Mathematica [A] time = 0.257894, size = 162, normalized size = 0.94

$$\frac{x\left(315a^4f-105a^3b\left(3e+fx^2\right)+21a^2b^2\left(15d+5ex^2+3fx^4\right)-3ab^3\left(105c+35dx^2+21ex^4+15fx^6\right)+b^4x^2\left(105c+63dx\right.\right.}{315b^5}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*
x^2 + 3*f*x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)
 + b^4*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) -
 (a^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(11/2)
```

Maple [A] time = 0.004, size = 230, normalized size = 1.3

$$\frac{fx^9}{9\,b}-\frac{x^7af}{7\,b^2}+\frac{x^7e}{7\,b}+\frac{x^5a^2f}{5\,b^3}-\frac{x^5ae}{5\,b^2}+\frac{x^5d}{5\,b}-\frac{x^3a^3f}{3\,b^4}+\frac{x^3a^2e}{3\,b^3}-\frac{ax^3d}{3\,b^2}+\frac{x^3c}{3\,b}+\frac{a^4fx}{b^5}-\frac{a^3ex}{b^4}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}-\frac{a^5f}{b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^4e}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3d}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2c}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/9*f*x^9/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/5/b^3*x^5*a^2*f-1/5/b^2
*x^5*a*e+1/5/b*x^5*d-1/3/b^4*x^3*a^3*f+1/3/b^3*x^3*a^2*e-1/3/b^2*
x^3*a*d+1/3/b*x^3*c+1/b^5*a^4*f*x-1/b^4*a^3*e*x+1/b^3*a^2*d*x-1/b
^2*a*c*x-a^5/b^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+a^4/b^4/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e-a^3/b^3/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*d+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237592, size = 1, normalized size = 0.01

$$\left[\frac{70\,b^4fx^9+90\,\left(b^4e-ab^3f\right)x^7+126\,\left(b^4d-ab^3e+a^2b^2f\right)x^5+210\,\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^3-315\,\left(ab^3c-a^2b^2d+\right.}{630\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*
b^3*e + a^2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b
*f)*x^3 - 315*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*
log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5, 1/315*(35*b^4*f*x^9 + 45*(b^
4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^2*b^2*f)*x^5 + 105*(
```

```
b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^3*c - a^2*b
^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 315*(a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

Sympy [A] time = 2.25389, size = 325, normalized size = 1.89

$$\frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2} + \frac{fx^9}{9b} - \frac{x^7\left(af-be\right)}{7b^2} + \frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3} - \frac{x^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^4} + \frac{x\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b*
*5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**
4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 - sqrt(-a**3/b**1
1)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**5*sqrt(-a**3/b*
*11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e +
 a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b) - x**7*(a*f - b*e)
/(7*b**2) + x**5*(a**2*f - a*b*e + b**2*d)/(5*b**3) - x**3*(a**3*
f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**4) + x*(a**4*f - a**3*b*e
 + a**2*b**2*d - a*b**3*c)/b**5
```

GIAC/XCAS [A] time = 0.222307, size = 270, normalized size = 1.57

$$\frac{\left(a^2b^3c-a^3b^2d-a^5f+a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8fx^9-45\,ab^7fx^7+45\,b^8x^7e+63\,b^8dx^5+63\,a^2b^6fx^5-63\,ab^7x^5e+105\,b^8cx^3-105\,ab^7dx^3-105\,a^3b^5fx^3+105\,a^2b}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^5) + 1/315*(35*b^8*f*x^9 - 45*a*b^7*f*x^7 + 45*b^8*x^
7*e + 63*b^8*d*x^5 + 63*a^2*b^6*f*x^5 - 63*a*b^7*x^5*e + 105*b^8*
c*x^3 - 105*a*b^7*d*x^3 - 105*a^3*b^5*f*x^3 + 105*a^2*b^6*x^3*e -
 315*a*b^7*c*x + 315*a^2*b^6*d*x + 315*a^4*b^4*f*x - 315*a^3*b^5*
x*e)/b^9
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=136

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^5(be-af)}{5b^2}+\frac{fx^7}{7b}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a
^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) -
(Sqrt[a]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(9/2)
```

Rubi [A] time = 0.215894, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^5(be-af)}{5b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a
^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) -
(Sqrt[a]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{a}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{9}{2}}}-\left(a^3f-a^2be+ab^2d-b^3c\right)\int\frac{1}{b^4}\,dx+\frac{fx^7}{7b}-\frac{x^5\left(af-be\right)}{5b^2}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(a)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/sq
rt(a))/b**(9/2) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*Integra
l(b**(-4), x) + f*x**7/(7*b) - x**5*(a*f - b*e)/(5*b**2) + x**3*(
a**2*f - a*b*e + b**2*d)/(3*b**3)
```

Mathematica [A] time = 0.19226, size = 128, normalized size = 0.94

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{9/2}}+\frac{x\left(-105a^3f+35a^2b\left(3e+fx^2\right)-7ab^2\left(15d+5ex^2+3fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2
 + 3*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105
*b^4) + (Sqrt[a]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.004, size = 182, normalized size = 1.3

$$\frac{fx^7}{7\,b}-\frac{x^5af}{5\,b^2}+\frac{x^5e}{5\,b}+\frac{x^3a^2f}{3\,b^3}-\frac{ax^3e}{3\,b^2}+\frac{x^3d}{3\,b}-\frac{a^3fx}{b^4}+\frac{a^2ex}{b^3}-\frac{adx}{b^2}+\frac{cx}{b}+\frac{a^4f}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{a^3e}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2d}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ac}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/7*f*x^7/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/3/b^3*x^3*a^2*f-1/3/b^2
*x^3*a*e+1/3/b*x^3*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+1/b*
c*x+a^4/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-a^3/b^3/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*e+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*d-a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231151, size = 1, normalized size = 0.01

$$\left[\frac{30\,b^3fx^7+42\left(b^3e-ab^2f\right)x^5+70\left(b^3d-ab^2e+a^2bf\right)x^3-105\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b
^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqr
t(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21
*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b^2*e + a^2*b*f)*x^3 - 105
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b)*arctan(x/sqrt(a/b)
) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4]
```

Sympy [A] time = 1.57485, size = 180, normalized size = 1.32

$$-\frac{\sqrt{-\frac{a}{b^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{fx^7}{7b}-\frac{x^5\left(af-be\right)}{5b^2}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*
sqrt(-a/b**9) + x)/2 + sqrt(-a/b**9)*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)*log(b**4*sqrt(-a/b**9) + x)/2 + f*x**7/(7*b) - x**5*(
a*f - b*e)/(5*b**2) + x**3*(a**2*f - a*b*e + b**2*d)/(3*b**3) - x
*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b**4
```

GIAC/XCAS [A] time = 0.219862, size = 205, normalized size = 1.51

$$-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{15\,b^6fx^7-21\,ab^5fx^5+21\,b^6x^5e+35\,b^6dx^3+35\,a^2b^4fx^3-35\,ab^5x^3e+105\,b^6cx-105\,ab^5dx-105\,a^3b^3fx+105\,a^2b^4xe}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^4) + 1/105*(15*b^6*f*x^7 - 21*a*b^5*f*x^5 + 21*b^6*x^5
*e + 35*b^6*d*x^3 + 35*a^2*b^4*f*x^3 - 35*a*b^5*x^3*e + 105*b^6*c
*x - 105*a*b^5*d*x - 105*a^3*b^3*f*x + 105*a^2*b^4*x*e)/b^7
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} \, dx$

Optimal. Leaf size=100

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*
x^5)/(5*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.139868, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*
x^5)/(5*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\left(a^2f-abe+b^2d\right)\int \frac{1}{b^3}\,dx+\frac{fx^5}{5b}-\frac{x^3\left(af-be\right)}{3b^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] (a**2*f - a*b*e + b**2*d)*Integral(b**(-3), x) + f*x**5/(5*b) - x
**3*(a*f - b*e)/(3*b**2) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c
)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.161167, size = 98, normalized size = 0.98

$$\frac{x\left(15a^2f-5ab\left(3e+fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^
4)))/(15*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.005, size = 135, normalized size = 1.4

$$\frac{fx^5}{5b} - \frac{ax^3f}{3b^2} + \frac{x^3e}{3b} + \frac{a^2fx}{b^3} - \frac{aex}{b^2} + \frac{dx}{b} - \frac{a^3f}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{a^2e}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*f*x^5/b-1/3/b^2*x^3*a*f+1/3/b*x^3*e+1/b^3*a^2*f*x-1/b^2*a*e*x
+1/b*d*x-1/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^3*f+1/b^2/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*e-1/b/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*a*d+c/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233446, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(-\frac{2\,abx - (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) - 2\left(3\,b^2fx^5 + 5\left(b^2e - abf\right)x^3 + 15\left(b^2d - abe + a^2f\right)x\right)\sqrt{-ab}}{30\sqrt{-ab}b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(-(2*a*b*x - (b
*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(3*b^2*f*x^5 + 5*(b^2*e -
a*b*f)*x^3 + 15*(b^2*d - a*b*e + a^2*f)*x)*sqrt(-a*b))/(sqrt(-a*b
)*b^3), 1/15*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(sqrt(
a*b)*x/a) + (3*b^2*f*x^5 + 5*(b^2*e - a*b*f)*x^3 + 15*(b^2*d - a*
b*e + a^2*f)*x)*sqrt(a*b))/(sqrt(a*b)*b^3)]
```

Sympy [A] time = 1.60759, size = 158, normalized size = 1.58

$$\frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(ab^3\sqrt{-\frac{1}{ab^7}} + x\right)}{2} + \frac{fx^5}{5b} - \frac{x^3\left(af - be\right)}{3b^2} + \frac{x\left(a^2f - abe + b^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*
b**3*sqrt(-1/(a*b**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*
b*e + a*b**2*d - b**3*c)*log(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*
x**5/(5*b) - x**3*(a*f - b*e)/(3*b**2) + x*(a**2*f - a*b*e + b**2
*d)/b**3
```

GIAC/XCAS [A] time = 0.217753, size = 143, normalized size = 1.43

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4fx^5 - 5\,ab^3fx^3 + 5\,b^4x^3e + 15\,b^4dx + 15\,a^2b^2fx - 15\,ab^3xe}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^3) + 1/15*(3*b^4*f*x^5 - 5*a*b^3*f*x^3 + 5*b^4*x^3*e + 15*b
^4*d*x + 15*a^2*b^2*f*x - 15*a*b^3*x*e)/b^5
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5
/2))
```

Rubi [A] time = 0.187443, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5
/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-(af-be)\int\frac{1}{b^2}\,dx+\frac{fx^3}{3b}-\frac{c}{ax}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -(a*f - b*e)*Integral(b**(-2), x) + f*x**3/(3*b) - c/(a*x) + (a**
3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/sqrt(a))/(a**(
3/2)*b**(5/2))
```

Mathematica [A] time = 0.129317, size = 83, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b
^(5/2))
```

Maple [A] time = 0.008, size = 114, normalized size = 1.4

$$\frac{fx^3}{3b} - \frac{afx}{b^2} + \frac{ex}{b} + \frac{a^2f}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{ae}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + d\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{bc}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/b-1/b^2*a*f*x+1/b*e*x+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a
*b)^(1/2))*f-a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e+1/(a*b)^(1
/2)*arctan(x*b/(a*b)^(1/2))*d-1/a*b/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*c-c/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23268, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3c - ab^2d + a^2be - a^3f\right)x\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right) - 2\left(abfx^4 - 3\,b^2c + 3\left(abe - a^2f\right)x^2\right)\sqrt{-ab}}{6\,\sqrt{-ab}ab^2x}, -\frac{3\left(b^3c - ab^2d + a^2be - a^3f\right)x\arctan\left(\frac{\sqrt{ab}x}{a}\right) - \left(abfx^4 - 3\,b^2c + 3\left(abe - a^2f\right)x^2\right)\sqrt{ab}}{3\,\sqrt{ab}ab^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*log((2*a*b*x + (b*
x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(a*b*f*x^4 - 3*b^2*c + 3*(a
*b*e - a^2*f)*x^2)*sqrt(-a*b))/(sqrt(-a*b)*a*b^2*x), -1/3*(3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x*arctan(sqrt(a*b)*x/a) - (a*b*f*
x^4 - 3*b^2*c + 3*(a*b*e - a^2*f)*x^2)*sqrt(a*b))/(sqrt(a*b)*a*b^
2*x)]
```

Sympy [A] time = 2.31059, size = 150, normalized size = 1.79

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{fx^3}{3b} - \frac{x\left(af - be\right)}{b^2} - \frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log
(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(a
**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5)) + x)/2 + f*x**3/(3*b) - x*(a*f - b*e)/b**2 - c/(a*x)
```

GIAC/XCAS [A] time = 0.217326, size = 116, normalized size = 1.38

$$-\frac{c}{ax}-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}+\frac{b^2fx^3-3\,abfx+3\,b^2xe}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*f*x^3 - 3*a*b*f*x + 3*b^2*x*e)/b
^3
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=82

$$\frac{bc-ad}{a^2x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.174724, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{a^2x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int f\,dx}{b}-\frac{c}{3ax^3}-\frac{ad-bc}{a^2x}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] Integral(f, x)/b - c/(3*a*x**3) - (a*d - b*c)/(a**2*x) - (a**3*f
- a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/sqrt(a))/(a**(5/2)
*b**(3/2))
```

Mathematica [A] time = 0.176651, size = 83, normalized size = 1.01

$$\frac{bc-ad}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/
2))
```

Maple [A] time = 0.01, size = 115, normalized size = 1.4

$$\frac{fx}{b}-\frac{c}{3\,ax^{3}}-\frac{d}{ax}+\frac{bc}{xa^{2}}-\frac{af}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+e\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^{2}c}{a^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] f*x/b-1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c-1/b*a/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*f+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e-b/a/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+b^2/a^2/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233999, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{3}\log\left(-\frac{2\,abx-\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)-2\left(3\,a^{2}fx^{4}-abc+3\left(b^{2}c-abd\right)x^{2}\right)\sqrt{-ab}}{6\sqrt{-ab}a^{2}bx^{3}},\ \frac{3\left(b^{3}c-ab^{2}d+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(-(2*a*b*x -
(b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(3*a^2*f*x^4 - a*b*c + 3
*(b^2*c - a*b*d)*x^2)*sqrt(-a*b))/(sqrt(-a*b)*a^2*b*x^3), 1/3*(3*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*arctan(sqrt(a*b)*x/a) + (
3*a^2*f*x^4 - a*b*c + 3*(b^2*c - a*b*d)*x^2)*sqrt(a*b))/(sqrt(a*b
)*a^2*b*x^3)]
```

Sympy [A] time = 4.4074, size = 151, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(-a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{2}+\frac{fx}{b}-\frac{ac+x^{2}\left(3ad-3bc\right)}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(
-a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*
f - a**2*b*e + a*b**2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3))
 + x)/2 + f*x/b - (a*c + x**2*(3*a*d - 3*b*c))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.21612, size = 109, normalized size = 1.33

$$\frac{fx}{b} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,bcx^2 - 3\,adx^2 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=104

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/
(a^3*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.208074, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/
(a^3*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(a^(7/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 41.9197, size = 90, normalized size = 0.87

$$-\frac{c}{5ax^5}-\frac{ad-bc}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{7}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -c/(5*a*x**5) - (a*d - b*c)/(3*a**2*x**3) - (a**2*e - a*b*d + b**
2*c)/(a**3*x) + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt
(b)*x/sqrt(a))/(a**(7/2)*sqrt(b))
```

Mathematica [A] time = 0.181875, size = 103, normalized size = 0.99

$$\frac{bc-ad}{3a^2x^3}+\frac{a^2(-e)+abd-b^2c}{a^3x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*
e)/(a^3*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.01, size = 142, normalized size = 1.4

$$-\frac{c}{5\,a x^{5}}-\frac{d}{3\,a x^{3}}+\frac{bc}{3\,x^{3}a^{2}}-\frac{e}{ax}+\frac{bd}{xa^{2}}-\frac{b^{2}c}{a^{3}x}+f\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{be}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^{2}d}{a^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^{3}c}{a^{3}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x)
```

```
[Out] -1/5*c/a/x^5-1/3/a/x^3*d+1/3/a^2/x^3*b*c-1/a/x*e+1/a^2/x*b*d-1/a^
3/x*b^2*c+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-1/a/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*b*e+1/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*b^2*d-1/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^3*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234085, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{5}\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(15\left(b^{2}c-abd+a^{2}e\right)x^{4}+3\,a^{2}c-5\left(abc-a^{2}d\right)x^{2}\right)\sqrt{-ab}}{30\sqrt{-ab}a^{3}x^{5}},\ -\frac{15\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{5}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(15\left(b^{2}c-abd+a^{2}e\right)x^{4}+3\,a^{2}c-5\left(abc-a^{2}d\right)x^{2}\right)\sqrt{ab}}{15\sqrt{ab}a^{3}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^6),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5*log((2*a*b*x +
 (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15*(b^2*c - a*b*d + a^
2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)*sqrt(-a*b))/(sqrt(-a*
b)*a^3*x^5), -1/15*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5*ar
ctan(sqrt(a*b)*x/a) + (15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c -
 5*(a*b*c - a^2*d)*x^2)*sqrt(a*b))/(sqrt(a*b)*a^3*x^5)]
```

Sympy [A] time = 10.5453, size = 167, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^{7}b}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(-a^{4}\sqrt{-\frac{1}{a^{7}b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^{7}b}}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(a^{4}\sqrt{-\frac{1}{a^{7}b}}+x\right)}{2}-\frac{3a^{2}c+x^{4}\left(15a^{2}e-15abd+15b^{2}c\right)+x^{2}\left(5a^{2}d-5abc\right)}{15a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a
**4*sqrt(-1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b
*e + a*b**2*d - b**3*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 - (3*a*
*2*c + x**4*(15*a**2*e - 15*a*b*d + 15*b**2*c) + x**2*(5*a**2*d -
 5*a*b*c))/(15*a**3*x**5)
```

GIAC/XCAS [A] time = 0.218994, size = 142, normalized size = 1.37

$$-\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,b^2cx^4 - 15\,abdx^4 + 15\,a^2x^4e - 5\,abcx^2 + 5\,a^2dx^2 + 3\,a^2c}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^3) - 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*x^4*e - 5*
a*b*c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^3*x^5)
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=137

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/
(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt
[b]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/a^(9/2)
```

Rubi [A] time = 0.276617, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/
(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt
[b]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/a^(9/2)
```

Rubi in Sympy [A] time = 51.9742, size = 124, normalized size = 0.91

$$-\frac{c}{7ax^7}-\frac{ad-bc}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}-\frac{a^3f-a^2be+ab^2d-b^3c}{a^4x}-\frac{\sqrt{b}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] -c/(7*a*x**7) - (a*d - b*c)/(5*a**2*x**5) - (a**2*e - a*b*d + b**
2*c)/(3*a**3*x**3) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**
4*x) - sqrt(b)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(
b)*x/sqrt(a))/a**(9/2)
```

Mathematica [A] time = 0.235325, size = 139, normalized size = 1.01

$$\frac{bc-ad}{5a^2x^5}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*
e)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (S
qrt[b]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(9/2)
```

Maple [A] time = 0.012, size = 190, normalized size = 1.4

$$-\frac{c}{7\,a x^{7}}-\frac{d}{5\,a x^{5}}+\frac{bc}{5\,x^{5}a^{2}}-\frac{e}{3\,a x^{3}}+\frac{bd}{3\,x^{3}a^{2}}-\frac{b^{2}c}{3\,a^{3}x^{3}}-\frac{f}{ax}+\frac{be}{xa^{2}}-\frac{b^{2}d}{a^{3}x}+\frac{b^{3}c}{a^{4}x}-\frac{bf}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^{2}e}{a^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^{3}d}{a^{3}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^{4}c}{a^{4}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x)
```

```
[Out] -1/7*c/a/x^7-1/5/a/x^5*d+1/5/a^2/x^5*b*c-1/3/a/x^3*e+1/3/a^2/x^3*
b*d-1/3/a^3/x^3*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3/x*b^2*d+1/a^4/x*b
^3*c-b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+b^2/a^2/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*e-b^3/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*d+b^4/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240428, size = 1, normalized size = 0.01

$$\left[-\frac{105\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{7}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-210\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{6}+70\left(ab^{2}c-a^{2}bd+a^{3}e\right.}{210\,a^{4}x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^8),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*l
og((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4
+ 30*a^3*c - 42*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(b/a)*arctan(b*x/(a*sqrt(
b/a))) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*
c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(
a^4*x^7)]
```

Sympy [A] time = 25.5437, size = 301, normalized size = 2.2

$$\frac{\sqrt{-\frac{b}{a^9}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c} + x\right)}{2} - \frac{\sqrt{-\frac{b}{a^9}}\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c} + x\right)}{2} - \frac{15a^3c + x^6\left(105a^3f - 105a^2be + 105ab^2d - 105b^3c\right) + x^4\left(35a^3e - 35a^2bd + 35ab^2c\right) + x^2\left(21a^3d - 21a^2bc\right)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*s
qrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f -
 a**2*b*e + a*b**2*d - b**3*c)*log(a**5*sqrt(-b/a**9)*(a**3*f - a
**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3*d -
 b**4*c) + x)/2 - (15*a**3*c + x**6*(105*a**3*f - 105*a**2*b*e +
105*a*b**2*d - 105*b**3*c) + x**4*(35*a**3*e - 35*a**2*b*d + 35*a
*b**2*c) + x**2*(21*a**3*d - 21*a**2*b*c))/(105*a**4*x**7)
```

GIAC/XCAS [A] time = 0.223187, size = 204, normalized size = 1.49

$$\frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,b^3cx^6 - 105\,ab^2dx^6 - 105\,a^3fx^6 + 105\,a^2bx^6e - 35\,ab^2cx^4 + 35\,a^2bdx^4 - 35\,a^3x^4e + 21\,a^2bcx^2 - 21\,a^3dx^2 - 15\,a^3c}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^8),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 - 105*a^3*f
*x^6 + 105*a^2*b*x^6*e - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3
*x^4*e + 21*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$

Optimal. Leaf size=175

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{11/2}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/
(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (
b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2
)
```

Rubi [A] time = 0.329932, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{11/2}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/
(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (
b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2
)
```

Rubi in Sympy [A] time = 60.8512, size = 156, normalized size = 0.89

$$-\frac{c}{9ax^9}-\frac{ad-bc}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{a^3f-a^2be+ab^2d-b^3c}{3a^4x^3}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x}+\frac{b^{\frac{3}{2}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -c/(9*a*x**9) - (a*d - b*c)/(7*a**2*x**7) - (a**2*e - a*b*d + b**
2*c)/(5*a**3*x**5) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a
**4*x**3) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*x) +
b**(3/2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/s
qrt(a))/a**(11/2)
```

Mathematica [A] time = 0.348364, size = 174, normalized size = 0.99

$$\frac{bc-ad}{7a^2x^7}+\frac{a^2(-e)+abd-b^2c}{5a^3x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{11/2}}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*
e)/(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3)
+ (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(
-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/a^(11/2)
```

Maple [A] time = 0.014, size = 238, normalized size = 1.4

$$-\frac{c}{9\,a x^9}-\frac{d}{7\,a x^7}+\frac{bc}{7\,a^2x^7}-\frac{e}{5\,a x^5}+\frac{bd}{5\,x^5a^2}-\frac{b^2c}{5\,a^3x^5}-\frac{f}{3\,a x^3}+\frac{be}{3\,x^3a^2}-\frac{b^2d}{3\,a^3x^3}+\frac{b^3c}{3\,a^4x^3}+\frac{bf}{xa^2}-\frac{b^2e}{a^3x}+\frac{b^3d}{a^4x}-\frac{b^4c}{a^5x}+\frac{b^2f}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^3e}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^4d}{a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^5c}{a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x)
```

```
[Out] -1/9*c/a/x^9-1/7/a/x^7*d+1/7/a^2/x^7*b*c-1/5/a/x^5*e+1/5/a^2/x^5*
b*d-1/5/a^3/x^5*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3/x^3*b^2
*d+1/3/a^4/x^3*b^3*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b^3/x*d-1/a^
5*b^4/x*c+b^2/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-b^3/a^3/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e+b^4/a^4/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))*d-b^5/a^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^10),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236087, size = 1, normalized size = 0.01

$$\left[-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-210\left(ab^3c-a^2b^2\right.}{630\,a^5x^9}\right.$$

$$-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-105\left(ab^3c-a^2b^2d+a^3be\right.}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^10),x, algorithm="fricas")
```

```
[Out] [-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/
a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c -
 a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d +
a^3*b*e - a^4*f)*x^6 + 70*a^4*c + 126*(a^2*b^2*c - a^3*b*d + a^4*
e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9), -1/315*(315*(b^4*c
- a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a)*arctan(b*x/(a*sqrt
(b/a))) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b
^2*c - a^3*b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
]
```

Sympy [A] time = 52.5361, size = 354, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{-35a^4c+x^8\left(315a^3bf-315a^2b^2e+315ab^3d-315b^4c\right)+x^6\left(-105a^4f+105a^3be-105a^2b^2d+105ab^3c\right)+x^4\left(-63a^4e+\right.}{315a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a
**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/
a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**6*sqrt(-b**
3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a
**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4*c + x**8*(315*
a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x**6*(-
105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**
4*(-63*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d
+ 45*a**3*b*c))/(315*a**5*x**9)
```

GIAC/XCAS [A] time = 0.220088, size = 271, normalized size = 1.55

$$-\frac{\left(b^5c-ab^4d-a^3b^2f+a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}-\frac{315\,b^4cx^8-315\,ab^3dx^8-315\,a^3bfx^8+315\,a^2b^2x^8e-105\,ab^3cx^6+105\,a^2b^2dx^6+105\,a^4fx^6-105\,a^3bx^6e+63\,a^2b^2cx^4\cdots}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^10),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 - 315*a^
3*b*f*x^8 + 315*a^2*b^2*x^8*e - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x
^6 + 105*a^4*f*x^6 - 105*a^3*b*x^6*e + 63*a^2*b^2*c*x^4 - 63*a^3*
b*d*x^4 + 63*a^4*x^4*e - 45*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c
)/(a^5*x^9)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$

Optimal. Leaf size=211

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{11ax^{11}}$$

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e
)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) -
 (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(13/2)
```

Rubi [A] time = 0.390927, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e
)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) -
 (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(13/2)
```

Rubi in Sympy [A] time = 71.6551, size = 192, normalized size = 0.91

$$-\frac{c}{11ax^{11}}-\frac{ad-bc}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{a^3f-a^2be+ab^2d-b^3c}{5a^4x^5}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^5x^3}-\frac{b^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^6x}-\frac{b^{\frac{5}{2}}\left(a^3f-a^2be+ab^2d-b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] -c/(11*a*x**11) - (a*d - b*c)/(9*a**2*x**9) - (a**2*e - a*b*d + b
**2*c)/(7*a**3*x**7) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(5
*a**4*x**5) + b*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**5*x
**3) - b**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**6*x) - b*
*(5/2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*atan(sqrt(b)*x/sqr
t(a))/a**(13/2)
```

Mathematica [A] time = 0.363152, size = 211, normalized size = 1.

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e
)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) +
 (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(13/2)
```

Maple [A] time = 0.013, size = 286, normalized size = 1.4

$$-\frac{c}{11\,ax^{11}}-\frac{d}{9\,ax^9}+\frac{bc}{9\,a^2x^9}-\frac{e}{7\,ax^7}+\frac{bd}{7\,a^2x^7}-\frac{b^2c}{7\,a^3x^7}-\frac{f}{5\,ax^5}+\frac{be}{5\,x^5a^2}-\frac{b^2d}{5\,a^3x^5}+\frac{b^3c}{5\,a^4x^5}-\frac{b^2f}{a^3x}+\frac{b^3e}{a^4x}-\frac{b^4d}{a^5x}+\frac{b^5c}{a^6x}+\frac{bf}{3\,x^3a^2}-\frac{b^2e}{3\,a^3x^3}+\frac{b^3d}{3\,a^4x^3}-\frac{b^4c}{3\,a^5x^3}-\frac{b^3f}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^4e}{a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^5d}{a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^6c}{a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x)
```

```
[Out] -1/11*c/a/x^11-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/7/a/x^7*e+1/7/a^2/x^
7*b*d-1/7/a^3/x^7*b^2*c-1/5/a/x^5*f+1/5/a^2/x^5*b*e-1/5/a^3/x^5*b
^2*d+1/5/a^4/x^5*b^3*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e-1/a^5*b^4/x*d+
1/a^6*b^5/x*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*b^3/x^3*d
-1/3/a^5*b^4/x^3*c-b^3/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+
b^4/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e-b^5/a^5/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*d+b^6/a^6/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^12),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238313, size = 1, normalized size = 0.

$$\left[-\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6930\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10}+2310\left(ab^4c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^12),x, algorithm="fricas")
```

```
[Out] [-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqr
t(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b
^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d
+ a^4*b*e - a^5*f)*x^6 + 630*a^5*c + 990*(a^3*b^2*c - a^4*b*d + a
^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11), 1/3465*(3465*(
b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a)*arctan(b*
x/(a*sqrt(b/a))) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)
*x^10 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 69
3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495
*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(
a^6*x^11)]
```

Sympy [A] time = 90.1979, size = 398, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{315a^5c+x^{10}\left(3465a^3b^2f-3465a^2b^3e+3465ab^4d-3465b^5c\right)+x^8\left(-1155a^4bf+1155a^3b^2e-1155a^2b^3d+1155ab^4c\right)+}{3465a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*
*7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**
3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a
**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**7*sqrt(-b**5
/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a*
*2*b**4*e + a*b**5*d - b**6*c) + x)/2 - (315*a**5*c + x**10*(3465
*a**3*b**2*f - 3465*a**2*b**3*e + 3465*a*b**4*d - 3465*b**5*c) +
x**8*(-1155*a**4*b*f + 1155*a**3*b**2*e - 1155*a**2*b**3*d + 1155
*a*b**4*c) + x**6*(693*a**5*f - 693*a**4*b*e + 693*a**3*b**2*d -
693*a**2*b**3*c) + x**4*(495*a**5*e - 495*a**4*b*d + 495*a**3*b**
2*c) + x**2*(385*a**5*d - 385*a**4*b*c))/(3465*a**6*x**11)
```

GIAC/XCAS [A] time = 0.217344, size = 336, normalized size = 1.59

$$\frac{\left(b^6c-ab^5d-a^3b^3f+a^2b^4e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6}+\frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}-3465\,a^3b^2fx^{10}+3465\,a^2b^3x^{10}e-1155\,ab^4cx^8+1155\,a^2b^3dx^8+1155\,a^4bfx^8-1155\,a^3b^2x^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)*x^12),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 - 34
65*a^3*b^2*f*x^10 + 3465*a^2*b^3*x^10*e - 1155*a*b^4*c*x^8 + 1155
*a^2*b^3*d*x^8 + 1155*a^4*b*f*x^8 - 1155*a^3*b^2*x^8*e + 693*a^2*
b^3*c*x^6 - 693*a^3*b^2*d*x^6 - 693*a^5*f*x^6 + 693*a^4*b*x^6*e -
 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*x^4*e + 385*a^4*b*
```

```
c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=240

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{ax\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^6}+\frac{x^3\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{6b^5}-\frac{x^5\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{10ab^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^{13/2}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

```
[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5
*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3
*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e -
2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) + ((c - (a*(b^2*d - a*b*e +
 a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (a^(3/2)*(5*b^3*c - 7*a*b^
2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13
/2))
```

Rubi [A] time = 0.648624, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{ax\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^6}+\frac{x^3\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{6b^5}-\frac{x^5\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{10ab^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^{13/2}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5
*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3
*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e -
2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) + ((c - (a*(b^2*d - a*b*e +
 a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (a^(3/2)*(5*b^3*c - 7*a*b^
2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13
/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.237628, size = 227, normalized size = 0.95

$$\frac{x^5\left(3a^2f-2abe+b^2d\right)}{5b^4}+\frac{ax\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{b^6}+\frac{x^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(11a^3f-9a^2be+7ab^2d-5b^3c\right)}{2b^{13/2}}-\frac{x\left(a^5(-f)+a^4be-a^3b^2d+a^2b^3c\right)}{2b^6\left(a+bx^2\right)}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c
- 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b
*e + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9
)/(9*b^2) - ((a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*
(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*b^2*d - 9*a^2*b*e + 11*a^
3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Maple [A] time = 0.017, size = 309, normalized size = 1.3

$$\frac{fx^9}{9\,b^2}-\frac{2\,x^7af}{7\,b^3}+\frac{x^7e}{7\,b^2}+\frac{3\,x^5a^2f}{5\,b^4}-\frac{2\,x^5ae}{5\,b^3}+\frac{x^5d}{5\,b^2}-\frac{4\,x^3a^3f}{3\,b^5}+\frac{x^3a^2e}{b^4}-\frac{2\,ax^3d}{3\,b^3}+\frac{x^3c}{3\,b^2}+5\,\frac{a^4fx}{b^6}-4\,\frac{a^3ex}{b^5}+3\,\frac{a^2dx}{b^4}-2\,\frac{acx}{b^3}+\frac{a^5xf}{2\,b^6\,(bx^2+a)}-\frac{a^4xe}{2\,b^5\,(bx^2+a)}+\frac{a^3xd}{2\,b^4\,(bx^2+a)}-\frac{xa^2c}{2\,b^3\,(bx^2+a)}-\frac{11\,a^5f}{2\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,a^4e}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,a^3d}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,a^2c}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-2/7/b^3*x^7*a*f+1/7/b^2*x^7*e+3/5/b^4*x^5*a^2*f-2/5
/b^3*x^5*a*e+1/5/b^2*x^5*d-4/3/b^5*x^3*a^3*f+1/b^4*x^3*a^2*e-2/3/
b^3*x^3*a*d+1/3/b^2*x^3*c+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d
*x-2/b^3*a*c*x+1/2*a^5/b^6*x/(b*x^2+a)*f-1/2*a^4/b^5*x/(b*x^2+a)*
e+1/2*a^3/b^4*x/(b*x^2+a)*d-1/2*a^2/b^3*x/(b*x^2+a)*c-11/2*a^5/b^
6/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+9/2*a^4/b^5/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*e-7/2*a^3/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*d+5/2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236943, size = 1, normalized size = 0.

$$\left[\frac{140\,b^5 f x^{11} + 20\left(9\,b^5 e - 11\,ab^4 f\right)x^9 + 36\left(7\,b^5 d - 9\,ab^4 e + 11\,a^2 b^3 f\right)x^7 + 84\left(5\,b^5 c - 7\,ab^4 d + 9\,a^2 b^3 e - 11\,a^3 b^2 f\right)x^5 - 4\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b
^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d +
9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d +
9*a^3*b^2*e - 11*a^4*b*f)*x^3 - 315*(5*a^2*b^3*c - 7*a^3*b^2*d +
9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 1
1*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*
x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f
)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11*a
*b^4*f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5
*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b
^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b
^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^2*b^
3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)
) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*x)/(b^
7*x^2 + a*b^6)]
```

Sympy [A] time = 4.61709, size = 430, normalized size = 1.79

$$\frac{x\left(a^5 f - a^4 be + a^3 b^2 d - a^2 b^3 c\right)}{2ab^6 + 2b^7 x^2} + \frac{\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3 f - 9a^2 be + 7ab^2 d - 5b^3 c\right)\log\left(-\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3 f - 9a^2 be + 7ab^2 d - 5b^3 c\right)}{11a^4 f - 9a^3 be + 7a^2 b^2 d - 5ab^3 c} + x\right)}{4} - \frac{\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3 f - 9a^2 be + 7ab^2 d - 5b^3 c\right)\log\left(\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\left(11a^3 f - 9a^2 be + 7ab^2 d - 5b^3 c\right)}{11a^4 f - 9a^3 be + 7a^2 b^2 d - 5ab^3 c} + x\right)}{4} + \frac{fx^9}{9b^2} - \frac{x^7\left(2af - be\right)}{7b^3} + \frac{x^5\left(3a^2 f - 2abe + b^2 d\right)}{5b^4} - \frac{x^3\left(4a^3 f - 3a^2 be + 2ab^2 d - b^3 c\right)}{3b^5} + \frac{x\left(5a^4 f - 4a^3 be + 3a^2 b^2 d - 2ab^3 c\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b
**7*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*
d - 5*b**3*c)*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e
 + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d
 - 5*a*b**3*c) + x)/4 - sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e
 + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a**3/b**13)*(11*a**3*f -
 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a**3*b*e + 7*
a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2) - x**7*(2*a*f
- b*e)/(7*b**3) + x**5*(3*a**2*f - 2*a*b*e + b**2*d)/(5*b**4) - x
**3*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(3*b**5) + x*(5
*a**4*f - 4*a**3*b*e + 3*a**2*b**2*d - 2*a*b**3*c)/b**6
```

GIAC/XCAS [A] time = 0.216489, size = 340, normalized size = 1.42

$$\frac{\left(5\,a^2b^3c - 7\,a^3b^2d - 11\,a^5f + 9\,a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} - \frac{a^2b^3cx - a^3b^2dx - a^5fx + a^4bxe}{2\,(bx^2 + a)b^6} + \frac{35\,b^{16}fx^9 - 90\,ab^{15}fx^7 + 45\,b^{16}x^7e + 63\,b^{16}dx^5 + 189\,a^2b^{14}fx^5 - 126\,ab^{15}x^5e + 105\,b^{16}cx^3 - 210\,ab^{15}dx^3 - 420\,a^3b^{13}fx^3}{315\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x - a^
5*f*x + a^4*b*x*e)/((b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 - 90*
a*b^15*f*x^7 + 45*b^16*x^7*e + 63*b^16*d*x^5 + 189*a^2*b^14*f*x^5
 - 126*a*b^15*x^5*e + 105*b^16*c*x^3 - 210*a*b^15*d*x^3 - 420*a^3
*b^13*f*x^3 + 315*a^2*b^14*x^3*e - 630*a*b^15*c*x + 945*a^2*b^14*
d*x + 1575*a^4*b^12*f*x - 1260*a^3*b^13*x*e)/b^18
```

3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2} dx$

Optimal. Leaf size=202

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^{11/2}}+\frac{x\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^5}-\frac{x^3\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3
*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*
a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a
^2*f))/b^3)*x^5)/(2*a*(a + b*x^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*
d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2)
)
```

Rubi [A] time = 0.544807, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^{11/2}}+\frac{x\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^5}-\frac{x^3\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3
*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*
a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a
^2*f))/b^3)*x^5)/(2*a*(a + b*x^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*
d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2)
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.187002, size = 187, normalized size = 0.93

$$\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)}{2b^{11/2}}+\frac{x\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{b^5}+\frac{x\left(a^4(-f)+a^3be-a^2b^2d+ab^3c\right)}{2b^5(a+bx^2)}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a
*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x
^7)/(7*b^2) + ((a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*
(a + b*x^2)) + (Sqrt[a]*(-3*b^3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3
*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A] time = 0.017, size = 258, normalized size = 1.3

$$\frac{fx^7}{7b^2} - \frac{2x^5af}{5b^3} + \frac{x^5e}{5b^2} + \frac{x^3a^2f}{b^4} - \frac{2ax^3e}{3b^3} + \frac{dx^3}{3b^2} - 4\frac{a^3fx}{b^5} + 3\frac{a^2ex}{b^4} - 2\frac{adx}{b^3} + \frac{cx}{b^2} - \frac{a^4xf}{2b^5(bx^2+a)} + \frac{a^3xe}{2b^4(bx^2+a)} - \frac{xa^2d}{2b^3(bx^2+a)} + \frac{axc}{2b^2(bx^2+a)} + \frac{9a^4f}{2b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{7a^3e}{2b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{5a^2d}{2b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3ac}{2b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+1/b^4*x^3*a^2*f-2/3/b
^3*x^3*a*e+1/3*d*x^3/b^2-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x+
1/b^2*x*c-1/2*a^4/b^5*x/(b*x^2+a)*f+1/2*a^3/b^4*x/(b*x^2+a)*e-1/2
*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b^2*x/(b*x^2+a)*c+9/2*a^4/b^5/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*f-7/2*a^3/b^4/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*e+5/2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*
d-3/2*a/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234452, size = 1, normalized size = 0.

$$\left[\frac{60\,b^4fx^9 + 12\left(7\,b^4e - 9\,ab^3f\right)x^7 + 28\left(5\,b^4d - 7\,ab^3e + 9\,a^2b^2f\right)x^5 + 140\left(3\,b^4c - 5\,ab^3d + 7\,a^2b^2e - 9\,a^3bf\right)x^3 - 105\left(3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d
 - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^
2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b
*e - 9*a^4*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^
2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2
10*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*x)/(b^6*x^2 +
a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*x^7 + 14*(5
*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d +
 7*a^2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*
a^3*b*e - 9*a^4*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*
```

```
f)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 105*(3*a*b^3*c - 5*a^2*b^
2*d + 7*a^3*b*e - 9*a^4*f)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 4.44068, size = 250, normalized size = 1.24

$$-\frac{x\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{2ab^5+2b^6x^2}-\frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)\log\left(-b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)\log\left(b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{fx^7}{7b^2}$$
$$-\frac{x^5\left(2af-be\right)}{5b^3}+\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{x\left(4a^3f-3a^2be+2ab^2d-b^3c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(2*a*b**5 + 2*b**
6*x**2) - sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*
b**3*c)*log(-b**5*sqrt(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*
f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(b**5*sqrt(-a/b**11) +
 x)/4 + f*x**7/(7*b**2) - x**5*(2*a*f - b*e)/(5*b**3) + x**3*(3*a
**2*f - 2*a*b*e + b**2*d)/(3*b**4) - x*(4*a**3*f - 3*a**2*b*e + 2
*a*b**2*d - b**3*c)/b**5
```

GIAC/XCAS [A] time = 0.216364, size = 271, normalized size = 1.34

$$-\frac{\left(3\,ab^3c-5\,a^2b^2d-9\,a^4f+7\,a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}+\frac{ab^3cx-a^2b^2dx-a^4fx+a^3bxe}{2\,(bx^2+a)b^5}$$
$$+\frac{15\,b^{12}fx^7-42\,ab^{11}fx^5+21\,b^{12}x^5e+35\,b^{12}dx^3+105\,a^2b^{10}fx^3-70\,ab^{11}x^3e+105\,b^{12}cx-210\,ab^{11}dx-420\,a^3b^9fx+31}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*
x + a^3*b*x*e)/((b*x^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 - 42*a*b^
11*f*x^5 + 21*b^12*x^5*e + 35*b^12*d*x^3 + 105*a^2*b^10*f*x^3 - 7
0*a*b^11*x^3*e + 105*b^12*c*x - 210*a*b^11*d*x - 420*a^3*b^9*f*x
+ 315*a^2*b^10*x*e)/b^14
```

3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=163

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-7a^3f+5a^2be-3ab^2d+b^3c)}{2\sqrt{a}b^{9/2}}-\frac{x(-7a^3f+5a^2be-3ab^2d+b^3c)}{2ab^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^5}{5b^2}$$

```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e
- 2*a*f)*x^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e
 + a^2*f))/b^3)*x^3)/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*
a^2*b*e - 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2
))
```

Rubi [A] time = 0.509814, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-7a^3f+5a^2be-3ab^2d+b^3c)}{2\sqrt{a}b^{9/2}}-\frac{x(-7a^3f+5a^2be-3ab^2d+b^3c)}{2ab^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```


```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e
- 2*a*f)*x^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e
 + a^2*f))/b^3)*x^3)/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*
a^2*b*e - 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2
))
```

Rubi in Sympy [A] time = 138.425, size = 141, normalized size = 0.87

$$\frac{fx^5}{5b^2}-\frac{x^3(2af-be)}{3b^3}+\frac{x(3a^2f-2abe+b^2d)}{b^4}+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{2b^4(a+bx^2)}-\frac{(7a^3f-5a^2be+3ab^2d-b^3c)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] f*x**5/(5*b**2) - x**3*(2*a*f - b*e)/(3*b**3) + x*(3*a**2*f - 2*a
*b*e + b**2*d)/b**4 + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(
2*b**4*(a + b*x**2)) - (7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3
*c)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(9/2))
```

Mathematica [A] time = 0.150438, size = 148, normalized size = 0.91

$$\frac{x(3a^2f-2abe+b^2d)}{b^4}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(7a^3f-5a^2be+3ab^2d-b^3c)}{2\sqrt{a}b^{9/2}}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^4(a+bx^2)}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3)
 + (f*x^5)/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b
^4*(a + b*x^2)) - ((-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Maple [A] time = 0.018, size = 212, normalized size = 1.3

$$\frac{fx^5}{5\,b^2}-\frac{2\,ax^3f}{3\,b^3}+\frac{x^3e}{3\,b^2}+3\,\frac{a^2fx}{b^4}-2\,\frac{aex}{b^3}+\frac{dx}{b^2}+\frac{xa^3f}{2\,b^4\,(bx^2+a)}-\frac{xa^2e}{2\,b^3\,(bx^2+a)}+\frac{axd}{2\,b^2\,(bx^2+a)}-\frac{cx}{2\,b\,(bx^2+a)}-\frac{7\,a^3f}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,a^2e}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,ad}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+3/b^4*a^2*f*x-2/b^3*a
*e*x+x*d/b^2+1/2/b^4*x/(b*x^2+a)*a^3*f-1/2/b^3*x/(b*x^2+a)*a^2*e+
1/2/b^2*x/(b*x^2+a)*a*d-1/2/b*x/(b*x^2+a)*c-7/2/b^4/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*a^3*f+5/2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*a^2*e-3/2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/2
/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237878, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+\left(b^4c-3\,ab^3d+5\,a^2b^2e-7\,a^3bf\right)x^2\right)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(6\,b^3fx^7+2\left(\right.\right.}{60\left(b^5x^2+ab^4\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c
- 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*log(-(2*a*b*x - (b*x^
2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(6*b^3*f*x^7 + 2*(5*b^3*e - 7
*a*b^2*f)*x^5 + 10*(3*b^3*d - 5*a*b^2*e + 7*a^2*b*f)*x^3 - 15*(b^
3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)*sqrt(-a*b))/((b^5*x^2 +
 a*b^4)*sqrt(-a*b)), 1/30*(15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e
- 7*a^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*ar
```

```
ctan(sqrt(a*b)*x/a) + (6*b^3*f*x^7 + 2*(5*b^3*e - 7*a*b^2*f)*x^5
+ 10*(3*b^3*d - 5*a*b^2*e + 7*a^2*b*f)*x^3 - 15*(b^3*c - 3*a*b^2*
d + 5*a^2*b*e - 7*a^3*f)*x)*sqrt(a*b))/((b^5*x^2 + a*b^4)*sqrt(a*
b))]
```

Sympy [A] time = 4.23291, size = 216, normalized size = 1.33

$$\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\left(7a^3f-5a^2be+3ab^2d-b^3c\right)\log\left(-ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^9}}\left(7a^3f-5a^2be+3ab^2d-b^3c\right)\log\left(ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4}+\frac{fx^5}{5b^2}-\frac{x^3\left(2af-be\right)}{3b^3}+\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2
) + sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*
c)*log(-a*b**4*sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a*
*3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b*
*9)) + x)/4 + f*x**5/(5*b**2) - x**3*(2*a*f - b*e)/(3*b**3) + x*(
3*a**2*f - 2*a*b*e + b**2*d)/b**4
```

GIAC/XCAS [A] time = 0.215669, size = 205, normalized size = 1.26

$$\frac{\left(b^3c-3\,ab^2d-7\,a^3f+5\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\,(bx^2+a)b^4}+\frac{3\,b^8fx^5-10\,ab^7fx^3+5\,b^8x^3e+15\,b^8dx+45\,a^2b^6fx-30\,ab^7xe}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x
*e)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*f*x^5 - 10*a*b^7*f*x^3 + 5*b^
8*x^3*e + 15*b^8*d*x + 45*a^2*b^6*f*x - 30*a*b^7*x*e)/b^10
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=118

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e
 + a^2*f))/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*
b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.276915, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e
 + a^2*f))/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*
b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi in Sympy [A] time = 81.0225, size = 114, normalized size = 0.97

$$\frac{fx^3}{3b^2}-\frac{x\left(2af-be\right)}{b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^3\left(a+bx^2\right)}+\frac{\left(5a^3f-3a^2be+ab^2d+b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] f*x**3/(3*b**2) - x*(2*a*f - b*e)/b**3 - x*(a**3*f - a**2*b*e + a
*b**2*d - b**3*c)/(2*a*b**3*(a + b*x**2)) + (5*a**3*f - 3*a**2*b*
e + a*b**2*d + b**3*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*b**(7/
2))
```

Mathematica [A] time = 0.165728, size = 122, normalized size = 1.03

$$-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3
*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/
2))
```

Maple [A] time = 0.014, size = 177, normalized size = 1.5

$$\frac{fx^3}{3\,b^2}-2\,\frac{afx}{b^3}+\frac{ex}{b^2}-\frac{xa^2f}{2\,b^3\,(bx^2+a)}+\frac{axe}{2\,b^2\,(bx^2+a)}-\frac{dx}{2\,b\,(bx^2+a)}+\frac{cx}{2\,a\,(bx^2+a)}+\frac{5\,a^2f}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,ae}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2-2/b^3*a*f*x+1/b^2*e*x-1/2/b^3*x*a^2/(b*x^2+a)*f+1/2
/b^2*x*a/(b*x^2+a)*e-1/2/b*x/(b*x^2+a)*d+1/2*x/a/(b*x^2+a)*c+5/2/
b^3*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-3/2/b^2*a/(a*b)^(1/
2)*arctan(x*b/(a*b)^(1/2))*e+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*d+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239996, size = 1, normalized size = 0.01

$$\left[\frac{3\left(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+\left(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,ab^2fx^5+2\left(3\,ab^2e\right.\right.}{12\left(ab^4x^2+a^2b^3\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^3*c + a^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*
b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*x^2)*log((2*a*b*x + (b*x^2 - a)*
sqrt(-a*b))/(b*x^2 + a)) + 2*(2*a*b^2*f*x^5 + 2*(3*a*b^2*e - 5*a^
2*b*f)*x^3 + 3*(b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f)*x)*sqrt(-a
*b))/((a*b^4*x^2 + a^2*b^3)*sqrt(-a*b)), 1/6*(3*(a*b^3*c + a^2*b^
2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^
3*b*f)*x^2)*arctan(sqrt(a*b)*x/a) + (2*a*b^2*f*x^5 + 2*(3*a*b^2*e
 - 5*a^2*b*f)*x^3 + 3*(b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f)*x)*
sqrt(a*b))/((a*b^4*x^2 + a^2*b^3)*sqrt(a*b))]
```

Sympy [A] time = 3.68975, size = 199, normalized size = 1.69

$$-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f-3a^2be+ab^2d+b^3c\right)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f-3a^2be+ab^2d+b^3c\right)\log\left(a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}+\frac{fx^3}{3b^2}-\frac{x\left(2af-be\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a**2*b**3 + 2*a*b**
4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d
+ b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a
**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b
**3*sqrt(-1/(a**3*b**7)) + x)/4 + f*x**3/(3*b**2) - x*(2*a*f - b*
e)/b**3
```

GIAC/XCAS [A] time = 0.215635, size = 170, normalized size = 1.44

$$\frac{\left(b^3c + ab^2d + 5\,a^3f - 3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3} + \frac{b^3cx - ab^2dx - a^3fx + a^2bxe}{2\,(bx^2 + a)ab^3} + \frac{b^4fx^3 - 6\,ab^3fx + 3\,b^4xe}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x
*e)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*f*x^3 - 6*a*b^3*f*x + 3*b^4*x*
e)/b^6
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=112

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\,(a+bx^2)}-\frac{c}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)
*x)/(2*a*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rubi [A] time = 0.322123, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\,(a+bx^2)}-\frac{c}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)
*x)/(2*a*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rubi in Sympy [A] time = 140.885, size = 122, normalized size = 1.09

$$\frac{fx}{b^2}-\frac{x\left(\frac{a^3f}{x^2}-\frac{a^2be}{x^2}+\frac{ab^2d}{x^2}-\frac{b^3c}{x^2}\right)}{2ab^3\,(a+bx^2)}-\frac{a^2f-abe+b^2d}{ab^3x}-\frac{\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] f*x/b**2 - x*(a**3*f/x**2 - a**2*b*e/x**2 + a*b**2*d/x**2 - b**3*
c/x**2)/(2*a*b**3*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(a*b*
*3*x) - (3*a**2*f - 2*a*b*e + b**2*d)*atan(sqrt(b)*x/sqrt(a))/(a*
*(3/2)*b**(5/2))
```

Mathematica [A] time = 0.114736, size = 115, normalized size = 1.03

$$-\frac{c}{a^2x}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^2b^2\,(a+bx^2)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x)/(2*a^2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Maple [A] time = 0.017, size = 165, normalized size = 1.5

$$\frac{fx}{b^2}-\frac{c}{xa^2}+\frac{axf}{2\,b^2\,(bx^2+a)}-\frac{ex}{2\,b\,(bx^2+a)}+\frac{dx}{2\,a\,(bx^2+a)}-\frac{bxc}{2\,a^2\,(bx^2+a)}-\frac{3\,af}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{e}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{d}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,bc}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] f*x/b^2-c/x/a^2+1/2*a/b^2*x/(b*x^2+a)*f-1/2/b*x/(b*x^2+a)*e+1/2/a
*x/(b*x^2+a)*d-1/2/a^2*b*x/(b*x^2+a)*c-3/2*a/b^2/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*f+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e
+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-3/2/a^2*b/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234765, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b^2d-a^3be+3\,a^4f\right)x\right)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,a^2bfx^4-2\,ab^2c \ldots\right.}{4\,(a^2b^3x^3+a^3b^2x)\sqrt{-ab}},\right.$$
$$-\frac{\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b^2d-a^3be+3\,a^4f\right)x\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(2\,a^2bfx^4-2\,ab^2c-\left(3\,b^3c-\ldots\right.\right.}{2\,(a^2b^3x^3+a^3b^2x)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3
*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*log(-(2*a*b*x - (b*x^2 - a
)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*a^2*b*f*x^4 - 2*a*b^2*c - (3*b^
3*c - a*b^2*d + a^2*b*e - 3*a^3*f)*x^2)*sqrt(-a*b))/((a^2*b^3*x^3
 + a^3*b^2*x)*sqrt(-a*b)), -1/2*(((3*b^4*c - a*b^3*d - a^2*b^2*e
+ 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)
*arctan(sqrt(a*b)*x/a) - (2*a^2*b*f*x^4 - 2*a*b^2*c - (3*b^3*c -
a*b^2*d + a^2*b*e - 3*a^3*f)*x^2)*sqrt(a*b))/((a^2*b^3*x^3 + a^3*
b^2*x)*sqrt(a*b))]
```

Sympy [A] time = 9.2556, size = 197, normalized size = 1.76

$$\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4} + \frac{-2ab^2c+x^2\left(a^3f-a^2be+ab^2d-3b^3c\right)}{2a^3b^2x+2a^2b^3x^3}+\frac{fx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*
log(-a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))
*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-
1/(a**5*b**5)) + x)/4 + (-2*a*b**2*c + x**2*(a**3*f - a**2*b*e +
a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + f*x/b*
*2
```

GIAC/XCAS [A] time = 0.216176, size = 165, normalized size = 1.47

$$\frac{fx}{b^2}-\frac{\left(3\,b^3c-ab^2d+3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2}-\frac{3\,b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e+2\,ab^2c}{2\,(bx^3+ax)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 -
 a^3*f*x^2 + a^2*b*x^2*e + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=121

$$\frac{2bc-ad}{a^3x}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}-\frac{c}{3a^2x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a
+ e - (a*f)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.364802, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2bc-ad}{a^3x}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}-\frac{c}{3a^2x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a
+ e - (a*f)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi in Sympy [A] time = 143.952, size = 146, normalized size = 1.21

$$-\frac{x\left(\frac{a^3f}{x^4}-\frac{a^2be}{x^4}+\frac{ab^2d}{x^4}-\frac{b^3c}{x^4}\right)}{2ab^3\left(a+bx^2\right)}-\frac{a^2f-abe+b^2d}{3ab^3x^3}+\frac{2a^2f-2abe+b^2d}{a^2b^2x}+\frac{\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f/x**4 - a**2*b*e/x**4 + a*b**2*d/x**4 - b**3*c/x**4)/(2
*a*b**3*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(3*a*b**3*x**3)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(a**2*b**2*x) + (3*a**2*f - 2*a*
b*e + b**2*d)*atan(sqrt(b)*x/sqrt(a))/(a**(5/2)*b**(3/2))
```

Mathematica [A] time = 0.137094, size = 125, normalized size = 1.03

$$\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^3b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d
+ a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2
))
```

Maple [A] time = 0.019, size = 182, normalized size = 1.5

$$-\frac{c}{3x^3a^2}-\frac{d}{xa^2}+2\frac{bc}{a^3x}-\frac{fx}{2b(bx^2+a)}+\frac{ex}{2a(bx^2+a)}-\frac{bxd}{2a^2(bx^2+a)}+\frac{b^2xc}{2a^3(bx^2+a)}+\frac{f}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{e}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3bd}{2a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5b^2c}{2a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c/a^2/x^3-1/a^2/x*d+2/a^3/x*b*c-1/2/b*x/(b*x^2+a)*f+1/2/a*x/
(b*x^2+a)*e-1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^2*x/(b*x^2+a)*c+1/2
/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+1/2/a/(a*b)^(1/2)*arctan
(x*b/(a*b)^(1/2))*e-3/2/a^2*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241104, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(5b^4c-3ab^3d+a^2b^2e+a^3bf\right)x^5+\left(5ab^3c-3a^2b^2d+a^3be+a^4f\right)x^3\right)\log\left(\frac{2abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\left(5b^3c-3ab^2d\right.\right.}{12(a^3b^2x^5+a^4bx^3)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e + a^3*b*f)*x^5 + (5*a*
b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*log((2*a*b*x + (b*x^2
 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(3*(5*b^3*c - 3*a*b^2*d + a^2*
b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^2*c - 3*a^2*b*d)*x^2)*sqr
t(-a*b))/((a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a*b)), 1/6*(3*((5*b^4*c
 - 3*a*b^3*d + a^2*b^2*e + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*
d + a^3*b*e + a^4*f)*x^3)*arctan(sqrt(a*b)*x/a) + (3*(5*b^3*c - 3
*a*b^2*d + a^2*b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^2*c - 3*a^
2*b*d)*x^2)*sqrt(a*b))/((a^3*b^2*x^5 + a^4*b*x^3)*sqrt(a*b))]
```

Sympy [A] time = 23.8513, size = 212, normalized size = 1.75

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}-\frac{2a^2bc+x^4\left(3a^3f-3a^2be+9ab^2d-15b^3c\right)+x^2\left(6a^2bd-10ab^2c\right)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)
*log(-a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(
a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a*
*7*b**3)) + x)/4 - (2*a**2*b*c + x**4*(3*a**3*f - 3*a**2*b*e + 9*
a*b**2*d - 15*b**3*c) + x**2*(6*a**2*b*d - 10*a*b**2*c))/(6*a**4*
b*x**3 + 6*a**3*b**2*x**5)
```

GIAC/XCAS [A] time = 0.216214, size = 166, normalized size = 1.37

$$\frac{\left(5\,b^{3}c - 3\,ab^{2}d + a^{3}f + a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^{3}b} + \frac{b^{3}cx - ab^{2}dx - a^{3}fx + a^{2}bxe}{2\,(bx^{2} + a)a^{3}b} + \frac{6\,bcx^{2} - 3\,adx^{2} - ac}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x
*e)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*
x^3)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^2}\,dx$

Optimal. Leaf size=152

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*
(a + b*x^2)) - ((7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*Sqrt[b])
```

Rubi [A] time = 0.417367, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*
(a + b*x^2)) - ((7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 149.568, size = 175, normalized size = 1.15

$$-\frac{x\left(\frac{a^3f}{x^6}-\frac{a^2be}{x^6}+\frac{ab^2d}{x^6}-\frac{b^3c}{x^6}\right)}{2ab^3\left(a+bx^2\right)}-\frac{a^2f-abe+b^2d}{5ab^3x^5}+\frac{2a^2f-2abe+b^2d}{3a^2b^2x^3}-\frac{3a^2f-2abe+b^2d}{a^3bx}-\frac{\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{7}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f/x**6 - a**2*b*e/x**6 + a*b**2*d/x**6 - b**3*c/x**6)/(2
*a*b**3*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(5*a*b**3*x**5)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(3*a**2*b**2*x**3) - (3*a**2*f -
 2*a*b*e + b**2*d)/(a**3*b*x) - (3*a**2*f - 2*a*b*e + b**2*d)*ata
n(sqrt(b)*x/sqrt(a))/(a**(7/2)*sqrt(b))
```

Mathematica [A] time = 0.168984, size = 151, normalized size = 0.99

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}+\frac{a^2(-e)+2abd-3b^2c}{a^4x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-3a^2be+5ab^2d-7b^3c\right)}{2a^{9/2}\sqrt{b}}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d
- a^2*e)/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*
a^4*(a + b*x^2)) + ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*Sqrt[b])
```

Maple [A] time = 0.021, size = 219, normalized size = 1.4

$$-\frac{c}{5x^5a^2}-\frac{d}{3x^3a^2}+\frac{2bc}{3a^3x^3}-\frac{e}{xa^2}+2\frac{bd}{a^3x}-3\frac{b^2c}{a^4x}+\frac{fx}{2a(bx^2+a)}-\frac{bxe}{2a^2(bx^2+a)}+\frac{xb^2d}{2a^3(bx^2+a)}-\frac{xb^3c}{2a^4(bx^2+a)}+\frac{f}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3be}{2a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5db^2}{2a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7b^3c}{2a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5*c/x^5/a^2-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c-1/a^2/x*e+2/a^3/x*b*
d-3/a^4/x*b^2*c+1/2/a*x/(b*x^2+a)*f-1/2/a^2*x/(b*x^2+a)*b*e+1/2/a
^3*x/(b*x^2+a)*b^2*d-1/2/a^4*x/(b*x^2+a)*b^3*c+1/2/a/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*f-3/2/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*b*e+5/2/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^2*d-7/2/a^
4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^3*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247554, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(\left(7b^4c-5ab^3d+3a^2b^2e-a^3bf\right)x^7+\left(7ab^3c-5a^2b^2d+3a^3be-a^4f\right)x^5\right)\log\left(\frac{2abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(15\left(7b^3c-\right.\right.}{60\left(a^4bx^7+a^5x^5\right)\sqrt{-ab}}\right.$$

$$-\frac{15\left(\left(7b^4c-5ab^3d+3a^2b^2e-a^3bf\right)x^7+\left(7ab^3c-5a^2b^2d+3a^3be-a^4f\right)x^5\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(15\left(7b^3c-5ab^2d+3\right.\right.}{30\left(a^4bx^7+a^5x^5\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^6),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (
7*a*b^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*log((2*a*b*x +
(b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15*(7*b^3*c - 5*a*b^2*d
```

```
 + 3*a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c - 5*a^2*b*d + 3*a^3*e)*
x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)*sqrt(-a*b))/((a^4*b*
x^7 + a^5*x^5)*sqrt(-a*b)), -1/30*(15*((7*b^4*c - 5*a*b^3*d + 3*a
^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d + 3*a^3*b*e -
a^4*f)*x^5)*arctan(sqrt(a*b)*x/a) + (15*(7*b^3*c - 5*a*b^2*d + 3*
a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c - 5*a^2*b*d + 3*a^3*e)*x^4 +
 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)*sqrt(a*b))/((a^4*b*x^7 +
a^5*x^5)*sqrt(a*b))]
```

Sympy [A] time = 56.1094, size = 226, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^9b}}\left(a^3f-3a^2be+5ab^2d-7b^3c\right)\log\left(-a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^9b}}\left(a^3f-3a^2be+5ab^2d-7b^3c\right)\log\left(a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+\frac{-6a^3c+x^6\left(15a^3f-45a^2be+75ab^2d-105b^3c\right)+x^4\left(-30a^3e+50a^2bd-70ab^2c\right)+x^2\left(-10a^3d+14a^2bc\right)}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*
log(-a**5*sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f -
3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) +
x)/4 + (-6*a**3*c + x**6*(15*a**3*f - 45*a**2*b*e + 75*a*b**2*d -
 105*b**3*c) + x**4*(-30*a**3*e + 50*a**2*b*d - 70*a*b**2*c) + x*
*2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*a**4*b*x**7)
```

GIAC/XCAS [A] time = 0.217092, size = 204, normalized size = 1.34

$$-\frac{\left(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\,(bx^2+a)a^4}-\frac{45\,b^2cx^4-30\,abdx^4+15\,a^2x^4e-10\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^6),x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*
x*e)/((b*x^2 + a)*a^4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a
^2*x^4*e - 10*a*b*c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx$

Optimal. Leaf size=189

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+5a^2be-7ab^2d+9b^3c\right)}{2a^{11/2}}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(
a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*
x^2)) + (Sqrt[b]*(9*b^3*c - 7*a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rubi [A] time = 0.577026, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+5a^2be-7ab^2d+9b^3c\right)}{2a^{11/2}}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(
a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*
x^2)) + (Sqrt[b]*(9*b^3*c - 7*a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rubi in Sympy [A] time = 160.894, size = 202, normalized size = 1.07

$$-\frac{x\left(\frac{a^3f}{x^8}-\frac{a^2be}{x^8}+\frac{ab^2d}{x^8}-\frac{b^3c}{x^8}\right)}{2ab^3\left(a+bx^2\right)}-\frac{a^2f-abe+b^2d}{7ab^3x^7}+\frac{2a^2f-2abe+b^2d}{5a^2b^2x^5}-\frac{3a^2f-2abe+b^2d}{3a^3bx^3}+\frac{3a^2f-2abe+b^2d}{a^4x}+\frac{\sqrt{b}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f/x**8 - a**2*b*e/x**8 + a*b**2*d/x**8 - b**3*c/x**8)/(2
*a*b**3*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(7*a*b**3*x**7)
 + (2*a**2*f - 2*a*b*e + b**2*d)/(5*a**2*b**2*x**5) - (3*a**2*f -
 2*a*b*e + b**2*d)/(3*a**3*b*x**3) + (3*a**2*f - 2*a*b*e + b**2*d
)/(a**4*x) + sqrt(b)*(3*a**2*f - 2*a*b*e + b**2*d)*atan(sqrt(b)*x
/sqrt(a))/a**(9/2)
```

Mathematica [A] time = 0.194289, size = 190, normalized size = 1.01

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}+\frac{a^2(-e)+2abd-3b^2c}{3a^4x^3}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{2a^{11/2}}-\frac{bx\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d
- a^2*e)/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/
(a^5*x) - (b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a
+ b*x^2)) - (Sqrt[b]*(-9*b^3*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Maple [A] time = 0.023, size = 268, normalized size = 1.4

$$-\frac{c}{7\,a^2x^7}-\frac{d}{5\,x^5a^2}+\frac{2\,bc}{5\,a^3x^5}-\frac{e}{3\,x^3a^2}+\frac{2\,bd}{3\,a^3x^3}-\frac{b^2c}{a^4x^3}-\frac{f}{xa^2}+2\,\frac{be}{a^3x}-3\,\frac{db^2}{a^4x}+4\,\frac{b^3c}{a^5x}-\frac{bxf}{2\,a^2\left(bx^2+a\right)}+\frac{b^2xe}{2\,a^3\left(bx^2+a\right)}-\frac{b^3xd}{2\,a^4\left(bx^2+a\right)}+\frac{b^4xc}{2\,a^5\left(bx^2+a\right)}-\frac{3\,fb}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2e}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,db^3}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,b^4c}{2\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/7*c/a^2/x^7-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/3/a^2/x^3*e+2/3/a^
3/x^3*b*d-1/a^4/x^3*b^2*c-1/a^2/x*f+2/a^3/x*b*e-3/a^4/x*b^2*d+4/a
^5/x*b^3*c-1/2*b/a^2*x/(b*x^2+a)*f+1/2*b^2/a^3*x/(b*x^2+a)*e-1/2*
b^3/a^4*x/(b*x^2+a)*d+1/2*b^4/a^5*x/(b*x^2+a)*c-3/2*b/a^2/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*f+5/2*b^2/a^3/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))*e-7/2*b^3/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d
+9/2*b^4/a^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239315, size = 1, normalized size = 0.01

$$\left[\frac{210\left(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf\right)x^8+140\left(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f\right)x^6-60\,a^4c-28\left(9\,a^2b^2c-7\,a^3bd+5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^8),x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 +
 140*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4
*c - 28*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c -
 7*a^4*d)*x^2 - 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b
*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sq
rt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b*
x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3
*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f
)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6
*(9*a^3*b*c - 7*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^
2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a
^4*f)*x^7)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^5*b*x^9 + a^6*
x^7)]
```

Sympy [A] time = 127.943, size = 394, normalized size = 2.08

$$\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{3a^3bf-5a^2b^2e+7ab^3d-9b^4c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)\log\left(\frac{a^6\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{3a^3bf-5a^2b^2e+7ab^3d-9b^4c}+x\right)}{4}-\frac{30a^4c+x^8\left(315a^3bf-525a^2b^2e+735ab^3d-945b^4c\right)+x^6\left(210a^4f-350a^3be+490a^2b^2d-630ab^3c\right)+x^4\left(70a^4e-98a\right.}{210a^6x^7+210a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*lo
g(-a**6*sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b*
*3*c)/(3*a**3*b*f - 5*a**2*b**2*e + 7*a*b**3*d - 9*b**4*c) + x)/4
 - sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)
*log(a**6*sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*
b**3*c)/(3*a**3*b*f - 5*a**2*b**2*e + 7*a*b**3*d - 9*b**4*c) + x)
/4 - (30*a**4*c + x**8*(315*a**3*b*f - 525*a**2*b**2*e + 735*a*b*
*3*d - 945*b**4*c) + x**6*(210*a**4*f - 350*a**3*b*e + 490*a**2*b
**2*d - 630*a*b**3*c) + x**4*(70*a**4*e - 98*a**3*b*d + 126*a**2*
b**2*c) + x**2*(42*a**4*d - 54*a**3*b*c))/(210*a**6*x**7 + 210*a*
*5*b*x**9)
```

GIAC/XCAS [A] time = 0.217014, size = 271, normalized size = 1.43

$$\frac{\left(9\,b^4c-7\,ab^3d-3\,a^3bf+5\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{2\,(bx^2+a)a^5}+\frac{420\,b^3cx^6-315\,ab^2dx^6-105\,a^3fx^6+210\,a^2bx^6e-105\,ab^2cx^4+70\,a^2bdx^4-35\,a^3x^4e+42\,a^2bcx^2-21\,a^3dx^2-15\,a^3c}{105\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^8),x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x - a^3*b*f*x +
 a^2*b^2*x*e)/((b*x^2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^
2*d*x^6 - 105*a^3*f*x^6 + 210*a^2*b*x^6*e - 105*a*b^2*c*x^4 + 70*
a^2*b*d*x^4 - 35*a^3*x^4*e + 42*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a
^3*c)/(a^5*x^7)
```

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx$

Optimal. Leaf size=230

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}-\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(
3*a^5*x^3) - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6
*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x
^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d + 7*a^2*b*e - 5*a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rubi [A] time = 0.73867, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}-\frac{b\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d +
 a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(
3*a^5*x^3) - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6
*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x
^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d + 7*a^2*b*e - 5*a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rubi in Sympy [A] time = 168.751, size = 231, normalized size = 1.

$$-\frac{x\left(\frac{a^3f}{x^{10}}-\frac{a^2be}{x^{10}}+\frac{ab^2d}{x^{10}}-\frac{b^3c}{x^{10}}\right)}{2ab^3\left(a+bx^2\right)}-\frac{a^2f-abe+b^2d}{9ab^3x^9}+\frac{2a^2f-2abe+b^2d}{7a^2b^2x^7}-\frac{3a^2f-2abe+b^2d}{5a^3bx^5}+\frac{3a^2f-2abe+b^2d}{3a^4x^3}-\frac{b\left(3a^2f-2abe+b^2d\right)}{a^5x}-\frac{b^{\frac{3}{2}}\left(3a^2f-2abe+b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*f/x**10 - a**2*b*e/x**10 + a*b**2*d/x**10 - b**3*c/x**10
)/(2*a*b**3*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(9*a*b**3*x
**9) + (2*a**2*f - 2*a*b*e + b**2*d)/(7*a**2*b**2*x**7) - (3*a**2
*f - 2*a*b*e + b**2*d)/(5*a**3*b*x**5) + (3*a**2*f - 2*a*b*e + b*
*2*d)/(3*a**4*x**3) - b*(3*a**2*f - 2*a*b*e + b**2*d)/(a**5*x) -
b**(3/2)*(3*a**2*f - 2*a*b*e + b**2*d)*atan(sqrt(b)*x/sqrt(a))/a*
*(11/2)
```

Mathematica [A] time = 0.221184, size = 230, normalized size = 1.

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}+\frac{a^2(-e)+2abd-3b^2c}{5a^4x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-7a^2be+9ab^2d-11b^3c\right)}{2a^{13/2}}+\frac{b^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d
- a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/
(3*a^5*x^3) + (b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a
^6*x) + (b^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a
+ b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*b^2*d - 7*a^2*b*e + 5*a^3*f
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Maple [A] time = 0.024, size = 318, normalized size = 1.4

$$-\frac{c}{9\,a^2x^9}-\frac{d}{7\,a^2x^7}+\frac{2\,bc}{7\,a^3x^7}-\frac{e}{5\,x^5a^2}+\frac{2\,bd}{5\,a^3x^5}-\frac{3\,b^2c}{5\,a^4x^5}-\frac{f}{3\,x^3a^2}+\frac{2\,be}{3\,a^3x^3}-\frac{db^2}{a^4x^3}+\frac{4\,b^3c}{3\,a^5x^3}+2\,\frac{fb}{a^3x}-3\,\frac{eb^2}{a^4x}+4\,\frac{db^3}{a^5x}-5\,\frac{cb^4}{a^6x}+\frac{b^2xf}{2\,a^3\left(bx^2+a\right)}-\frac{b^3xe}{2\,a^4\left(bx^2+a\right)}+\frac{b^4xd}{2\,a^5\left(bx^2+a\right)}-\frac{b^5xc}{2\,a^6\left(bx^2+a\right)}+\frac{5\,fb^2}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,b^3e}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,db^4}{2\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{11\,b^5c}{2\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x)
```

```
[Out] -1/9*c/a^2/x^9-1/7/a^2/x^7*d+2/7/a^3/x^7*b*c-1/5/a^2/x^5*e+2/5/a^
3/x^5*b*d-3/5/a^4/x^5*b^2*c-1/3/a^2/x^3*f+2/3/a^3/x^3*b*e-1/a^4/x
^3*b^2*d+4/3/a^5/x^3*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*
d-5*b^4/a^6/x*c+1/2*b^2/a^3*x/(b*x^2+a)*f-1/2*b^3/a^4*x/(b*x^2+a)
*e+1/2*b^4/a^5*x/(b*x^2+a)*d-1/2*b^5/a^6*x/(b*x^2+a)*c+5/2*b^2/a^
3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-7/2*b^3/a^4/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*e+9/2*b^4/a^5/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*d-11/2*b^5/a^6/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^10),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241076, size = 1, normalized size = 0.

$$\left[-\frac{630\left(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f\right)x^{10}+420\left(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf\right)x^8-84\left(11\,a^2b^3c-9\,a^3b^2d+\cdots\right.}{\cdots}\right.$$

$$-\frac{315\left(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f\right)x^{10}+210\left(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf\right)x^8-42\left(11\,a^2b^3c-9\,a^3b^2d+7\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^10),x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*
x^10 + 420*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x
^8 - 84*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 +
140*a^5*c + 36*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*
a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e
 - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e -
5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*b^
4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b
^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^
2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*
a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11
*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*
c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(b/a)*arctan(
b*x/(a*sqrt(b/a))))/(a^6*b*x^11 + a^7*x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.213703, size = 340, normalized size = 1.48

$$-\frac{\left(11\,b^5c-9\,ab^4d-5\,a^3b^2f+7\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6}-\frac{b^5cx-ab^4dx-a^3b^2fx+a^2b^3xe}{2\,(bx^2+a)a^6}$$
$$-\frac{1575\,b^4cx^8-1260\,ab^3dx^8-630\,a^3bfx^8+945\,a^2b^2x^8e-420\,ab^3cx^6+315\,a^2b^2dx^6+105\,a^4fx^6-210\,a^3bx^6e+189\,a^2b^2c\cdots}{315\,a^6x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^2*x^10),x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x - a^3*b^2
*f*x + a^2*b^3*x*e)/((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1
260*a*b^3*d*x^8 - 630*a^3*b*f*x^8 + 945*a^2*b^2*x^8*e - 420*a*b^3
*c*x^6 + 315*a^2*b^2*d*x^6 + 105*a^4*f*x^6 - 210*a^3*b*x^6*e + 18
9*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*x^4*e - 90*a^3*b*c*x^2
 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

3.133 $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx$

Optimal. Leaf size=311

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^9\left(-17a^3f+13a^2be-9ab^2d+5b^3c\right)}{8a^2b^3(a+bx^2)}-\frac{ax\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{8b^7}+\frac{x^3\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{24b^6}-\frac{x^5\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{40ab^5}+\frac{x^7\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{56a^2b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{8b^{15/2}}+\frac{fx^9}{9b^3}$$

```
[Out] -(a*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x)/(8*b^7) +
 ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^3)/(24*b^6)
- ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^5)/(40*a*b^
5) + ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^7)/(56*a
^2*b^4) + (f*x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3
)*x^9)/(4*a*(a + b*x^2)^2) - ((5*b^3*c - 9*a*b^2*d + 13*a^2*b*e -
 17*a^3*f)*x^9)/(8*a^2*b^3*(a + b*x^2)) + (a^(3/2)*(35*b^3*c - 63
*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*b^(15/2))
```

Rubi [A] time = 1.08233, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^9\left(-17a^3f+13a^2be-9ab^2d+5b^3c\right)}{8a^2b^3(a+bx^2)}-\frac{ax\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{8b^7}+\frac{x^3\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{24b^6}-\frac{x^5\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{40ab^5}+\frac{x^7\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{56a^2b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{8b^{15/2}}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```


```
[Out] -(a*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x)/(8*b^7) +
 ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^3)/(24*b^6)
- ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^5)/(40*a*b^
5) + ((35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*x^7)/(56*a
^2*b^4) + (f*x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3
)*x^9)/(4*a*(a + b*x^2)^2) - ((5*b^3*c - 9*a*b^2*d + 13*a^2*b*e -
 17*a^3*f)*x^9)/(8*a^2*b^3*(a + b*x^2)) + (a^(3/2)*(35*b^3*c - 63
*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*b^(15/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.312003, size = 272, normalized size = 0.87

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{a^2x\left(25a^3f-21a^2be+17ab^2d-13b^3c\right)}{8b^7\left(a+bx^2\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^7\left(a+bx^2\right)^2}+\frac{ax\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)}{b^7}+\frac{x^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{3b^6}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{8b^{15/2}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*
c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*
a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7
*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a*b^2*d - 21*a^2*b*e + 25*
a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*c + 63*a*b^2*d
- 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(15/2
))
```

Maple [A] time = 0.02, size = 394, normalized size = 1.3

$$\frac{25\,a^5x^3f}{8\,b^6\left(bx^2+a\right)^2}+\frac{x^7e}{7\,b^3}+\frac{x^5d}{5\,b^3}+\frac{x^3c}{3\,b^3}+\frac{15\,a^4dx}{8\,b^5\left(bx^2+a\right)^2}-\frac{11\,a^3cx}{8\,b^4\left(bx^2+a\right)^2}-\frac{143\,a^5f}{8\,b^7}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{99\,a^4e}{8\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,a^3d}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,a^2c}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{fx^9}{9\,b^3}-\frac{3\,x^7af}{7\,b^4}+\frac{6\,x^5a^2f}{5\,b^5}-\frac{3\,x^5ae}{5\,b^4}-\frac{10\,x^3a^3f}{3\,b^6}+2\,\frac{x^3a^2e}{b^5}-\frac{ax^3d}{b^4}+15\,\frac{a^4fx}{b^7}-10\,\frac{a^3ex}{b^6}+6\,\frac{a^2dx}{b^5}-3\,\frac{acx}{b^4}-\frac{21\,a^4x^3e}{8\,b^5\left(bx^2+a\right)^2}+\frac{17\,x^3a^3d}{8\,b^4\left(bx^2+a\right)^2}-\frac{13\,x^3a^2c}{8\,b^3\left(bx^2+a\right)^2}+\frac{23\,a^6fx}{8\,b^7\left(bx^2+a\right)^2}-\frac{19\,a^5ex}{8\,b^6\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 25/8*a^5/b^6/(b*x^2+a)^2*x^3*f+1/7/b^3*x^7*e+1/5/b^3*x^5*d+1/3/b^
3*x^3*c+15/8*a^4/b^5/(b*x^2+a)^2*d*x-11/8*a^3/b^4/(b*x^2+a)^2*c*x
-143/8*a^5/b^7/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+99/8*a^4/b^6
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e-63/8*a^3/b^5/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*d+35/8*a^2/b^4/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))*c+1/9*f*x^9/b^3-3/7/b^4*x^7*a*f+6/5/b^5*x^5*a^2*f-3/5/b^
4*x^5*a*e-10/3/b^6*x^3*a^3*f+2/b^5*x^3*a^2*e-1/b^4*x^3*a*d+15/b^7
*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x-21/8*a^4/b^5/(b
*x^2+a)^2*x^3*e+17/8*a^3/b^4/(b*x^2+a)^2*x^3*d-13/8*a^2/b^3/(b*x^
2+a)^2*x^3*c+23/8*a^6/b^7/(b*x^2+a)^2*f*x-19/8*a^5/b^6/(b*x^2+a)^
2*e*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^8/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241515, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^8/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63
*b^6*d - 99*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^
5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^
2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c
- 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 - 315*(35*a^3*b^
3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c - 63*a^
2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 6
3*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((b*
x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63
*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 +
a^2*b^7), 1/2520*(280*b^6*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11
 + 8*(63*b^6*d - 99*a*b^5*e + 143*a^2*b^4*f)*x^9 + 24*(35*b^6*c -
 63*a*b^5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x^7 - 168*(35*a*b^5*c
 - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 525*(35*a^2
*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 + 315*(35
*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c
- 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^
4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*a
rctan(x/sqrt(a/b)) - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*
e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)]
```

Sympy [A] time = 25.6735, size = 491, normalized size = 1.58

$$\frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16} - \frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)\log\left(\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16} + \frac{x^3\left(25a^5bf-21a^4b^2e+17a^3b^3d-13a^2b^4c\right)+x\left(23a^6f-19a^5be+15a^4b^2d-11a^3b^3c\right)}{8a^2b^7+16ab^8x^2+8b^9x^4} + \frac{fx^9}{9b^3} - \frac{x^7\left(3af-be\right)}{7b^4} + \frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5} - \frac{x^3\left(10a^3f-6a^2be+3ab^2d-b^3c\right)}{3b^6} + \frac{x\left(15a^4f-10a^3be+6a^2b^2d-3ab^3c\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b*
*3*c)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*
a*b**2*d - 35*b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d
- 35*a*b**3*c) + x)/16 - sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*
b*e + 63*a*b**2*d - 35*b**3*c)*log(b**7*sqrt(-a**3/b**15)*(143*a*
*3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)/(143*a**4*f - 99*a*
*3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3*(25*a**5*b
```

```
*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a*
*6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b*
*7 + 16*a*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3) - x**7*(3*a*
f - b*e)/(7*b**4) + x**5*(6*a**2*f - 3*a*b*e + b**2*d)/(5*b**5) -
 x**3*(10*a**3*f - 6*a**2*b*e + 3*a*b**2*d - b**3*c)/(3*b**6) + x
*(15*a**4*f - 10*a**3*b*e + 6*a**2*b**2*d - 3*a*b**3*c)/b**7
```

GIAC/XCAS [A] time = 0.220473, size = 406, normalized size = 1.31

$$\frac{\left(35\,a^2b^3c - 63\,a^3b^2d - 143\,a^5f + 99\,a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} - \frac{13\,a^2b^4cx^3 - 17\,a^3b^3dx^3 - 25\,a^5bfx^3 + 21\,a^4b^2x^3e + 11\,a^3b^3cx - 15\,a^4b^2dx - 23\,a^6fx + 19\,a^5bxe}{8\,(bx^2+a)^2b^7} + \frac{35\,b^{24}fx^9 - 135\,ab^{23}fx^7 + 45\,b^{24}x^7e + 63\,b^{24}dx^5 + 378\,a^2b^{22}fx^5 - 189\,ab^{23}x^5e + 105\,b^{24}cx^3 - 315\,ab^{23}dx^3 - 1050\,a^3b^{21}f}{315\,b^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^8/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*
b^3*d*x^3 - 25*a^5*b*f*x^3 + 21*a^4*b^2*x^3*e + 11*a^3*b^3*c*x -
15*a^4*b^2*d*x - 23*a^6*f*x + 19*a^5*b*x*e)/((b*x^2 + a)^2*b^7) +
 1/315*(35*b^24*f*x^9 - 135*a*b^23*f*x^7 + 45*b^24*x^7*e + 63*b^2
4*d*x^5 + 378*a^2*b^22*f*x^5 - 189*a*b^23*x^5*e + 105*b^24*c*x^3
- 315*a*b^23*d*x^3 - 1050*a^3*b^21*f*x^3 + 630*a^2*b^22*x^3*e - 9
45*a*b^23*c*x + 1890*a^2*b^22*d*x + 4725*a^4*b^20*f*x - 3150*a^3*
b^21*x*e)/b^27
```

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=273

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^7\left(-15a^3f+11a^2be-7ab^2d+3b^3c\right)}{8a^2b^3(a+bx^2)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^{13/2}}+\frac{x\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^6}-\frac{x^3\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{24ab^5}+\frac{x^5\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{40a^2b^4}+\frac{fx^7}{7b^3}$$

```
[Out] ((15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x)/(8*b^6) - ((1
5*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x^3)/(24*a*b^5) + (
(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x^5)/(40*a^2*b^4)
 + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/
(4*a*(a + b*x^2)^2) - ((3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3
*f)*x^7)/(8*a^2*b^3*(a + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*
d + 63*a^2*b*e - 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/
2))
```

Rubi [A] time = 0.966788, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^7\left(-15a^3f+11a^2be-7ab^2d+3b^3c\right)}{8a^2b^3(a+bx^2)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^{13/2}}+\frac{x\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^6}-\frac{x^3\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{24ab^5}+\frac{x^5\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{40a^2b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```


```
[Out] ((15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x)/(8*b^6) - ((1
5*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x^3)/(24*a*b^5) + (
(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*x^5)/(40*a^2*b^4)
 + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/
(4*a*(a + b*x^2)^2) - ((3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3
*f)*x^7)/(8*a^2*b^3*(a + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*
d + 63*a^2*b*e - 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/
2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.291528, size = 232, normalized size = 0.85

$$\frac{x^3\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^3f-63a^2be+35ab^2d-15b^3c\right)}{8b^{13/2}}+\frac{ax\left(-21a^3f+17a^2be-13ab^2d+9b^3c\right)}{8b^6\left(a+bx^2\right)}+\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^6\left(a+bx^2\right)^2}+\frac{x\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{b^6}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*
a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*
x^7)/(7*b^3) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*
b^6*(a + b*x^2)^2) + (a*(9*b^3*c - 13*a*b^2*d + 17*a^2*b*e - 21*a
^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*(-15*b^3*c + 35*a*b^2*d -
 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Maple [A] time = 0.019, size = 343, normalized size = 1.3

$$\frac{fx^7}{7\,b^3}-\frac{3\,x^5af}{5\,b^4}+\frac{x^5e}{5\,b^3}+2\,\frac{x^3a^2f}{b^5}-\frac{ax^3e}{b^4}+\frac{x^3d}{3\,b^3}-10\,\frac{a^3fx}{b^6}+6\,\frac{a^2ex}{b^5}-3\,\frac{adx}{b^4}+\frac{cx}{b^3}-\frac{21\,a^4x^3f}{8\,b^5\left(bx^2+a\right)^2}+\frac{17\,a^3x^3e}{8\,b^4\left(bx^2+a\right)^2}-\frac{13\,x^3a^2d}{8\,b^3\left(bx^2+a\right)^2}+\frac{9\,ax^3c}{8\,b^2\left(bx^2+a\right)^2}-\frac{19\,a^5fx}{8\,b^6\left(bx^2+a\right)^2}+\frac{15\,a^4ex}{8\,b^5\left(bx^2+a\right)^2}-\frac{11\,a^3dx}{8\,b^4\left(bx^2+a\right)^2}+\frac{7\,a^2cx}{8\,b^3\left(bx^2+a\right)^2}+\frac{99\,fa^4}{8\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,a^3e}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,a^2d}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,ac}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*f*x^7/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+2/b^5*x^3*a^2*f-1/b^4
*x^3*a*e+1/3/b^3*x^3*d-10/b^6*a^3*f*x+6/b^5*a^2*e*x-3/b^4*a*d*x+1
/b^3*c*x-21/8*a^4/b^5/(b*x^2+a)^2*x^3*f+17/8*a^3/b^4/(b*x^2+a)^2*
x^3*e-13/8*a^2/b^3/(b*x^2+a)^2*x^3*d+9/8*a/b^2/(b*x^2+a)^2*x^3*c-
19/8*a^5/b^6/(b*x^2+a)^2*f*x+15/8*a^4/b^5/(b*x^2+a)^2*e*x-11/8*a^
3/b^4/(b*x^2+a)^2*d*x+7/8*a^2/b^3/(b*x^2+a)^2*c*x+99/8*a^4/b^6/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-63/8*a^3/b^5/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*e+35/8*a^2/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*d-15/8*a/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235125, size = 1, normalized size = 0.

$$\left[\frac{240\,b^5 f x^{11} + 48\left(7\,b^5 e - 11\,ab^4 f\right)x^9 + 16\left(35\,b^5 d - 63\,ab^4 e + 99\,a^2b^3 f\right)x^7 + 112\left(15\,b^5 c - 35\,ab^4 d + 63\,a^2b^3 e - 99\,a^3b^2 f\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*
b^5*d - 63*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4
*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*
b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a
^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^
2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a
^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4
*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120
*b^5*f*x^11 + 24*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*
b^4*e + 99*a^2*b^3*f)*x^7 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^
3*e - 99*a^3*b^2*f)*x^5 + 175*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3
*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*
a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a
^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*
a^4*b*f)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 105*(15*a^2*b^3*c -
 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2
+ a^2*b^6)]
```

Sympy [A] time = 25.271, size = 311, normalized size = 1.14

$$- \frac{\sqrt{- \frac{a}{b^{13}}} \left(99a^3 f - 63a^2 be + 35ab^2 d - 15b^3 c\right) \log\left(-b^6 \sqrt{- \frac{a}{b^{13}}} + x\right)}{16} + \frac{\sqrt{- \frac{a}{b^{13}}} \left(99a^3 f - 63a^2 be + 35ab^2 d - 15b^3 c\right) \log\left(b^6 \sqrt{- \frac{a}{b^{13}}} + x\right)}{16} - \frac{x^3 \left(21a^4 bf - 17a^3 b^2 e + 13a^2 b^3 d - 9ab^4 c\right) + x\left(19a^5 f - 15a^4 be + 11a^3 b^2 d - 7a^2 b^3 c\right)}{8a^2 b^6 + 16ab^7 x^2 + 8b^8 x^4} + \frac{fx^7}{7b^3} - \frac{x^5 (3af - be)}{5b^4} + \frac{x^3\left(6a^2 f - 3abe + b^2 d\right)}{3b^5} - \frac{x\left(10a^3 f - 6a^2 be + 3ab^2 d - b^3 c\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*
c)*log(-b**6*sqrt(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f -
 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(b**6*sqrt(-a/b**13) +
 x)/16 - (x**3*(21*a**4*b*f - 17*a**3*b**2*e + 13*a**2*b**3*d - 9
*a*b**4*c) + x*(19*a**5*f - 15*a**4*b*e + 11*a**3*b**2*d - 7*a**2
*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4) + f*x**7/(
7*b**3) - x**5*(3*a*f - b*e)/(5*b**4) + x**3*(6*a**2*f - 3*a*b*e
+ b**2*d)/(3*b**5) - x*(10*a**3*f - 6*a**2*b*e + 3*a*b**2*d - b**
3*c)/b**6
```

GIAC/XCAS [A] time = 0.218793, size = 338, normalized size = 1.24

$$-\frac{\left(15\,ab^{3}c - 35\,a^{2}b^{2}d - 99\,a^{4}f + 63\,a^{3}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^{6}} + \frac{9\,ab^{4}cx^{3} - 13\,a^{2}b^{3}dx^{3} - 21\,a^{4}bfx^{3} + 17\,a^{3}b^{2}x^{3}e + 7\,a^{2}b^{3}cx - 11\,a^{3}b^{2}dx - 19\,a^{5}fx + 15\,a^{4}bxe}{8\,(bx^{2}+a)^{2}b^{6}} + \frac{15\,b^{18}fx^{7} - 63\,ab^{17}fx^{5} + 21\,b^{18}x^{5}e + 35\,b^{18}dx^{3} + 210\,a^{2}b^{16}fx^{3} - 105\,ab^{17}x^{3}e + 105\,b^{18}cx - 315\,ab^{17}dx - 1050\,a^{3}b^{15}fx +}{105\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^6/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d
*x^3 - 21*a^4*b*f*x^3 + 17*a^3*b^2*x^3*e + 7*a^2*b^3*c*x - 11*a^3
*b^2*d*x - 19*a^5*f*x + 15*a^4*b*x*e)/((b*x^2 + a)^2*b^6) + 1/105
*(15*b^18*f*x^7 - 63*a*b^17*f*x^5 + 21*b^18*x^5*e + 35*b^18*d*x^3
 + 210*a^2*b^16*f*x^3 - 105*a*b^17*x^3*e + 105*b^18*c*x - 315*a*b
^17*d*x - 1050*a^3*b^15*f*x + 630*a^2*b^16*x*e)/b^21
```

3.135 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=235

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(-13a^3f+9a^2be-5ab^2d+b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}b^{11/2}}-\frac{x\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8ab^5}+\frac{x^3\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{24a^2b^4}+\frac{fx^5}{5b^3}$$

```
[Out] -((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*x)/(8*a*b^5) + (
(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*x^3)/(24*a^2*b^4)
+ (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(
4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*
x^5)/(8*a^2*b^3*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*
e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rubi [A] time = 0.867842, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(-13a^3f+9a^2be-5ab^2d+b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}b^{11/2}}-\frac{x\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8ab^5}+\frac{x^3\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{24a^2b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```


```
[Out] -((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*x)/(8*a*b^5) + (
(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*x^3)/(24*a^2*b^4)
+ (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(
4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*
x^5)/(8*a^2*b^3*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*
e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.306987, size = 176, normalized size = 0.75

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}b^{11/2}}+\frac{x\left(945a^4f-525a^3b\left(e-3fx^2\right)+a^2b^2\left(225d-875ex^2+504fx^4\right)-ab^3\left(45c-375dx^2+280ex^4+72fx^6\right)+b^4x^2\left(8\left(15d\right.\right.}{120b^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*
x^2 + 504*f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6
) + b^4*x^2*(-75*c + 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5
*(a + b*x^2)^2) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Maple [A] time = 0.018, size = 294, normalized size = 1.3

$$\frac{fx^5}{5\,b^3}-\frac{ax^3f}{b^4}+\frac{x^3e}{3\,b^3}+6\,\frac{a^2fx}{b^5}-3\,\frac{aex}{b^4}+\frac{dx}{b^3}+\frac{17\,x^3a^3f}{8\,b^4\,(bx^2+a)^2}-\frac{13\,x^3a^2e}{8\,b^3\,(bx^2+a)^2}+\frac{9\,ax^3d}{8\,b^2\,(bx^2+a)^2}-\frac{5\,x^3c}{8\,b\,(bx^2+a)^2}+\frac{15\,fa^4x}{8\,b^5\,(bx^2+a)^2}-\frac{11\,a^3ex}{8\,b^4\,(bx^2+a)^2}+\frac{7\,a^2dx}{8\,b^3\,(bx^2+a)^2}-\frac{3\,acx}{8\,b^2\,(bx^2+a)^2}-\frac{63\,a^3f}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,a^2e}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,ad}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e+6/b^5*a^2*f*x-3/b^4*a*e
*x+1/b^3*d*x+17/8/b^4/(b*x^2+a)^2*x^3*a^3*f-13/8/b^3/(b*x^2+a)^2*
x^3*a^2*e+9/8/b^2/(b*x^2+a)^2*x^3*a*d-5/8/b/(b*x^2+a)^2*x^3*c+15/
8/b^5/(b*x^2+a)^2*a^4*f*x-11/8/b^4/(b*x^2+a)^2*a^3*e*x+7/8/b^3/(b
*x^2+a)^2*a^2*d*x-3/8/b^2/(b*x^2+a)^2*a*c*x-63/8/b^5/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*a^3*f+35/8/b^4/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*a^2*e-15/8/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+
3/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24066, size = 1, normalized size = 0.

$$\left[-\frac{15\left(3\,a^2b^3c-15\,a^3b^2d+35\,a^4be-63\,a^5f+\left(3\,b^5c-15\,ab^4d+35\,a^2b^3e-63\,a^3b^2f\right)x^4+2\left(3\,ab^4c-15\,a^2b^3d+35\,a^3b^2e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f +
 (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*
```

```
a*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*log(-(2*
a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(24*b^4*f*x^9 +
8*(5*b^4*e - 9*a*b^3*f)*x^7 + 8*(15*b^4*d - 35*a*b^3*e + 63*a^2*b
^2*f)*x^5 - 25*(3*b^4*c - 15*a*b^3*d + 35*a^2*b^2*e - 63*a^3*b*f)
*x^3 - 15*(3*a*b^3*c - 15*a^2*b^2*d + 35*a^3*b*e - 63*a^4*f)*x)*s
qrt(-a*b))/((b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)*sqrt(-a*b)), 1/120*
(15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f + (3*b^5*
c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c
- 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*arctan(sqrt(a*b)
*x/a) + (24*b^4*f*x^9 + 8*(5*b^4*e - 9*a*b^3*f)*x^7 + 8*(15*b^4*d
 - 35*a*b^3*e + 63*a^2*b^2*f)*x^5 - 25*(3*b^4*c - 15*a*b^3*d + 35
*a^2*b^2*e - 63*a^3*b*f)*x^3 - 15*(3*a*b^3*c - 15*a^2*b^2*d + 35*
a^3*b*e - 63*a^4*f)*x)*sqrt(a*b))/((b^7*x^4 + 2*a*b^6*x^2 + a^2*b
^5)*sqrt(a*b))]
```

Sympy [A] time = 24.5198, size = 279, normalized size = 1.19

$$\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}+\frac{x^3\left(17a^3bf-13a^2b^2e+9ab^3d-5b^4c\right)+x\left(15a^4f-11a^3be+7a^2b^2d-3ab^3c\right)}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{fx^5}{5b^3}-\frac{x^3\left(3af-be\right)}{3b^4}+\frac{x\left(6a^2f-3abe+b^2d\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**
3*c)*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*
(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqr
t(-1/(a*b**11)) + x)/16 + (x**3*(17*a**3*b*f - 13*a**2*b**2*e + 9
*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 11*a**3*b*e + 7*a**2*b**2*
d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4) + f
*x**5/(5*b**3) - x**3*(3*a*f - b*e)/(3*b**4) + x*(6*a**2*f - 3*a*
b*e + b**2*d)/b**5
```

GIAC/XCAS [A] time = 0.216108, size = 270, normalized size = 1.15

$$\frac{\left(3\,b^3c-15\,ab^2d-63\,a^3f+35\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}-\frac{5\,b^4cx^3-9\,ab^3dx^3-17\,a^3bfx^3+13\,a^2b^2x^3e+3\,ab^3cx-7\,a^2b^2dx-15\,a^4fx+11\,a^3bxe}{8\left(bx^2+a\right)^2b^5}+\frac{3\,b^{12}fx^5-15\,ab^{11}fx^3+5\,b^{12}x^3e+15\,b^{12}dx+90\,a^2b^{10}fx-45\,ab^{11}xe}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 - 17*a
^3*b*f*x^3 + 13*a^2*b^2*x^3*e + 3*a*b^3*c*x - 7*a^2*b^2*d*x - 15*
a^4*f*x + 11*a^3*b*x*e)/((b*x^2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5
- 15*a*b^11*f*x^3 + 5*b^12*x^3*e + 15*b^12*d*x + 90*a^2*b^10*f*x
- 45*a*b^11*x*e)/b^15
```

3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx$

Optimal. Leaf size=193

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}+\frac{x^3(11a^3f-7a^2be+3ab^2d+b^3c)}{8a^2b^3(a+bx^2)}-\frac{x(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^2b^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}}+\frac{fx^3}{3b^3}$$

```
[Out] -((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*x)/(8*a^2*b^4) + (f
*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)/(4*a*
(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f)*x^3)
/(8*a^2*b^3*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.739767, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}+\frac{x^3(11a^3f-7a^2be+3ab^2d+b^3c)}{8a^2b^3(a+bx^2)}-\frac{x(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^2b^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*x)/(8*a^2*b^4) + (f
*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)/(4*a*
(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f)*x^3)
/(8*a^2*b^3*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.254623, size = 156, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}}+\frac{x\left(-105a^4f+5a^3b(9e-35fx^2)+a^2b^2(-9d+75ex^2-56fx^4)+ab^3(-3c-15dx^2+24ex^4+8fx^6)+3b^4cx^2\right)}{24ab^4(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2
*(-9*d + 75*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4
 + 8*f*x^6)))/(24*a*b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15
*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9
/2))
```

Maple [A] time = 0.016, size = 259, normalized size = 1.3

$$\frac{f x^3}{3 b^3}-3 \frac{afx}{b^4}+\frac{ex}{b^3}-\frac{13 x^3 a^2 f}{8 b^3 (bx^2+a)^2}+\frac{9 a x^3 e}{8 b^2 (bx^2+a)^2}-\frac{5 x^3 d}{8 b (bx^2+a)^2}+\frac{x^3 c}{8 (bx^2+a)^2 a} -\frac{11 a^3 fx}{8 b^4 (bx^2+a)^2}+\frac{7 a^2 ex}{8 b^3 (bx^2+a)^2}-\frac{3 adx}{8 b^2 (bx^2+a)^2}-\frac{cx}{8 b (bx^2+a)^2}+\frac{35 a^2 f}{8 b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} -\frac{15 ae}{8 b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3 d}{8 b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{8 ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3-3/b^4*a*f*x+1/b^3*e*x-13/8/b^3/(b*x^2+a)^2*x^3*a^2*
f+9/8/b^2/(b*x^2+a)^2*x^3*a*e-5/8/b/(b*x^2+a)^2*x^3*d+1/8/(b*x^2+
a)^2/a*x^3*c-11/8/b^4/(b*x^2+a)^2*a^3*f*x+7/8/b^3/(b*x^2+a)^2*a^2
*e*x-3/8/b^2/(b*x^2+a)^2*a*d*x-1/8/b/(b*x^2+a)^2*c*x+35/8/b^4*a^2
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-15/8/b^3*a/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))*e+3/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*d+1/8/b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236339, size = 1, normalized size = 0.01

$$\left[\frac{3\left(a^2 b^3 c+3 a^3 b^2 d-15 a^4 be+35 a^5 f+\left(b^5 c+3 ab^4 d-15 a^2 b^3 e+35 a^3 b^2 f\right)x^4+2\left(ab^4 c+3 a^2 b^3 d-15 a^3 b^2 e+35 a^4 bf\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(a^2*b^3*c + 3*a^3*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*
c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*b^2*f)*x^4 + 2*(a*b^4*c + 3
*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*log((2*a*b*x + (b*x^
2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(8*a*b^3*f*x^7 + 8*(3*a*b^3*e
 - 7*a^2*b^2*f)*x^5 + (3*b^4*c - 15*a*b^3*d + 75*a^2*b^2*e - 175*
a^3*b*f)*x^3 - 3*(a*b^3*c + 3*a^2*b^2*d - 15*a^3*b*e + 35*a^4*f)*
x)*sqrt(-a*b))/((a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)*sqrt(-a*b))
, 1/24*(3*(a^2*b^3*c + 3*a^3*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5
*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*b^2*f)*x^4 + 2*(a*b^4*c +
3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*arctan(sqrt(a*b)*x/
```

```
a) + (8*a*b^3*f*x^7 + 8*(3*a*b^3*e - 7*a^2*b^2*f)*x^5 + (3*b^4*c
- 15*a*b^3*d + 75*a^2*b^2*e - 175*a^3*b*f)*x^3 - 3*(a*b^3*c + 3*a
^2*b^2*d - 15*a^3*b*e + 35*a^4*f)*x)*sqrt(a*b))/((a*b^6*x^4 + 2*a
^2*b^5*x^2 + a^3*b^4)*sqrt(a*b))]
```

Sympy [A] time = 21.23, size = 258, normalized size = 1.34

$$-\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}-\frac{x^3\left(13a^3bf-9a^2b^2e+5ab^3d-b^4c\right)+x\left(11a^4f-7a^3be+3a^2b^2d+ab^3c\right)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}+\frac{fx^3}{3b^3}-\frac{x\left(3af-be\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**
3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/(a**3*
b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b
**4*sqrt(-1/(a**3*b**9)) + x)/16 - (x**3*(13*a**3*b*f - 9*a**2*b*
*2*e + 5*a*b**3*d - b**4*c) + x*(11*a**4*f - 7*a**3*b*e + 3*a**2*
b**2*d + a*b**3*c))/(8*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x
**4) + f*x**3/(3*b**3) - x*(3*a*f - b*e)/b**4
```

GIAC/XCAS [A] time = 0.217039, size = 234, normalized size = 1.21

$$\frac{\left(b^3c+3\,ab^2d+35\,a^3f-15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4}+\frac{b^4cx^3-5\,ab^3dx^3-13\,a^3bfx^3+9\,a^2b^2x^3e-ab^3cx-3\,a^2b^2dx-11\,a^4fx+7\,a^3bxe}{8\,(bx^2+a)^2ab^4}+\frac{b^6fx^3-9\,ab^5fx+3\,b^6xe}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 15*a^2*b*e)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 - 13*a^3*
b*f*x^3 + 9*a^2*b^2*x^3*e - a*b^3*c*x - 3*a^2*b^2*d*x - 11*a^4*f*
x + 7*a^3*b*x*e)/((b*x^2 + a)^2*a*b^4) + 1/3*(b^6*f*x^3 - 9*a*b^5
*f*x + 3*b^6*x*e)/b^9
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=147

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{x\left(9a^3f-5a^2be+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b
*x^2)^2) + ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b
^3*(a + b*x^2)) + ((3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rubi [A] time = 0.354676, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{x\left(9a^3f-5a^2be+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b
*x^2)^2) + ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b
^3*(a + b*x^2)) + ((3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rubi in Sympy [A] time = 84.4624, size = 146, normalized size = 0.99

$$\frac{fx}{b^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4ab^3\left(a+bx^2\right)^2}+\frac{x\left(9a^3f-5a^2be+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}-\frac{\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] f*x/b**3 - x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(4*a*b**3*(a
 + b*x**2)**2) + x*(9*a**3*f - 5*a**2*b*e + a*b**2*d + 3*b**3*c)/
(8*a**2*b**3*(a + b*x**2)) - (15*a**3*f - 3*a**2*b*e - a*b**2*d -
 3*b**3*c)*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*b**(7/2))
```

Mathematica [A] time = 0.223779, size = 141, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{x\left(15a^4f+a^3b\left(25fx^2-3e\right)-a^2b^2\left(d+5ex^2-8fx^4\right)+ab^3\left(5c+dx^2\right)+3b^4cx^2\right)}{8a^2b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e +
25*f*x^2) - a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x
^2)^2) + ((3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Maple [A] time = 0.018, size = 234, normalized size = 1.6

$$\frac{fx}{b^3}+\frac{9\,ax^3f}{8\,b^2\,(bx^2+a)^2}-\frac{5\,x^3e}{8\,b\,(bx^2+a)^2}+\frac{x^3d}{8\,(bx^2+a)^2\,a}+\frac{3\,bx^3c}{8\,(bx^2+a)^2\,a^2}+\frac{7\,a^2fx}{8\,b^3\,(bx^2+a)^2}-\frac{3\,aex}{8\,b^2\,(bx^2+a)^2}-\frac{dx}{8\,b\,(bx^2+a)^2}+\frac{5\,cx}{8\,(bx^2+a)^2\,a}-\frac{15\,af}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,e}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{d}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] f*x/b^3+9/8/b^2/(b*x^2+a)^2*x^3*a*f-5/8/b/(b*x^2+a)^2*x^3*e+1/8/(
b*x^2+a)^2/a*x^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c+7/8/b^3/(b*x^2+a)^
2*a^2*f*x-3/8/b^2/(b*x^2+a)^2*a*e*x-1/8/b/(b*x^2+a)^2*d*x+5/8/(b*
x^2+a)^2/a*x*c-15/8/b^3*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+3
/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e+1/8/b/a/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*d+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239641, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+\left(3\,b^5c+ab^4d+3\,a^2b^3e-15\,a^3b^2f\right)x^4+2\left(3\,ab^4c+a^2b^3d+3\,a^3b^2e-15\,a^4bf\right)x^2\right)}{16\,(a^2b^5x^4+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*
c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^
2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*log(-(2*a*b*x - (b*x^2 -
 a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(8*a^2*b^2*f*x^5 + (3*b^4*c + a*
b^3*d - 5*a^2*b^2*e + 25*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*b^2*d -
3*a^3*b*e + 15*a^4*f)*x)*sqrt(-a*b))/((a^2*b^5*x^4 + 2*a^3*b^4*x^
```

```
2 + a^4*b^3)*sqrt(-a*b)), 1/8*((3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b
*e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*
x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*a
rctan(sqrt(a*b)*x/a) + (8*a^2*b^2*f*x^5 + (3*b^4*c + a*b^3*d - 5*
a^2*b^2*e + 25*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*b^2*d - 3*a^3*b*e
+ 15*a^4*f)*x)*sqrt(a*b))/((a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3
)*sqrt(a*b))]
```

Sympy [A] time = 15.3698, size = 243, normalized size = 1.65

$$\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}+\frac{x^3\left(9a^3bf-5a^2b^2e+ab^3d+3b^4c\right)+x\left(7a^4f-3a^3be-a^2b^2d+5ab^3c\right)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{fx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*
c)*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b*
*7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3
*sqrt(-1/(a**5*b**7)) + x)/16 + (x**3*(9*a**3*b*f - 5*a**2*b**2*e
 + a*b**3*d + 3*b**4*c) + x*(7*a**4*f - 3*a**3*b*e - a**2*b**2*d
+ 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**5*x**
4) + f*x/b**3
```

GIAC/XCAS [A] time = 0.218685, size = 201, normalized size = 1.37

$$\frac{fx}{b^3}+\frac{\left(3\,b^3c+ab^2d-15\,a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}+\frac{3\,b^4cx^3+ab^3dx^3+9\,a^3bfx^3-5\,a^2b^2x^3e+5\,ab^3cx-a^2b^2dx+7\,a^4fx-3\,a^3bxe}{8\,(bx^2+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^
3 + 9*a^3*b*f*x^3 - 5*a^2*b^2*x^3*e + 5*a*b^3*c*x - a^2*b^2*d*x +
 7*a^4*f*x - 3*a^3*b*x*e)/((b*x^2 + a)^2*a^2*b^3)
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx$

Optimal. Leaf size=150

$$-\frac{c}{a^3x}-\frac{x\left(\frac{5a^2f}{b^2}+\frac{7bc}{a}-\frac{ae}{b}-3d\right)}{8a^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f-a^2be-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}$$

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a
+ b*x^2)^2) - (((7*b*c)/a - 3*d - (a*e)/b + (5*a^2*f)/b^2)*x)/(8*
a^2*(a + b*x^2)) - ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*b^(5/2))
```

Rubi [A] time = 0.464153, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{c}{a^3x}-\frac{x\left(\frac{5a^2f}{b^2}+\frac{7bc}{a}-\frac{ae}{b}-3d\right)}{8a^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f-a^2be-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a
+ b*x^2)^2) - (((7*b*c)/a - 3*d - (a*e)/b + (5*a^2*f)/b^2)*x)/(8*
a^2*(a + b*x^2)) - ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*b^(5/2))
```

Rubi in Sympy [A] time = 151.552, size = 160, normalized size = 1.07

$$-\frac{x\left(\frac{a^3f}{x^2}-\frac{a^2be}{x^2}+\frac{ab^2d}{x^2}-\frac{b^3c}{x^2}\right)}{4ab^3\left(a+bx^2\right)^2}-\frac{x\left(3a^2f-2abe+b^2d\right)}{2a^2b^2\left(a+bx^2\right)}-\frac{a^2f-abe+b^2d}{a^2b^3x}-\frac{\left(3a^2f-4abe+3b^2d\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -x*(a**3*f/x**2 - a**2*b*e/x**2 + a*b**2*d/x**2 - b**3*c/x**2)/(4
*a*b**3*(a + b*x**2)**2) - x*(3*a**2*f - 2*a*b*e + b**2*d)/(2*a**
2*b**2*(a + b*x**2)) - (a**2*f - a*b*e + b**2*d)/(a**2*b**3*x) -
(3*a**2*f - 4*a*b*e + 3*b**2*d)*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/
2)*b**(5/2))
```

Mathematica [A] time = 0.236805, size = 155, normalized size = 1.03

$$-\frac{c}{a^3x}-\frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^2b^2\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f+a^2be+3ab^2d-15b^3c\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*
b^2*(a + b*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x
)/(8*a^3*b^2*(a + b*x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3
*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*b^(5/2))
```

Maple [A] time = 0.02, size = 237, normalized size = 1.6

$$-\frac{c}{a^3x}-\frac{5\,x^3f}{8\,(bx^2+a)^2\,b}+\frac{x^3e}{8\,a\,(bx^2+a)^2}+\frac{3\,x^3bd}{8\,a^2\,(bx^2+a)^2}-\frac{7\,x^3b^2c}{8\,a^3\,(bx^2+a)^2}-\frac{3\,axf}{8\,(bx^2+a)^2\,b^2}-\frac{ex}{8\,(bx^2+a)^2\,b}+\frac{5\,dx}{8\,a\,(bx^2+a)^2}-\frac{9\,bxc}{8\,a^2\,(bx^2+a)^2}+\frac{3\,f}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{e}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,d}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,bc}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -c/a^3/x-5/8/(b*x^2+a)^2/b*x^3*f+1/8/a/(b*x^2+a)^2*x^3*e+3/8/a^2/
(b*x^2+a)^2*x^3*b*d-7/8/a^3/(b*x^2+a)^2*x^3*b^2*c-3/8*a/(b*x^2+a)
^2/b^2*x*f-1/8/(b*x^2+a)^2/b*x*e+5/8/a/(b*x^2+a)^2*x*d-9/8/a^2/(b
*x^2+a)^2*x*b*c+3/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+1/8
/a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*e+3/8/a^2/(a*b)^(1/2)*ar
ctan(x*b/(a*b)^(1/2))*d-15/8/a^3*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238737, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(15\,b^5c-3\,ab^4d-a^2b^3e-3\,a^3b^2f\right)x^5+2\left(15\,ab^4c-3\,a^2b^3d-a^3b^2e-3\,a^4bf\right)x^3+\left(15\,a^2b^3c-3\,a^3b^2d-a^4be-3\,a^5\right.}{16\left(a^3b^4x^5+2\right.}\right.$$

$$-\frac{\left(\left(15\,b^5c-3\,ab^4d-a^2b^3e-3\,a^3b^2f\right)x^5+2\left(15\,ab^4c-3\,a^2b^3d-a^3b^2e-3\,a^4bf\right)x^3+\left(15\,a^2b^3c-3\,a^3b^2d-a^4be-3\,a^5f\right.}{8\left(a^3b^4x^5+2\,a^4b^3x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2
*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2
```

```
*b^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*log((2*a*b*x + (b*x^
2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(8*a^2*b^2*c + (15*b^4*c - 3*
a*b^3*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (25*a*b^3*c - 5*a^2*b^2*d
+ a^3*b*e + 3*a^4*f)*x^2)*sqrt(-a*b))/((a^3*b^4*x^5 + 2*a^4*b^3*x
^3 + a^5*b^2*x)*sqrt(-a*b)), -1/8*(((15*b^5*c - 3*a*b^4*d - a^2*b
^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e
 - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5
*f)*x)*arctan(sqrt(a*b)*x/a) + (8*a^2*b^2*c + (15*b^4*c - 3*a*b^3
*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (25*a*b^3*c - 5*a^2*b^2*d + a^3
*b*e + 3*a^4*f)*x^2)*sqrt(a*b))/((a^3*b^4*x^5 + 2*a^4*b^3*x^3 + a
^5*b^2*x)*sqrt(a*b))]
```

Sympy [A] time = 40.3625, size = 250, normalized size = 1.67

$$-\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}-\frac{8a^2b^2c+x^4\left(5a^3bf-a^2b^2e-3ab^3d+15b^4c\right)+x^2\left(3a^4f+a^3be-5a^2b^2d+25ab^3c\right)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3
*c)*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b
**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**
2*sqrt(-1/(a**7*b**5)) + x)/16 - (8*a**2*b**2*c + x**4*(5*a**3*b*
f - a**2*b**2*e - 3*a*b**3*d + 15*b**4*c) + x**2*(3*a**4*f + a**3
*b*e - 5*a**2*b**2*d + 25*a*b**3*c))/(8*a**5*b**2*x + 16*a**4*b**
3*x**3 + 8*a**3*b**4*x**5)
```

GIAC/XCAS [A] time = 0.217715, size = 207, normalized size = 1.38

$$-\frac{c}{a^3x}-\frac{\left(15\,b^3c-3\,ab^2d-3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}-\frac{7\,b^4cx^3-3\,ab^3dx^3+5\,a^3bfx^3-a^2b^2x^3e+9\,ab^3cx-5\,a^2b^2dx+3\,a^4fx+a^3bxe}{8\,(bx^2+a)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3
*d*x^3 + 5*a^3*b*f*x^3 - a^2*b^2*x^3*e + 9*a*b^3*c*x - 5*a^2*b^2*
d*x + 3*a^4*f*x + a^3*b*x*e)/((b*x^2 + a)^2*a^3*b^2)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{x\left(a^3f+3a^2be-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}$$

```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a
+ e - (a*f)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d +
3*a^2*b*e + a^3*f)*x)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b
^2*d + 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)
*b^(3/2))
```

Rubi [A] time = 0.556753, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{x\left(a^3f+3a^2be-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a
+ e - (a*f)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d +
3*a^2*b*e + a^3*f)*x)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b
^2*d + 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)
*b^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.293769, size = 169, normalized size = 1.01

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{-3a^4fx^4+a^3b\left(3x^2\left(-8d+5ex^2+fx^4\right)-8c\right)+a^2b^2x^2\left(56c-75dx^2+9ex^4\right)+5ab^3x^4\left(35c-9dx^2\right)+105b^4cx^6}{24a^4bx^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^
2*b^2*x^2*(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d
 + 5*e*x^2 + f*x^4)))/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c -
 15*a*b^2*d + 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*
a^(9/2)*b^(3/2))
```

Maple [A] time = 0.022, size = 264, normalized size = 1.6

$$-\frac{c}{3a^3x^3}-\frac{d}{a^3x}+3\frac{bc}{a^4x}+\frac{x^3f}{8a(bx^2+a)^2}+\frac{3x^3be}{8a^2(bx^2+a)^2}-\frac{7x^3b^2d}{8a^3(bx^2+a)^2}+\frac{11x^3b^3c}{8a^4(bx^2+a)^2}-\frac{fx}{8(bx^2+a)^2b}+\frac{5ex}{8a(bx^2+a)^2}-\frac{9bxd}{8a^2(bx^2+a)^2}+\frac{13xb^2c}{8a^3(bx^2+a)^2}+\frac{f}{8ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3e}{8a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15bd}{8a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35b^2c}{8a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3*c/a^3/x^3-1/a^3/x*d+3/a^4/x*b*c+1/8/a/(b*x^2+a)^2*x^3*f+3/8/
a^2/(b*x^2+a)^2*x^3*b*e-7/8/a^3/(b*x^2+a)^2*x^3*b^2*d+11/8/a^4/(b
*x^2+a)^2*x^3*b^3*c-1/8/(b*x^2+a)^2*x/b*f+5/8/a/(b*x^2+a)^2*x*e-9
/8/a^2/(b*x^2+a)^2*x*b*d+13/8/a^3/(b*x^2+a)^2*x*b^2*c+1/8/a/b/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+3/8/a^2/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))*e-15/8/a^3*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+
35/8/a^4*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240968, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(35\,b^5c-15\,ab^4d+3\,a^2b^3e+a^3b^2f\right)x^7+2\left(35\,ab^4c-15\,a^2b^3d+3\,a^3b^2e+a^4bf\right)x^5+\left(35\,a^2b^3c-15\,a^3b^2d+3\,a^4be+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^4),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3*e + a^3*b^2*f)*x^7 +
 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)*x^5 + (35*
a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*log((2*a*b*x +
 (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(3*(35*b^4*c - 15*a*b^3
*d + 3*a^2*b^2*e + a^3*b*f)*x^6 - 8*a^3*b*c + (175*a*b^3*c - 75*a
^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c - 3*a^3*b*d
)*x^2)*sqrt(-a*b))/((a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3)*sqr
t(-a*b)), 1/24*(3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3*e + a^3*b^2
*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)*x
^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*arcta
```

```
n(sqrt(a*b)*x/a) + (3*(35*b^4*c - 15*a*b^3*d + 3*a^2*b^2*e + a^3*
b*f)*x^6 - 8*a^3*b*c + (175*a*b^3*c - 75*a^2*b^2*d + 15*a^3*b*e -
 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c - 3*a^3*b*d)*x^2)*sqrt(a*b))/((a^4
*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3)*sqrt(a*b))]
```

Sympy [A] time = 99.1656, size = 270, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{-8a^3bc+x^6\left(3a^3bf+9a^2b^2e-45ab^3d+105b^4c\right)+x^4\left(-3a^4f+15a^3be-75a^2b^2d+175ab^3c\right)+x^2\left(-24a^3bd+56a^2b^2c\right)}{24a^6bx^3+48a^5b^2x^5+24a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2*b*e - 15*a*b**2*d + 35*b**
3*c)*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + sqrt(-1/(a**9*b**
3))*(a**3*f + 3*a**2*b*e - 15*a*b**2*d + 35*b**3*c)*log(a**5*b*sq
rt(-1/(a**9*b**3)) + x)/16 + (-8*a**3*b*c + x**6*(3*a**3*b*f + 9*
a**2*b**2*e - 45*a*b**3*d + 105*b**4*c) + x**4*(-3*a**4*f + 15*a*
*3*b*e - 75*a**2*b**2*d + 175*a*b**3*c) + x**2*(-24*a**3*b*d + 56
*a**2*b**2*c))/(24*a**6*b*x**3 + 48*a**5*b**2*x**5 + 24*a**4*b**3
*x**7)
```

GIAC/XCAS [A] time = 0.220755, size = 230, normalized size = 1.37

$$\frac{\left(35\,b^3c-15\,ab^2d+a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b}+\frac{11\,b^4cx^3-7\,ab^3dx^3+a^3bfx^3+3\,a^2b^2x^3e+13\,ab^3cx-9\,a^2b^2dx-a^4fx+5\,a^3bxe}{8\,(bx^2+a)^2a^4b}+\frac{9\,bcx^2-3\,adx^2-ac}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^4),x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + a^3*
b*f*x^3 + 3*a^2*b^2*x^3*e + 13*a*b^3*c*x - 9*a^2*b^2*d*x - a^4*f*
x + 5*a^3*b*x*e)/((b*x^2 + a)^2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x
^2 - a*c)/(a^4*x^3)
```

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3}\,dx$

Optimal. Leaf size=196

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*
(a + b*x^2)^2) - ((15*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x
)/(8*a^5*(a + b*x^2)) - ((63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2)*Sqrt[b])
```

Rubi [A] time = 0.667145, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*
(a + b*x^2)^2) - ((15*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x
)/(8*a^5*(a + b*x^2)) - ((63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2)*Sqrt[b])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.228875, size = 196, normalized size = 1.

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}+\frac{a^2(-e)+3abd-6b^2c}{a^5x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)}{8a^{11/2}\sqrt{b}}+\frac{x\left(3a^3f-7a^2be+11ab^2d-15b^3c\right)}{8a^5\left(a+bx^2\right)}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d
- a^2*e)/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*
a^4*(a + b*x^2)^2) + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3
*f)*x)/(8*a^5*(a + b*x^2)) + ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*
e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2)*Sqrt[b])
```

Maple [A] time = 0.024, size = 300, normalized size = 1.5

$$-\frac{c}{5\,a^3x^5}-\frac{d}{3\,a^3x^3}+\frac{bc}{a^4x^3}-\frac{e}{a^3x}+3\,\frac{bd}{a^4x}-6\,\frac{b^2c}{a^5x}+\frac{3\,x^3bf}{8\,a^2\,(bx^2+a)^2}-\frac{7\,x^3b^2e}{8\,a^3\,(bx^2+a)^2}+\frac{11\,x^3b^3d}{8\,a^4\,(bx^2+a)^2}-\frac{15\,x^3b^4c}{8\,a^5\,(bx^2+a)^2}+\frac{5\,fx}{8\,a\,(bx^2+a)^2}-\frac{9\,bex}{8\,a^2\,(bx^2+a)^2}+\frac{13\,db^2x}{8\,a^3\,(bx^2+a)^2}-\frac{17\,b^3cx}{8\,a^4\,(bx^2+a)^2}+\frac{3\,f}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,be}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,db^2}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,b^3c}{8\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5*c/a^3/x^5-1/3/a^3/x^3*d+1/a^4/x^3*b*c-1/a^3/x*e+3/a^4/x*b*d-
6/a^5/x*b^2*c+3/8/a^2/(b*x^2+a)^2*x^3*b*f-7/8/a^3/(b*x^2+a)^2*x^3
*b^2*e+11/8/a^4/(b*x^2+a)^2*x^3*b^3*d-15/8/a^5/(b*x^2+a)^2*x^3*b^
4*c+5/8/a/(b*x^2+a)^2*f*x-9/8/a^2/(b*x^2+a)^2*b*e*x+13/8/a^3/(b*x
^2+a)^2*b^2*d*x-17/8/a^4/(b*x^2+a)^2*b^3*c*x+3/8/a^2/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*f-15/8/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*b*e+35/8/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^2*d-63/8
/a^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^3*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236501, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(\left(63\,b^5c-35\,ab^4d+15\,a^2b^3e-3\,a^3b^2f\right)x^9+2\left(63\,ab^4c-35\,a^2b^3d+15\,a^3b^2e-3\,a^4bf\right)x^7+\left(63\,a^2b^3c-35\,a^3b^2d+\right.}{}\right.$$

$$-\frac{15\left(\left(63\,b^5c-35\,ab^4d+15\,a^2b^3e-3\,a^3b^2f\right)x^9+2\left(63\,ab^4c-35\,a^2b^3d+15\,a^3b^2e-3\,a^4bf\right)x^7+\left(63\,a^2b^3c-35\,a^3b^2d+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^6),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)
*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x
```

```
^7 + (63*a^2*b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*lo
g((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15*(63*b^4
*c - 35*a*b^3*d + 15*a^2*b^2*e - 3*a^3*b*f)*x^8 + 25*(63*a*b^3*c
- 35*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2
*b^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2
)*sqrt(-a*b))/((a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)*sqrt(-a*b)),
 -1/120*(15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)
*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x
^7 + (63*a^2*b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*ar
ctan(sqrt(a*b)*x/a) + (15*(63*b^4*c - 35*a*b^3*d + 15*a^2*b^2*e -
 3*a^3*b*f)*x^8 + 25*(63*a*b^3*c - 35*a^2*b^2*d + 15*a^3*b*e - 3*
a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2*b^2*c - 35*a^3*b*d + 15*a^4*e)*
x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)*sqrt(a*b))/((a^5*b^2*x^9 + 2*a
^6*b*x^7 + a^7*x^5)*sqrt(a*b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216964, size = 267, normalized size = 1.36

$$-\frac{\left(63\,b^3c-35\,ab^2d-3\,a^3f+15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}-\frac{15\,b^4cx^3-11\,ab^3dx^3-3\,a^3bfx^3+7\,a^2b^2x^3e+17\,ab^3cx-13\,a^2b^2dx-5\,a^4fx+9\,a^3bxe}{8\,(bx^2+a)^2a^5}-\frac{90\,b^2cx^4-45\,abdx^4+15\,a^2x^4e-15\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^6),x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2*b*e)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 - 3
*a^3*b*f*x^3 + 7*a^2*b^2*x^3*e + 17*a*b^3*c*x - 13*a^2*b^2*d*x -
5*a^4*f*x + 9*a^3*b*x*e)/((b*x^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4
 - 45*a*b*d*x^4 + 15*a^2*x^4*e - 15*a*b*c*x^2 + 5*a^2*d*x^2 + 3*a
^2*c)/(a^5*x^5)
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx$

Optimal. Leaf size=234

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2be-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}$$

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b
*x^2)^2) + (b*(19*b^3*c - 15*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(
8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c - 63*a*b^2*d + 35*a^2*b*e
 - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rubi [A] time = 0.912121, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2be-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b
*x^2)^2) + (b*(19*b^3*c - 15*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(
8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c - 63*a*b^2*d + 35*a^2*b*e
 - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.253282, size = 234, normalized size = 1.

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2be-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b
*x^2)^2) + (b*(19*b^3*c - 15*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(
8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c - 63*a*b^2*d + 35*a^2*b*e
 - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Maple [A] time = 0.026, size = 351, normalized size = 1.5

$$-\frac{c}{7\,a^3x^7}-\frac{d}{5\,a^3x^5}+\frac{3\,bc}{5\,a^4x^5}-\frac{e}{3\,a^3x^3}+\frac{bd}{a^4x^3}-2\,\frac{b^2c}{a^5x^3}-\frac{f}{a^3x}+3\,\frac{be}{a^4x}-6\,\frac{db^2}{a^5x}+10\,\frac{b^3c}{a^6x}-\frac{7\,b^2x^3f}{8\,a^3\left(bx^2+a\right)^2}+\frac{11\,b^3x^3e}{8\,a^4\left(bx^2+a\right)^2}-\frac{15\,b^4x^3d}{8\,a^5\left(bx^2+a\right)^2}+\frac{19\,b^5x^3c}{8\,a^6\left(bx^2+a\right)^2}-\frac{9\,fbx}{8\,a^2\left(bx^2+a\right)^2}+\frac{13\,b^2ex}{8\,a^3\left(bx^2+a\right)^2}-\frac{17\,db^3x}{8\,a^4\left(bx^2+a\right)^2}+\frac{21\,b^4cx}{8\,a^5\left(bx^2+a\right)^2}-\frac{15\,fb}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,b^2e}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,db^3}{8\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{99\,b^4c}{8\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x)
```

```
[Out] -1/7*c/a^3/x^7-1/5/a^3/x^5*d+3/5/a^4/x^5*b*c-1/3/a^3/x^3*e+1/a^4/
x^3*b*d-2/a^5/x^3*b^2*c-1/a^3/x*f+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^
6/x*b^3*c-7/8*b^2/a^3/(b*x^2+a)^2*x^3*f+11/8*b^3/a^4/(b*x^2+a)^2*
x^3*e-15/8*b^4/a^5/(b*x^2+a)^2*x^3*d+19/8*b^5/a^6/(b*x^2+a)^2*x^3
*c-9/8*b/a^2/(b*x^2+a)^2*f*x+13/8*b^2/a^3/(b*x^2+a)^2*e*x-17/8*b^
3/a^4/(b*x^2+a)^2*d*x+21/8*b^4/a^5/(b*x^2+a)^2*c*x-15/8*b/a^3/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f+35/8*b^2/a^4/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))*e-63/8*b^3/a^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*d+99/8*b^4/a^6/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24664, size = 1, normalized size = 0.

$$\left[\frac{210\left(99\,b^5c - 63\,ab^4d + 35\,a^2b^3e - 15\,a^3b^2f\right)x^{10} + 350\left(99\,ab^4c - 63\,a^2b^3d + 35\,a^3b^2e - 15\,a^4bf\right)x^8 + 112\left(99\,a^2b^3c - 63\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^8),x, algorithm="fricas")
```

```
[Out] [1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f
)*x^10 + 350*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b
*f)*x^8 + 112*(99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*
f)*x^6 - 240*a^5*c - 16*(99*a^3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^
4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 - 105*((99*b^5*c - 63*a*b^4*d +
 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d
 + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d
+ 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(
-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7),
1/840*(105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*
x^10 + 175*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f
)*x^8 + 56*(99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*
x^6 - 120*a^5*c - 8*(99*a^3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 +
24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^5*c - 63*a*b^4*d + 35*
a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d + 3
5*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d + 35
*a^4*b*e - 15*a^5*f)*x^7)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a
^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.218883, size = 338, normalized size = 1.44

$$\frac{\left(99\,b^4c - 63\,ab^3d - 15\,a^3bf + 35\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6} + \frac{19\,b^5cx^3 - 15\,ab^4dx^3 - 7\,a^3b^2fx^3 + 11\,a^2b^3x^3e + 21\,ab^4cx - 17\,a^2b^3dx - 9\,a^4bfx + 13\,a^3b^2xe}{8\,(bx^2 + a)^2a^6} + \frac{1050\,b^3cx^6 - 630\,ab^2dx^6 - 105\,a^3fx^6 + 315\,a^2bx^6e - 210\,ab^2cx^4 + 105\,a^2bdx^4 - 35\,a^3x^4e + 63\,a^2bcx^2 - 21\,a^3dx^2 - 15\,a^3c}{105\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^8),x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 35*a^2*b^2*e)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3
 - 7*a^3*b^2*f*x^3 + 11*a^2*b^3*x^3*e + 21*a*b^4*c*x - 17*a^2*b^3
*d*x - 9*a^4*b*f*x + 13*a^3*b^2*x*e)/((b*x^2 + a)^2*a^6) + 1/105*
(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 - 105*a^3*f*x^6 + 315*a^2*b*x^6
*e - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*x^4*e + 63*a^2*b*
c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx$

Optimal. Leaf size=277

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x\left(-11a^3f+15a^2be-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}$$

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(3*a^6*x^3) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(
a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^6*(a +
b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d + 15*a^2*b*e - 11*a^3*f)*
x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*a*b^2*d + 63*a^
2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Rubi [A] time = 1.16117, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x\left(-11a^3f+15a^2be-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(3*a^6*x^3) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(
a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^6*(a +
b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d + 15*a^2*b*e - 11*a^3*f)*
x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*a*b^2*d + 63*a^
2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.284251, size = 276, normalized size = 1.

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^3f-63a^2be+99ab^2d-143b^3c\right)}{8a^{15/2}}+\frac{b^2x\left(11a^3f-15a^2be+19ab^2d-23b^3c\right)}{8a^7\left(a+bx^2\right)}+\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}+\frac{b^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^6\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d +
 a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/
(3*a^6*x^3) + (b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/
(a^7*x) + (b^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(
a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a*b^2*d - 15*a^2*b*e + 11*a^
3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3*c + 99*a*b^2*d -
 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Maple [A] time = 0.028, size = 401, normalized size = 1.5

$$\frac{21\,db^4x}{8\,a^5\left(bx^2+a\right)^2}-\frac{25\,b^5cx}{8\,a^6\left(bx^2+a\right)^2}-\frac{d}{7\,a^3x^7}-\frac{e}{5\,a^3x^5}-\frac{f}{3\,a^3x^3}-\frac{c}{9\,a^3x^9}+\frac{11\,b^3x^3f}{8\,a^4\left(bx^2+a\right)^2}-\frac{15\,b^4x^3e}{8\,a^5\left(bx^2+a\right)^2}-\frac{17\,b^3ex}{8\,a^4\left(bx^2+a\right)^2}+3\,\frac{fb}{a^4x}-6\,\frac{eb^2}{a^5x}+10\,\frac{db^3}{a^6x}-15\,\frac{b^4c}{a^7x}+\frac{3\,bc}{7\,a^4x^7}+\frac{3\,bd}{5\,a^4x^5}-\frac{6\,b^2c}{5\,a^5x^5}+\frac{be}{a^4x^3}-2\,\frac{db^2}{a^5x^3}+\frac{10\,b^3c}{3\,a^6x^3}+\frac{19\,b^5x^3d}{8\,a^6\left(bx^2+a\right)^2}-\frac{23\,b^6x^3c}{8\,a^7\left(bx^2+a\right)^2}+\frac{13\,fb^2x}{8\,a^3\left(bx^2+a\right)^2}+\frac{35\,fb^2}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,b^3e}{8\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{99\,db^4}{8\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{143\,b^5c}{8\,a^7}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x)
```

```
[Out] 21/8*b^4/a^5/(b*x^2+a)^2*d*x-25/8*b^5/a^6/(b*x^2+a)^2*c*x-1/7/a^3
/x^7*d-1/5/a^3/x^5*e-1/3/a^3/x^3*f-1/9*c/a^3/x^9+11/8*b^3/a^4/(b*
x^2+a)^2*x^3*f-15/8*b^4/a^5/(b*x^2+a)^2*x^3*e-17/8*b^3/a^4/(b*x^2
+a)^2*e*x+3*b/a^4/x*f-6*b^2/a^5/x*e+10*b^3/a^6/x*d-15*b^4/a^7/x*c
+3/7/a^4/x^7*b*c+3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+1/a^4/x^3*b*e-
2/a^5/x^3*b^2*d+10/3/a^6/x^3*b^3*c+19/8*b^5/a^6/(b*x^2+a)^2*x^3*d
-23/8*b^6/a^7/(b*x^2+a)^2*x^3*c+13/8*b^2/a^3/(b*x^2+a)^2*f*x+35/8
*b^2/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*f-63/8*b^3/a^5/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))*e+99/8*b^4/a^6/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))*d-143/8*b^5/a^7/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^10),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23619, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^10),x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3
*f)*x^12 + 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a
^4*b^2*f)*x^10 + 336*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e
 - 35*a^5*b*f)*x^8 + 560*a^6*c - 48*(143*a^3*b^3*c - 99*a^4*b^2*d
 + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(143*a^4*b^2*c - 99*a^5*b*d +
63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2 + 315*((143*b^6*c -
 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c
- 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4
*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*lo
g((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*
a^8*b*x^11 + a^9*x^9), -1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*
a^2*b^4*e - 35*a^3*b^3*f)*x^12 + 525*(143*a*b^5*c - 99*a^2*b^4*d
+ 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(143*a^2*b^4*c - 99*a^3
*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c - 24*(143*a^3
*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4*b
^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2
 + 315*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^
13 + 2*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)
*x^11 + (143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f
)*x^9)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^7*b^2*x^13 + 2*a^8
*b*x^11 + a^9*x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216141, size = 406, normalized size = 1.47

$$-\frac{\left(143\,b^5c-99\,ab^4d-35\,a^3b^2f+63\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^7}-\frac{23\,b^6cx^3-19\,ab^5dx^3-11\,a^3b^3fx^3+15\,a^2b^4x^3e+25\,ab^5cx-21\,a^2b^4dx-13\,a^4b^2fx+17\,a^3b^3xe}{8\left(bx^2+a\right)^2a^7}-\frac{4725\,b^4cx^8-3150\,ab^3dx^8-945\,a^3bfx^8+1890\,a^2b^2x^8e-1050\,ab^3cx^6+630\,a^2b^2dx^6+105\,a^4fx^6-315\,a^3bx^6e+378\,a^2}{315\,a^7x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/((b*x^2 + a)^3*x^10),x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^2*f + 63*a^2*b^3*e)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d
*x^3 - 11*a^3*b^3*f*x^3 + 15*a^2*b^4*x^3*e + 25*a*b^5*c*x - 21*a^
2*b^4*d*x - 13*a^4*b^2*f*x + 17*a^3*b^3*x*e)/((b*x^2 + a)^2*a^7)
```

```
- 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x^8 - 945*a^3*b*f*x^8 + 18
90*a^2*b^2*x^8*e - 1050*a*b^3*c*x^6 + 630*a^2*b^2*d*x^6 + 105*a^4
*f*x^6 - 315*a^3*b*x^6*e + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x^4 +
63*a^4*x^4*e - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^
9)
```

3.143 $\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{(a+bx^2)^{7/2}(10a^2f-4abe+b^2d)}{7b^6}+\frac{(a+bx^2)^{5/2}(-10a^3f+6a^2be-3ab^2d+b^3c)}{5b^6}-\frac{a(a+bx^2)^{3/2}(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{a^2\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}+\frac{(a+bx^2)^{9/2}(be-5af)}{9b^6}+\frac{f(a+bx^2)^{11/2}}{11b^6}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (
a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/
(3*b^6) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)
^(5/2))/(5*b^6) + ((b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2)
)/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(9/2))/(9*b^6) + (f*(a + b
*x^2)^(11/2))/(11*b^6)
```

Rubi [A] time = 0.464928, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{(a+bx^2)^{7/2}(10a^2f-4abe+b^2d)}{7b^6}+\frac{(a+bx^2)^{5/2}(-10a^3f+6a^2be-3ab^2d+b^3c)}{5b^6}-\frac{a(a+bx^2)^{3/2}(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{a^2\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}+\frac{(a+bx^2)^{9/2}(be-5af)}{9b^6}+\frac{f(a+bx^2)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (
a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/
(3*b^6) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)
^(5/2))/(5*b^6) + ((b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2)
)/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(9/2))/(9*b^6) + (f*(a + b
*x^2)^(11/2))/(11*b^6)
```

Rubi in Sympy [A] time = 85.8513, size = 206, normalized size = 0.96

$$-\frac{a^2\sqrt{a+bx^2}(a^3f-a^2be+ab^2d-b^3c)}{b^6}+\frac{a(a+bx^2)^{\frac{3}{2}}(5a^3f-4a^2be+3ab^2d-2b^3c)}{3b^6}+\frac{f(a+bx^2)^{\frac{11}{2}}}{11b^6}-\frac{(a+bx^2)^{\frac{9}{2}}(5af-be)}{9b^6}+\frac{(a+bx^2)^{\frac{7}{2}}(10a^2f-4abe+b^2d)}{7b^6}-\frac{(a+bx^2)^{\frac{5}{2}}(10a^3f-6a^2be+3ab^2d-b^3c)}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -a**2*sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b*
*6 + a*(a + b*x**2)**(3/2)*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d -
2*b**3*c)/(3*b**6) + f*(a + b*x**2)**(11/2)/(11*b**6) - (a + b*x*
*2)**(9/2)*(5*a*f - b*e)/(9*b**6) + (a + b*x**2)**(7/2)*(10*a**2*
f - 4*a*b*e + b**2*d)/(7*b**6) - (a + b*x**2)**(5/2)*(10*a**3*f -
```

```
6*a**2*b*e + 3*a*b**2*d - b**3*c)/(5*b**6)
```

Mathematica [A] time = 0.220062, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-2\cdots\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a
^3*b^2*(99*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2
 + 66*e*x^4 + 50*f*x^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*
x^4 + 175*f*x^6) + b^5*x^4*(693*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f
*x^6))))/(3465*b^6)
```

Maple [A] time = 0.01, size = 193, normalized size = 0.9

$$-\frac{-315\,fx^{10}b^5+350\,ab^4fx^8-385\,b^5ex^8-400\,a^2b^3fx^6+440\,ab^4ex^6-495\,b^5dx^6+480\,a^3b^2fx^4-528\,a^2b^3ex^4+594\,ab^4d\cdots}{346\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*
e*x^8-400*a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2
*f*x^4-528*a^2*b^3*e*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*
f*x^2+704*a^3*b^2*e*x^2-792*a^2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^
5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2*b^3*c)/b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^5/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256504, size = 239, normalized size = 1.12

$$\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4be-1280\cdots\right.}{3465\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^5/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b
^5*d - 88*a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3
*b^2*d + 1408*a^4*b*e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d +
 176*a^2*b^3*e - 160*a^3*b^2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^
```

```
3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*sqrt(b*x^2 + a)/b^6
```

Sympy [A] time = 6.80497, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} + \frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3} \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*s
qrt(a + b*x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(69
3*b**5) - 16*a**3*d*sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*s
qrt(a + b*x**2)/(315*b**4) - 32*a**3*f*x**4*sqrt(a + b*x**2)/(231
*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b**3) + 8*a**2*d*x**2*sqrt
(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b*x**2)/(105*b**
3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2*sqrt
(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) -
8*a*e*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x*
*2)/(99*b**2) + c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b
*x**2)/(7*b) + e*x**8*sqrt(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b
*x**2)/(11*b), Ne(b, 0)), ((c*x**6/6 + d*x**8/8 + e*x**10/10 + f*
x**12/12)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.219536, size = 387, normalized size = 1.81

$$\frac{693\left(bx^2+a\right)^{\frac{5}{2}}b^3c - 2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c + 3465\sqrt{bx^2+a}a^2b^3c + 495\left(bx^2+a\right)^{\frac{7}{2}}b^2d - 2079\left(bx^2+a\right)^{\frac{5}{2}}ab^2d + 3465\left(bx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^5/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/3465*(693*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^
3*c + 3465*sqrt(b*x^2 + a)*a^2*b^3*c + 495*(b*x^2 + a)^(7/2)*b^2*
d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*b
^2*d - 3465*sqrt(b*x^2 + a)*a^3*b^2*d + 315*(b*x^2 + a)^(11/2)*f
- 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f - 693
0*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f - 3465*s
qrt(b*x^2 + a)*a^5*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 +
a)^(7/2)*a*b*e + 4158*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a
)^(3/2)*a^3*b*e + 3465*sqrt(b*x^2 + a)*a^4*b*e)/b^6
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{7/2}(be-4af)}{7b^5}+\frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) +
((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*
b^5) + ((b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) +
((b*e - 4*a*f)*(a + b*x^2)^(7/2))/(7*b^5) + (f*(a + b*x^2)^(9/2))
/(9*b^5)
```

Rubi [A] time = 0.354669, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{7/2}(be-4af)}{7b^5}+\frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) +
((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*
b^5) + ((b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) +
((b*e - 4*a*f)*(a + b*x^2)^(7/2))/(7*b^5) + (f*(a + b*x^2)^(9/2))
/(9*b^5)
```

Rubi in Sympy [A] time = 62.4364, size = 156, normalized size = 0.93

$$\frac{a\sqrt{a+bx^2}\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^5}+\frac{f\left(a+bx^2\right)^{\frac{9}{2}}}{9b^5}-\frac{\left(a+bx^2\right)^{\frac{7}{2}}(4af-be)}{7b^5}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}\left(6a^2f-3abe+b^2d\right)}{5b^5}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(4a^3f-3a^2be+2ab^2d-b^3c\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] a*sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b**5 +
 f*(a + b*x**2)**(9/2)/(9*b**5) - (a + b*x**2)**(7/2)*(4*a*f - b*
e)/(7*b**5) + (a + b*x**2)**(5/2)*(6*a**2*f - 3*a*b*e + b**2*d)/(
5*b**5) - (a + b*x**2)**(3/2)*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d
 - b**3*c)/(3*b**5)
```

Mathematica [A] time = 0.151845, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(105c\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b
^2*(7*d + 3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x
^4 + 20*f*x^6) + b^4*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)
))/(315*b^5)
```

Maple [A] time = 0.01, size = 145, normalized size = 0.9

$$\frac{35\,f x^8 b^4 - 40\,ab^3 f x^6 + 45\,b^4 e x^6 + 48\,a^2 b^2 f x^4 - 54\,ab^3 e x^4 + 63\,b^4 d x^4 - 64\,a^3 b f x^2 + 72\,a^2 b^2 e x^2 - 84\,ab^3 d x^2 + 105\,b^4 c x^2 +}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+4
8*a^2*b^2*f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2
*b^2*e*x^2-84*a*b^3*d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168
*a^2*b^2*d-210*a*b^3*c)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^3/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249499, size = 181, normalized size = 1.08

$$\frac{\left(35\,b^4 f x^8 + 5\left(9\,b^4 e - 8\,ab^3 f\right)x^6 - 210\,ab^3 c + 168\,a^2 b^2 d - 144\,a^3 be + 128\,a^4 f + 3\left(21\,b^4 d - 18\,ab^3 e + 16\,a^2 b^2 f\right)x^4 + (105}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^3/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c +
 168*a^2*b^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3
*e + 16*a^2*b^2*f)*x^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e -
 64*a^3*b*f)*x^2)*sqrt(b*x^2 + a)/b^5
```

Sympy [A] time = 4.04707, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4 f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3 e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3 f x^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2 d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2 e x^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2 f x^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - 6ae \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqr
t(a + b*x**2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b*
*4) + 8*a**2*d*sqrt(a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a
+ b*x**2)/(35*b**3) + 16*a**2*f*x**4*sqrt(a + b*x**2)/(105*b**3)
- 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*d*x**2*sqrt(a + b*x**2)/(
15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*f*x**6*sqr
t(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*x**4*
sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*s
qrt(a + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/
8 + f*x**10/10)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.223768, size = 296, normalized size = 1.77

$$\frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c-315\sqrt{bx^2+a}ab^3c+63\left(bx^2+a\right)^{\frac{5}{2}}b^2d-210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+315\sqrt{bx^2+a}a^2b^2d+35\left(bx^2+a\right)^{\frac{9}{2}}f-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^3/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*b^3*c - 315*sqrt(b*x^2 + a)*a*b^3*c
+ 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)*a*b^2*d + 31
5*sqrt(b*x^2 + a)*a^2*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2
 + a)^(7/2)*a*f + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(
3/2)*a^3*f + 315*sqrt(b*x^2 + a)*a^4*f + 45*(b*x^2 + a)^(7/2)*b*e
 - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e -
315*sqrt(b*x^2 + a)*a^3*b*e)/b^5
```

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=121

$$\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2
*d - 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*
f)*(a + b*x^2)^(5/2))/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.271466, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2
*d - 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*
f)*(a + b*x^2)^(5/2))/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi in Sympy [A] time = 45.7985, size = 110, normalized size = 0.91

$$\frac{f(a+bx^2)^{\frac{7}{2}}}{7b^4}-\frac{(a+bx^2)^{\frac{5}{2}}(3af-be)}{5b^4}+\frac{(a+bx^2)^{\frac{3}{2}}(3a^2f-2abe+b^2d)}{3b^4}-\frac{\sqrt{a+bx^2}\,(a^3f-a^2be+ab^2d-b^3c)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(7/2)/(7*b**4) - (a + b*x**2)**(5/2)*(3*a*f - b*e
)/(5*b**4) + (a + b*x**2)**(3/2)*(3*a**2*f - 2*a*b*e + b**2*d)/(3
*b**4) - sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
/b**4
```

Mathematica [A] time = 0.120952, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(
35*d + 14*e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 1
```

```
5*f*x^6)))/(105*b^4)
```

Maple [A] time = 0.007, size = 99, normalized size = 0.8

$$-\frac{-15\,f x^6 b^3+18\,a b^2 f x^4-21\,b^3 e x^4-24\,a^2 b f x^2+28\,a b^2 e x^2-35\,b^3 d x^2+48\,a^3 f-56\,a^2 b e+70\,a b^2 d-105\,b^3 c}{105\,b^4}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4
-24*a^2*b*f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+7
0*a*b^2*d-105*b^3*c)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238782, size = 127, normalized size = 1.05

$$\frac{\left(15\,b^3 f x^6+3\,\left(7\,b^3 e-6\,a b^2 f\right) x^4+105\,b^3 c-70\,a b^2 d+56\,a^2 b e-48\,a^3 f+\left(35\,b^3 d-28\,a b^2 e+24\,a^2 b f\right) x^2\right)\sqrt{b x^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 7
0*a*b^2*d + 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a
^2*b*f)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 2.54908, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3 f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2 e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2 f x^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6af x^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+ \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3)
- 2*a*d*sqrt(a + b*x**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(
15*b**2) - 6*a*f*x**4*sqrt(a + b*x**2)/(35*b**2) + c*sqrt(a + b*x
**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) + e*x**4*sqrt(a + b*x**2)/
(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((c*x**2/2 + d*
```

```
x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.218731, size = 207, normalized size = 1.71

$$\frac{105\sqrt{bx^2+a}b^3c + 35\left(bx^2+a\right)^{\frac{3}{2}}b^2d - 105\sqrt{bx^2+a}ab^2d + 15\left(bx^2+a\right)^{\frac{7}{2}}f - 63\left(bx^2+a\right)^{\frac{5}{2}}af + 105\left(bx^2+a\right)^{\frac{3}{2}}a^2f - 105}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(b*x^2 + a)*b^3*c + 35*(b*x^2 + a)^(3/2)*b^2*d - 1
05*sqrt(b*x^2 + a)*a*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 +
 a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2)*a^2*f - 105*sqrt(b*x^2 + a)
*a^3*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e +
105*sqrt(b*x^2 + a)*a^2*b*e)/b^4
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{a+bx^2}\,(a^2f-abe+b^2d)}{b^3}+\frac{(a+bx^2)^{3/2}\,(be-2af)}{3b^3}+\frac{f\,(a+bx^2)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a
 + b*x^2)^(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.262866, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{a+bx^2}\,(a^2f-abe+b^2d)}{b^3}+\frac{(a+bx^2)^{3/2}\,(be-2af)}{3b^3}+\frac{f\,(a+bx^2)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a
 + b*x^2)^(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 49.3283, size = 92, normalized size = 0.89

$$\frac{f\,(a+bx^2)^{\frac{5}{2}}}{5b^3}-\frac{(a+bx^2)^{\frac{3}{2}}\,(2af-be)}{3b^3}+\frac{\sqrt{a+bx^2}\,(a^2f-abe+b^2d)}{b^3}-\frac{c\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f - b*e
)/(3*b**3) + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3 - c*
atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.25925, size = 97, normalized size = 0.94

$$\frac{\sqrt{a+bx^2}\,\left(8a^2f-2ab\,(5e+2fx^2)+b^2\,(15d+5ex^2+3fx^4)\right)}{15b^3}-\frac{c\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{c\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5
*e*x^2 + 3*f*x^4)))/(15*b^3) + (c*Log[x])/Sqrt[a] - (c*Log[a + Sq
rt[a]*Sqrt[a + b*x^2]])/Sqrt[a]
```

Maple [A] time = 0.013, size = 134, normalized size = 1.3

$$\frac{d}{b}\sqrt{bx^{2}+a}-c\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{ex^{2}}{3\,b}\sqrt{bx^{2}+a}\\-\frac{2\,ae}{3\,b^{2}}\sqrt{bx^{2}+a}+\frac{fx^{4}}{5\,b}\sqrt{bx^{2}+a}-\frac{4\,afx^{2}}{15\,b^{2}}\sqrt{bx^{2}+a}+\frac{8\,a^{2}f}{15\,b^{3}}\sqrt{bx^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] d/b*(b*x^2+a)^(1/2)-c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+1/3*e*x^2/b*(b*x^2+a)^(1/2)-2/3*e*a/b^2*(b*x^2+a)^(1/2)+1/5*f*
x^4/b*(b*x^2+a)^(1/2)-4/15*f*a/b^2*x^2*(b*x^2+a)^(1/2)+8/15*f*a^2
/b^3*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249379, size = 1, normalized size = 0.01

$$\left[\frac{15\,b^{3}c\log\left(-\frac{\left(bx^{2}+2\,a\right)\sqrt{a}-2\,\sqrt{bx^{2}+a}a}{x^{2}}\right)+2\,\left(3\,b^{2}fx^{4}+15\,b^{2}d-10\,abe+8\,a^{2}f+\left(5\,b^{2}e-4\,abf\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{a}}{30\,\sqrt{a}b^{3}},\right.\\\left.-\frac{15\,b^{3}c\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(3\,b^{2}fx^{4}+15\,b^{2}d-10\,abe+8\,a^{2}f+\left(5\,b^{2}e-4\,abf\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{-a}}{15\,\sqrt{-a}b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^3*c*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a
)/x^2) + 2*(3*b^2*f*x^4 + 15*b^2*d - 10*a*b*e + 8*a^2*f + (5*b^2*
e - 4*a*b*f)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(sqrt(a)*b^3), -1/15*(
15*b^3*c*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*b^2*f*x^4 + 15*b^2
*d - 10*a*b*e + 8*a^2*f + (5*b^2*e - 4*a*b*f)*x^2)*sqrt(b*x^2 + a
)*sqrt(-a))/(sqrt(-a)*b^3)]
```

Sympy [A] time = 23.5296, size = 192, normalized size = 1.86

$$c\left(\begin{cases}\frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}} & \text{for } -\frac{1}{a}>0\\ -\frac{\operatorname{acoth}\left(\frac{1}{\sqrt{a+bx^2}\sqrt{\frac{1}{a}}}\right)}{a\sqrt{\frac{1}{a}}} & \text{for } -\frac{1}{a}<0\wedge\frac{1}{a}<\frac{1}{a+bx^2}\\ -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{a+bx^2}\sqrt{\frac{1}{a}}}\right)}{a\sqrt{\frac{1}{a}}} & \text{for } \frac{1}{a}>\frac{1}{a+bx^2}\wedge-\frac{1}{a}<0\end{cases}\right)+\frac{f\left(a+bx^2\right)^{\frac{5}{2}}}{5b^3}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}(2af-be)}{3b^3}+\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] c*Piecewise((atan(1/(sqrt(-1/a)*sqrt(a + b*x**2)))/(a*sqrt(-1/a))
, -1/a > 0), (-acoth(1/(sqrt(a + b*x**2)*sqrt(1/a)))/(a*sqrt(1/a)
), (-1/a < 0) & (1/a < 1/(a + b*x**2))), (-atanh(1/(sqrt(a + b*x*
*2)*sqrt(1/a)))/(a*sqrt(1/a)), (-1/a < 0) & (1/a > 1/(a + b*x**2)
))) + f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f
 - b*e)/(3*b**3) + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**
3
```

GIAC/XCAS [A] time = 0.220671, size = 171, normalized size = 1.66

$$\frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\sqrt{bx^2+a}b^{14}d+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15\sqrt{bx^2+a}a^2b^{12}f+5\left(bx^2+a\right)^{\frac{3}{2}}b^{13}e-15\sqrt{bx^2+a}ab^{13}e}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2
 + a)*b^14*d + 3*(b*x^2 + a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*
a*b^12*f + 15*sqrt(b*x^2 + a)*a^2*b^12*f + 5*(b*x^2 + a)^(3/2)*b^
13*e - 15*sqrt(b*x^2 + a)*a*b^13*e)/b^15
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2)
 + (f*(a + b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] time = 0.4077, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2)
 + (f*(a + b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi in Sympy [A] time = 156.743, size = 87, normalized size = 0.87

$$\frac{f\left(a+bx^2\right)^{\frac{3}{2}}}{3b^2}-\frac{\sqrt{a+bx^2}\,(af-be)}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{(2ad-bc)\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(3/2)/(3*b**2) - sqrt(a + b*x**2)*(a*f - b*e)/b**
2 - c*sqrt(a + b*x**2)/(2*a*x**2) - (2*a*d - b*c)*atanh(sqrt(a +
b*x**2)/sqrt(a))/(2*a**(3/2))
```

Mathematica [A] time = 0.302753, size = 101, normalized size = 1.01

$$\frac{1}{6}\left(\frac{3(bc-2ad)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{3/2}}+\frac{3\log(x)(2ad-bc)}{a^{3/2}}+\sqrt{a+bx^2}\left(-\frac{4af}{b^2}-\frac{3c}{ax^2}+\frac{6e+2fx^2}{b}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*((-4*a*f)/b^2 - (3*c)/(a*x^2) + (6*e + 2*f*x^2)/
b) + (3*(-(b*c) + 2*a*d)*Log[x])/a^(3/2) + (3*(b*c - 2*a*d)*Log[a
 + Sqrt[a]*Sqrt[a + b*x^2]])/a^(3/2))/6
```

Maple [A] time = 0.013, size = 127, normalized size = 1.3

$$-\frac{c}{2\,a x^2}\sqrt{b x^2+a}+\frac{bc}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}$$
$$-d\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{e}{b}\sqrt{b x^2+a}+\frac{f x^2}{3\,b}\sqrt{b x^2+a}-\frac{2\,a f}{3\,b^2}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*c*(b*x^2+a)^(1/2)/a/x^2+1/2*c*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)-d/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
+e/b*(b*x^2+a)^(1/2)+1/3*f*x^2/b*(b*x^2+a)^(1/2)-2/3*f*a/b^2*(b*x
^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^3),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260359, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3 c-2\,a b^2 d\right)x^2\log\left(-\frac{\left(b x^2+2\,a\right)\sqrt{a}-2\,\sqrt{b x^2+a}\,a}{x^2}\right)-2\left(2\,a b f x^4-3\,b^2 c+2\left(3\,a b e-2\,a^2 f\right)x^2\right)\sqrt{b x^2+a}\sqrt{a}}{12\,a^{\frac{3}{2}}b^2 x^2},\ \frac{3\left(b^3 c-2\,a b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^3),x, algorithm="fricas"
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*x^2*log(-((b*x^2 + 2*a)*sqrt(a) - 2
*sqrt(b*x^2 + a)*a)/x^2) - 2*(2*a*b*f*x^4 - 3*b^2*c + 2*(3*a*b*e
- 2*a^2*f)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(3/2)*b^2*x^2), 1/6*(
3*(b^3*c - 2*a*b^2*d)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*a
*b*f*x^4 - 3*b^2*c + 2*(3*a*b*e - 2*a^2*f)*x^2)*sqrt(b*x^2 + a)*s
qrt(-a))/(sqrt(-a)*a*b^2*x^2)]
```

Sympy [A] time = 37.2329, size = 138, normalized size = 1.38

$$e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, Tr
ue)) + f*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a
+ b*x**2)/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c
*sqrt(a/(b*x**2) + 1)/(2*a*x) - d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt
(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

GIAC/XCAS [A] time = 0.221151, size = 154, normalized size = 1.54

$$-\frac{\frac{3\left(b^{2}c-2\,abd\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3\,\sqrt{bx^{2}+a}bc}{ax^{2}}-\frac{2\left(\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}f-3\,\sqrt{bx^{2}+a}ab^{2}f+3\,\sqrt{bx^{2}+a}b^{3}e\right)}{b^{3}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(
-a)*a) + 3*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*((b*x^2 + a)^(3/2)*b^2
*f - 3*sqrt(b*x^2 + a)*a*b^2*f + 3*sqrt(b*x^2 + a)*b^3*e)/b^3)/b
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a+bx^2}(3bc-4ad)}{8a^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c -
 4*a*d)*Sqrt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^
2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] time = 0.534256, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\sqrt{a+bx^2}(3bc-4ad)}{8a^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c -
 4*a*d)*Sqrt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^
2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi in Sympy [A] time = 147.611, size = 104, normalized size = 0.91

$$\frac{f\sqrt{a+bx^2}}{b}-\frac{c\sqrt{a+bx^2}}{4ax^4}-\frac{\sqrt{a+bx^2}\,(4ad-3bc)}{8a^2x^2}-\frac{\left(8a^2e-4abd+3b^2c\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] f*sqrt(a + b*x**2)/b - c*sqrt(a + b*x**2)/(4*a*x**4) - sqrt(a + b
*x**2)*(4*a*d - 3*b*c)/(8*a**2*x**2) - (8*a**2*e - 4*a*b*d + 3*b*
*2*c)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8*a**(5/2))
```

Mathematica [A] time = 0.328466, size = 125, normalized size = 1.1

$$\sqrt{a+bx^2}\left(\frac{3bc-4ad}{8a^2x^2}-\frac{c}{4ax^4}+\frac{f}{b}\right)-\frac{\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}+\frac{\log(x)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f/b - c/(4*a*x^4) + (3*b*c - 4*a*d)/(8*a^2*x^2))*Sqrt[a + b*x^2]
 + ((3*b^2*c - 4*a*b*d + 8*a^2*e)*Log[x])/(8*a^(5/2)) - ((3*b^2*c
 - 4*a*b*d + 8*a^2*e)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(8*a^(5/2
))
```

Maple [A] time = 0.015, size = 162, normalized size = 1.4

$$\frac{f}{b}\sqrt{bx^2+a}-\frac{c}{4\,ax^4}\sqrt{bx^2+a}+\frac{3\,bc}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,b^2c}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{d}{2\,ax^2}\sqrt{bx^2+a}+\frac{bd}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-e\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] f*(b*x^2+a)^(1/2)/b-1/4*c*(b*x^2+a)^(1/2)/a/x^4+3/8*c*b/a^2/x^2*(
b*x^2+a)^(1/2)-3/8*c*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)-1/2*d/a/x^2*(b*x^2+a)^(1/2)+1/2*d*b/a^(3/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)-e/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^5),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274824, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,b^3c-4\,ab^2d+8\,a^2be\right)x^4\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(8\,a^2fx^4-2\,abc+\left(3\,b^2c-4\,abd\right)x^2\right)\sqrt{bx^2+a}\sqrt{a}}{16\,a^{\frac{5}{2}}bx^4},\ -\frac{\left(3\,b^3c-4\,ab^2d+8\,a^2be\right)x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(8\,a^2fx^4-2\,abc+\left(3\,b^2c-4\,abd\right)x^2\right)\sqrt{bx^2+a}\sqrt{-a}}{8\,\sqrt{-a}a^2bx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^5),x, algorithm="fricas"
```

```
[Out] [1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*x^4*log(-((b*x^2 + 2*a)*
sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(8*a^2*f*x^4 - 2*a*b*c +
(3*b^2*c - 4*a*b*d)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(5/2)*b*x^4)
, -1/8*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*x^4*arctan(sqrt(-a)/sqr
t(b*x^2 + a)) - (8*a^2*f*x^4 - 2*a*b*c + (3*b^2*c - 4*a*b*d)*x^2)
*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*b*x^4)]
```

Sympy [A] time = 62.4029, size = 194, normalized size = 1.7

$$f\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)-\frac{c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{3b^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, Tr
ue)) - c/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(8*a*x
**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x
) + 3*b**(3/2)*c/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - e*asinh(sqrt(a
)/(sqrt(b)*x))/sqrt(a) + b*d*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/
2)) - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2))
```

GIAC/XCAS [A] time = 0.223875, size = 190, normalized size = 1.67

$$\frac{8\sqrt{bx^2+a}f + \frac{(3b^3c-4ab^2d+8a^2be)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3(bx^2+a)^{\frac{3}{2}}b^3c-5\sqrt{bx^2+a}ab^3c-4(bx^2+a)^{\frac{3}{2}}ab^2d+4\sqrt{bx^2+a}a^2b^2d}{a^2b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arct
an(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2
)*b^3*c - 5*sqrt(b*x^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d
 + 4*sqrt(b*x^2 + a)*a^2*b^2*d)/(a^2*b^2*x^4))/b
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{a+bx^2}(5bc-6ad)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2]
)/(24*a^2*x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/
(16*a^3*x^2) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rubi [A] time = 0.59972, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{\sqrt{a+bx^2}(5bc-6ad)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2]
)/(24*a^2*x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/
(16*a^3*x^2) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rubi in Sympy [A] time = 152.04, size = 141, normalized size = 0.97

$$-\frac{c\sqrt{a+bx^2}}{6ax^6}-\frac{\sqrt{a+bx^2}\,(6ad-5bc)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}-\frac{\left(16a^3f-8a^2be+6ab^2d-5b^3c\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -c*sqrt(a + b*x**2)/(6*a*x**6) - sqrt(a + b*x**2)*(6*a*d - 5*b*c)
/(24*a**2*x**4) - sqrt(a + b*x**2)*(8*a**2*e - 6*a*b*d + 5*b**2*c
)/(16*a**3*x**2) - (16*a**3*f - 8*a**2*b*e + 6*a*b**2*d - 5*b**3*
c)*atanh(sqrt(a + b*x**2)/sqrt(a))/(16*a**(7/2))
```

Mathematica [A] time = 0.272577, size = 166, normalized size = 1.14

$$\sqrt{a+bx^2}\left(\frac{5bc-6ad}{24a^2x^4}+\frac{-8a^2e+6abd-5b^2c}{16a^3x^2}-\frac{c}{6ax^6}\right)-\frac{\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)\left(16a^3f-8a^2be+6ab^2d-5b^3c\right)}{16a^{7/2}}+\frac{\log(x)\left(16a^3f-8a^2be+6ab^2d-5b^3c\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (-c/(6*a*x^6) + (5*b*c - 6*a*d)/(24*a^2*x^4) + (-5*b^2*c + 6*a*b*
d - 8*a^2*e)/(16*a^3*x^2))*Sqrt[a + b*x^2] + ((-5*b^3*c + 6*a*b^2
*d - 8*a^2*b*e + 16*a^3*f)*Log[x])/(16*a^(7/2)) - ((-5*b^3*c + 6*
a*b^2*d - 8*a^2*b*e + 16*a^3*f)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])
/(16*a^(7/2))
```

Maple [A] time = 0.016, size = 238, normalized size = 1.6

$$-\frac{c}{6\,a x^6}\sqrt{bx^2+a}+\frac{5\,bc}{24\,a^2x^4}\sqrt{bx^2+a}-\frac{5\,b^2c}{16\,a^3x^2}\sqrt{bx^2+a}+\frac{5\,b^3c}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{d}{4\,a x^4}\sqrt{bx^2+a}+\frac{3\,bd}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,b^2d}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{e}{2\,a x^2}\sqrt{bx^2+a}+\frac{be}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-f\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*c*(b*x^2+a)^(1/2)/a/x^6+5/24*c*b/a^2/x^4*(b*x^2+a)^(1/2)-5/1
6*c*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*c*b^3/a^(7/2)*ln((2*a+2*a^(1
/2)*(b*x^2+a)^(1/2))/x)-1/4*d/a/x^4*(b*x^2+a)^(1/2)+3/8*d*b/a^2/x
^2*(b*x^2+a)^(1/2)-3/8*d*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)-1/2*e/a/x^2*(b*x^2+a)^(1/2)+1/2*e*b/a^(3/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x)-f/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^7),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.314747, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f\right)x^6\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(3\left(5\,b^2c-6\,abd+8\,a^2e\right)x^4+8\,a^2c-2\left(5\,abc-\right.\right.}{96\,a^{\frac{7}{2}}x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^7),x, algorithm="fricas"
```

```
[Out] [-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*x^6*log(-(
(b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(3*(5*b^2*c
 - 6*a*b*d + 8*a^2*e)*x^4 + 8*a^2*c - 2*(5*a*b*c - 6*a^2*d)*x^2)*
sqrt(b*x^2 + a)*sqrt(a))/(a^(7/2)*x^6), 1/48*(3*(5*b^3*c - 6*a*b^
2*d + 8*a^2*b*e - 16*a^3*f)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (3*(5*b^2*c - 6*a*b*d + 8*a^2*e)*x^4 + 8*a^2*c - 2*(5*a*b*c - 6
*a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^6)]
```

Sympy [A] time = 86.7126, size = 303, normalized size = 2.08

$$-\frac{c}{6\sqrt{b}x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{d}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{\sqrt{b}c}{24ax^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{\sqrt{b}d}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^{2}}+1}}{2ax}-\frac{5b^{\frac{3}{2}}c}{48a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b^{\frac{3}{2}}d}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5b^{\frac{5}{2}}c}{16a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+\frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^{2}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^{3}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - d/(4*sqrt(b)*x**5*sqrt
(a/(b*x**2) + 1)) + sqrt(b)*c/(24*a*x**5*sqrt(a/(b*x**2) + 1)) +
sqrt(b)*d/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*e*sqrt(a/(b*x
**2) + 1)/(2*a*x) - 5*b**(3/2)*c/(48*a**2*x**3*sqrt(a/(b*x**2) +
1)) + 3*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*c
/(16*a**3*x*sqrt(a/(b*x**2) + 1)) - f*asinh(sqrt(a)/(sqrt(b)*x))/
sqrt(a) + b*e*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*d*
asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*c*asinh(sqrt(a)/
(sqrt(b)*x))/(16*a**(7/2))
```

GIAC/XCAS [A] time = 0.229471, size = 313, normalized size = 2.14

$$-\frac{\frac{3\left(5b^{4}c-6ab^{3}d-16a^{3}bf+8a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}}+\frac{15\left(bx^{2}+a\right)^{\frac{5}{2}}b^{4}c-40\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{4}c+33\sqrt{bx^{2}+a}a^{2}b^{4}c-18\left(bx^{2}+a\right)^{\frac{5}{2}}ab^{3}d+48\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b^{3}d-30}{a^{3}b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d - 16*a^3*b*f + 8*a^2*b^2*e)*arctan(
sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*
b^4*c - 40*(b*x^2 + a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4
*c - 18*(b*x^2 + a)^(5/2)*a*b^3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*
d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b*x^2 + a)^(5/2)*a^2*b^2*e
 - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2 + a)*a^4*b^2*e)
/(a^3*b^3*x^6))/b
```

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{a+bx^2}(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)}{128a^{9/2}}+\frac{\sqrt{a+bx^2}\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{c\sqrt{a+bx^2}}{8ax^8}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2]
)/(48*a^2*x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2
])/(192*a^3*x^4) + ((35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*
f)*Sqrt[a + b*x^2])/(128*a^4*x^2) - (b*(35*b^3*c - 40*a*b^2*d + 4
8*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(9
/2))
```

Rubi [A] time = 0.8178, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219

$$\frac{\sqrt{a+bx^2}(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)}{128a^{9/2}}+\frac{\sqrt{a+bx^2}\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{c\sqrt{a+bx^2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2]
)/(48*a^2*x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2
])/(192*a^3*x^4) + ((35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*
f)*Sqrt[a + b*x^2])/(128*a^4*x^2) - (b*(35*b^3*c - 40*a*b^2*d + 4
8*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(9
/2))
```

Rubi in Sympy [A] time = 162.471, size = 190, normalized size = 0.97

$$-\frac{c\sqrt{a+bx^2}}{8ax^8}-\frac{\sqrt{a+bx^2}\,(8ad-7bc)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}-\frac{\sqrt{a+bx^2}\left(64a^3f-48a^2be+40ab^2d-35b^3c\right)}{128a^4x^2}+\frac{b\left(64a^3f-48a^2be+40ab^2d-35b^3c\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] -c*sqrt(a + b*x**2)/(8*a*x**8) - sqrt(a + b*x**2)*(8*a*d - 7*b*c)
/(48*a**2*x**6) - sqrt(a + b*x**2)*(48*a**2*e - 40*a*b*d + 35*b**
2*c)/(192*a**3*x**4) - sqrt(a + b*x**2)*(64*a**3*f - 48*a**2*b*e
+ 40*a*b**2*d - 35*b**3*c)/(128*a**4*x**2) + b*(64*a**3*f - 48*a*
*2*b*e + 40*a*b**2*d - 35*b**3*c)*atanh(sqrt(a + b*x**2)/sqrt(a))
```

```
/(128*a**(9/2))
```

Mathematica [A] time = 0.375695, size = 199, normalized size = 1.02

$$\frac{3bx^8\log(x)\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)-3bx^8\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)+\sqrt{a}\cdots}{384a^{9/2}x\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*x^2]*(105*b^3*c*x^6 - 10*a*b^2*x^4*(7*c + 12*
d*x^2) + 8*a^2*b*x^2*(7*c + 10*d*x^2 + 18*e*x^4) - 16*a^3*(3*c +
4*d*x^2 + 6*e*x^4 + 12*f*x^6)) + 3*b*(35*b^3*c - 40*a*b^2*d + 48*
a^2*b*e - 64*a^3*f)*x^8*Log[x] - 3*b*(35*b^3*c - 40*a*b^2*d + 48*
a^2*b*e - 64*a^3*f)*x^8*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(384*a^
(9/2)*x^8)
```

Maple [A] time = 0.021, size = 320, normalized size = 1.6

$$-\frac{c}{8\,a x^8}\sqrt{bx^2+a}+\frac{7\,bc}{48\,a^2x^6}\sqrt{bx^2+a}-\frac{35\,b^2c}{192\,a^3x^4}\sqrt{bx^2+a}+\frac{35\,b^3c}{128\,a^4x^2}\sqrt{bx^2+a}$$
$$-\frac{35\,b^4c}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}-\frac{d}{6\,a x^6}\sqrt{bx^2+a}+\frac{5\,bd}{24\,a^2x^4}\sqrt{bx^2+a}$$
$$-\frac{5\,db^2}{16\,a^3x^2}\sqrt{bx^2+a}+\frac{5\,db^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}$$
$$-\frac{e}{4\,a x^4}\sqrt{bx^2+a}+\frac{3\,be}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,eb^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$
$$-\frac{f}{2\,a x^2}\sqrt{bx^2+a}+\frac{bf}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/8*c*(b*x^2+a)^(1/2)/a/x^8+7/48*c*b/a^2/x^6*(b*x^2+a)^(1/2)-35/
192*c*b^2/a^3/x^4*(b*x^2+a)^(1/2)+35/128*c*b^3/a^4/x^2*(b*x^2+a)^
(1/2)-35/128*c*b^4/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-
1/6*d/a/x^6*(b*x^2+a)^(1/2)+5/24*d*b/a^2/x^4*(b*x^2+a)^(1/2)-5/16
*d*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*d*b^3/a^(7/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)-1/4*e/a/x^4*(b*x^2+a)^(1/2)+3/8*e*b/a^2/x^
2*(b*x^2+a)^(1/2)-3/8*e*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x)-1/2*f/a/x^2*(b*x^2+a)^(1/2)+1/2*f*b/a^(3/2)*ln((2*a+2*a^
(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^9),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.526919, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf\right)x^8\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}+2\,\sqrt{bx^2+a}a}{x^2}\right)-2\left(3\left(35\,b^3c-40\,ab^2d+48\,a^2be-64\,a^3f\right)x^6\right.}{768\,a^{\frac{9}{2}}x^8}\right.$$

$$-\frac{3\left(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf\right)x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(3\left(35\,b^3c-40\,ab^2d+48\,a^2be-64\,a^3f\right)x^6-2\left(35\,ab^2c\right.\right.}{384\,\sqrt{-a}a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^9),x, algorithm="fricas"
```

```
[Out] [-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*x^
8*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2) - 2*(3*
(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*x^6 - 2*(35*a*b^2
*c - 40*a^2*b*d + 48*a^3*e)*x^4 - 48*a^3*c + 8*(7*a^2*b*c - 8*a^3
*d)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(9/2)*x^8), -1/384*(3*(35*b^
4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*x^8*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) - (3*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a
^3*f)*x^6 - 2*(35*a*b^2*c - 40*a^2*b*d + 48*a^3*e)*x^4 - 48*a^3*c
 + 8*(7*a^2*b*c - 8*a^3*d)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-
a)*a^4*x^8)]
```

Sympy [A] time = 145.495, size = 444, normalized size = 2.28

$$-\frac{c}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{d}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{e}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}c}{48ax^7\sqrt{\frac{a}{bx^2}+1}}$$
$$+\frac{\sqrt{b}d}{24ax^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}e}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{7b^{\frac{3}{2}}c}{192a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{3}{2}}d}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}$$
$$+\frac{3b^{\frac{3}{2}}e}{8a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^{\frac{5}{2}}c}{384a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{5}{2}}d}{16a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^{\frac{7}{2}}c}{128a^4x\sqrt{\frac{a}{bx^2}+1}}$$
$$+\frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}-\frac{35b^4c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - d/(6*sqrt(b)*x**7*sqrt
(a/(b*x**2) + 1)) - e/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqr
t(b)*c/(48*a*x**7*sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(24*a*x**5*sq
rt(a/(b*x**2) + 1)) + sqrt(b)*e/(8*a*x**3*sqrt(a/(b*x**2) + 1)) -
 sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(2*a*x) - 7*b**(3/2)*c/(192*a**2*
x**5*sqrt(a/(b*x**2) + 1)) - 5*b**(3/2)*d/(48*a**2*x**3*sqrt(a/(b
*x**2) + 1)) + 3*b**(3/2)*e/(8*a**2*x*sqrt(a/(b*x**2) + 1)) + 35*
b**(5/2)*c/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*d/(1
6*a**3*x*sqrt(a/(b*x**2) + 1)) + 35*b**(7/2)*c/(128*a**4*x*sqrt(a
/(b*x**2) + 1)) + b*f*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3
*b**2*e*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*d*asinh(
sqrt(a)/(sqrt(b)*x))/(16*a**(7/2)) - 35*b**4*c*asinh(sqrt(a)/(sqr
t(b)*x))/(128*a**(9/2))
```

GIAC/XCAS [A] time = 0.2291, size = 487, normalized size = 2.5

$$\frac{3\left(35\,b^5c-40\,ab^4d-64\,a^3b^2f+48\,a^2b^3e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\left(bx^2+a\right)^{\frac{7}{2}}b^5c-385\left(bx^2+a\right)^{\frac{5}{2}}ab^5c+511\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5c-279\sqrt{bx^2+a}a^3b^5c-120\left(bx^2+a\right)^{\frac{7}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^9),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d - 64*a^3*b^2*f + 48*a^2*b^3*e)*ar
ctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^
(7/2)*b^5*c - 385*(b*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/
2)*a^2*b^5*c - 279*sqrt(b*x^2 + a)*a^3*b^5*c - 120*(b*x^2 + a)^(7
/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*b^4*d - 584*(b*x^2 + a)^(
3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d - 192*(b*x^2 + a)^
(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x^2 +
a)^(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f + 144*(b*x^2 +
 a)^(7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x^
2 + a)^(3/2)*a^4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e)/(a^4*b^4*
x^8))/b
```

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{x^5\sqrt{a+bx^2}\,(63a^2f-70abe+80b^2d)}{480b^3}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^{11/2}}-\frac{ax\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^5}+\frac{x^3\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{384b^4}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{80b^2}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*
x^2])/(256*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f
)*x^3*Sqrt[a + b*x^2])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2
*f)*x^5*Sqrt[a + b*x^2])/(480*b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a
 + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*x^2])/(10*b) + (a^2*(96*b
^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(256*b^(11/2))
```

Rubi [A] time = 0.649824, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{x^5\sqrt{a+bx^2}\,(63a^2f-70abe+80b^2d)}{480b^3}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^{11/2}}-\frac{ax\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^5}+\frac{x^3\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{384b^4}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{80b^2}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*
x^2])/(256*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f
)*x^3*Sqrt[a + b*x^2])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2
*f)*x^5*Sqrt[a + b*x^2])/(480*b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a
 + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*x^2])/(10*b) + (a^2*(96*b
^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(256*b^(11/2))
```

Rubi in Sympy [A] time = 70.8295, size = 238, normalized size = 0.97

$$-\frac{a^2\left(a\left(63a^2f-70abe+80b^2d\right)-96b^3c\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{11}{2}}}+\frac{ax\sqrt{a+bx^2}\left(a\left(63a^2f-70abe+80b^2d\right)-96b^3c\right)}{256b^5}+\frac{fx^9\sqrt{a+bx^2}}{10b}-\frac{x^7\sqrt{a+bx^2}\,(9af-10be)}{80b^2}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{480b^3}-\frac{x^3\sqrt{a+bx^2}\left(a\left(63a^2f-70abe+80b^2d\right)-96b^3c\right)}{384b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -a**2*(a*(63*a**2*f - 70*a*b*e + 80*b**2*d) - 96*b**3*c)*atanh(sq
rt(b)*x/sqrt(a + b*x**2))/(256*b**(11/2)) + a*x*sqrt(a + b*x**2)*
(a*(63*a**2*f - 70*a*b*e + 80*b**2*d) - 96*b**3*c)/(256*b**5) + f
```

```
*x**9*sqrt(a + b*x**2)/(10*b) - x**7*sqrt(a + b*x**2)*(9*a*f - 10
*b*e)/(80*b**2) + x**5*sqrt(a + b*x**2)*(63*a**2*f - 70*a*b*e + 8
0*b**2*d)/(480*b**3) - x**3*sqrt(a + b*x**2)*(a*(63*a**2*f - 70*a
*b*e + 80*b**2*d) - 96*b**3*c)/(384*b**4)
```

Mathematica [A] time = 0.242686, size = 188, normalized size = 0.77

$$\frac{15a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)-\sqrt{b}x\sqrt{a+bx^2}\left(-945a^4f+210a^3b\left(5e+3fx^2\right)-4a^2b\right.}{3840b^{11/2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (-(Sqrt[b]*x*Sqrt[a + b*x^2]*(-945*a^4*f + 210*a^3*b*(5*e + 3*f*x
^2) - 4*a^2*b^2*(300*d + 175*e*x^2 + 126*f*x^4) - 32*b^4*x^2*(30*
c + 20*d*x^2 + 15*e*x^4 + 12*f*x^6) + 16*a*b^3*(90*c + 50*d*x^2 +
 35*e*x^4 + 27*f*x^6))) + 15*a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*
b*e - 63*a^3*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(3840*b^(11/2
))
```

Maple [A] time = 0.032, size = 368, normalized size = 1.5

$$\begin{aligned}
&\frac{cx^3}{4\,b}\sqrt{bx^2+a}-\frac{3\,acx}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2c}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{dx^5}{6\,b}\sqrt{bx^2+a}\\
&-\frac{5\,adx^3}{24\,b^2}\sqrt{bx^2+a}+\frac{5\,a^2dx}{16\,b^3}\sqrt{bx^2+a}-\frac{5\,a^3d}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}\\
&+\frac{ex^7}{8\,b}\sqrt{bx^2+a}-\frac{7\,aex^5}{48\,b^2}\sqrt{bx^2+a}+\frac{35\,a^2ex^3}{192\,b^3}\sqrt{bx^2+a}-\frac{35\,ea^3x}{128\,b^4}\sqrt{bx^2+a}\\
&+\frac{35\,ea^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{fx^9}{10\,b}\sqrt{bx^2+a}-\frac{9\,afx^7}{80\,b^2}\sqrt{bx^2+a}+\frac{21\,a^2fx^5}{160\,b^3}\sqrt{bx^2+a}\\
&-\frac{21\,a^3fx^3}{128\,b^4}\sqrt{bx^2+a}+\frac{63\,fa^4x}{256\,b^5}\sqrt{bx^2+a}-\frac{63\,fa^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{11}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*c*x^3/b*(b*x^2+a)^(1/2)-3/8*c*a/b^2*x*(b*x^2+a)^(1/2)+3/8*c*a
^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*d*x^5/b*(b*x^2+a)^(1
/2)-5/24*d*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*d*a^2/b^3*x*(b*x^2+a)^(
1/2)-5/16*d*a^3/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/8*e*x^7/b
*(b*x^2+a)^(1/2)-7/48*e*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*e*a^2/b^
3*x^3*(b*x^2+a)^(1/2)-35/128*e*a^3/b^4*x*(b*x^2+a)^(1/2)+35/128*e
*a^4/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/10*f*x^9*(b*x^2+a)^(
1/2)/b-9/80*f*a/b^2*x^7*(b*x^2+a)^(1/2)+21/160*f*a^2/b^3*x^5*(b*x
^2+a)^(1/2)-21/128*f*a^3/b^4*x^3*(b*x^2+a)^(1/2)+63/256*f*a^4/b^5
*x*(b*x^2+a)^(1/2)-63/256*f*a^5/b^(11/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.557313, size = 1, normalized size = 0.

$$\left[\frac{2\left(384\,b^4fx^9+48\left(10\,b^4e-9\,ab^3f\right)x^7+8\left(80\,b^4d-70\,ab^3e+63\,a^2b^2f\right)x^5+10\left(96\,b^4c-80\,ab^3d+70\,a^2b^2e-63\,a^3bf\right)x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*b^4*f*x^9 + 48*(10*b^4*e - 9*a*b^3*f)*x^7 + 8*(80
*b^4*d - 70*a*b^3*e + 63*a^2*b^2*f)*x^5 + 10*(96*b^4*c - 80*a*b^3
*d + 70*a^2*b^2*e - 63*a^3*b*f)*x^3 - 15*(96*a*b^3*c - 80*a^2*b^2
*d + 70*a^3*b*e - 63*a^4*f)*x)*sqrt(b*x^2 + a)*sqrt(b) - 15*(96*a
^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*log(2*sqrt(b*x^2
 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(11/2), 1/3840*((384*b^4*f*
x^9 + 48*(10*b^4*e - 9*a*b^3*f)*x^7 + 8*(80*b^4*d - 70*a*b^3*e +
63*a^2*b^2*f)*x^5 + 10*(96*b^4*c - 80*a*b^3*d + 70*a^2*b^2*e - 63
*a^3*b*f)*x^3 - 15*(96*a*b^3*c - 80*a^2*b^2*d + 70*a^3*b*e - 63*a
^4*f)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 15*(96*a^2*b^3*c - 80*a^3*b^2
*d + 70*a^4*b*e - 63*a^5*f)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(
sqrt(-b)*b^5)]
```

Sympy [A] time = 52.5299, size = 586, normalized size = 2.39

$$\frac{63a^{\frac{9}{2}}fx}{256b^5\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{7}{2}}ex}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{21a^{\frac{7}{2}}fx^3}{256b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}dx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}ex^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^{\frac{5}{2}}fx^5}{640b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}cx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}dx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}ex^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{3}{2}}fx^7}{160b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}cx^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}dx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}ex^7}{48b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}fx^9}{80b\sqrt{1+\frac{bx^2}{a}}}-\frac{63a^5f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{11}{2}}}+\frac{35a^4e\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{cx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{dx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{ex^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 63*a**(9/2)*f*x/(256*b**5*sqrt(1 + b*x**2/a)) - 35*a**(7/2)*e*x/(
128*b**4*sqrt(1 + b*x**2/a)) + 21*a**(7/2)*f*x**3/(256*b**4*sqrt(
1 + b*x**2/a)) + 5*a**(5/2)*d*x/(16*b**3*sqrt(1 + b*x**2/a)) - 35
*a**(5/2)*e*x**3/(384*b**3*sqrt(1 + b*x**2/a)) - 21*a**(5/2)*f*x*
*5/(640*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*c*x/(8*b**2*sqrt(1
+ b*x**2/a)) + 5*a**(3/2)*d*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + 7
*a**(3/2)*e*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + 3*a**(3/2)*f*x**
7/(160*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*c*x**3/(8*b*sqrt(1 + b*
x**2/a)) - sqrt(a)*d*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*e*x
**7/(48*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**9/(80*b*sqrt(1 + b*x
**2/a)) - 63*a**5*f*asinh(sqrt(b)*x/sqrt(a))/(256*b**(11/2)) + 35
*a**4*e*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*d*asinh(
sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*c*asinh(sqrt(b)*x/sqrt(
a))/(8*b**(5/2)) + c*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + d*x**7
/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + e*x**9/(8*sqrt(a)*sqrt(1 + b*x*
*2/a)) + f*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.228277, size = 302, normalized size = 1.23

$$\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,fx^2}{b}-\frac{9\,ab^7f-10\,b^8e}{b^9}\right)x^2+\frac{80\,b^8d+63\,a^2b^6f-70\,ab^7e}{b^9}\right)x^2+\frac{5\left(96\,b^8c-80\,ab^7d-63\,a^3b^5f+70\,a^2b^6e\right.}{b^9}\right.\right.$$

$$-\frac{\left(96\,a^2b^3c-80\,a^3b^2d-63\,a^5f+70\,a^4be\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^4/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b - (9*a*b^7*f - 10*b^8*e)/b^9)*x^2 + (8
0*b^8*d + 63*a^2*b^6*f - 70*a*b^7*e)/b^9)*x^2 + 5*(96*b^8*c - 80*
a*b^7*d - 63*a^3*b^5*f + 70*a^2*b^6*e)/b^9)*x^2 - 15*(96*a*b^7*c
- 80*a^2*b^6*d - 63*a^4*b^4*f + 70*a^3*b^5*e)/b^9)*sqrt(b*x^2 + a
)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d - 63*a^5*f + 70*a^4*b*e)
*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x\sqrt{a+bx^2}\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^4}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2
])/(128*b^4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x
^2])/(192*b^3) + ((8*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) +
 (f*x^7*Sqrt[a + b*x^2])/(8*b) - (a*(64*b^3*c - 48*a*b^2*d + 40*a
^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(
9/2))
```

Rubi [A] time = 0.532242, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x\sqrt{a+bx^2}\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^4}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2
])/(128*b^4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x
^2])/(192*b^3) + ((8*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) +
 (f*x^7*Sqrt[a + b*x^2])/(8*b) - (a*(64*b^3*c - 48*a*b^2*d + 40*a
^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(
9/2))
```

Rubi in Sympy [A] time = 70.0503, size = 187, normalized size = 0.96

$$\frac{a\left(a\left(35a^2f-40abe+48b^2d\right)-64b^3c\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{9}{2}}}+\frac{fx^7\sqrt{a+bx^2}}{8b}-\frac{x^5\sqrt{a+bx^2}\,(7af-8be)}{48b^2}+\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}-\frac{x\sqrt{a+bx^2}\left(a\left(35a^2f-40abe+48b^2d\right)-64b^3c\right)}{128b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] a*(a*(35*a**2*f - 40*a*b*e + 48*b**2*d) - 64*b**3*c)*atanh(sqrt(b
)*x/sqrt(a + b*x**2))/(128*b**(9/2)) + f*x**7*sqrt(a + b*x**2)/(8
*b) - x**5*sqrt(a + b*x**2)*(7*a*f - 8*b*e)/(48*b**2) + x**3*sqrt
(a + b*x**2)*(35*a**2*f - 40*a*b*e + 48*b**2*d)/(192*b**3) - x*sq
rt(a + b*x**2)*(a*(35*a**2*f - 40*a*b*e + 48*b**2*d) - 64*b**3*c)
/(128*b**4)
```

Mathematica [A] time = 0.219571, size = 152, normalized size = 0.78

$$\frac{3a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(35a^3f-40a^2be+48ab^2d-64b^3c\right)+\sqrt{b}x\sqrt{a+bx^2}\left(-105a^3f+10a^2b\left(12e+7fx^2\right)-8ab^2\left(18\right.\right.}{384b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2
) - 8*a*b^2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2
+ 4*e*x^4 + 3*f*x^6)) + 3*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e
+ 35*a^3*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(384*b^(9/2))
```

Maple [A] time = 0.014, size = 284, normalized size = 1.5

$$\frac{dx^3}{4b}\sqrt{bx^2+a}-\frac{3\,adx}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2d}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cx}{2\,b}\sqrt{bx^2+a}$$
$$-\frac{ac}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{ex^5}{6\,b}\sqrt{bx^2+a}-\frac{5\,aex^3}{24\,b^2}\sqrt{bx^2+a}+\frac{5\,a^2ex}{16\,b^3}\sqrt{bx^2+a}$$
$$-\frac{5\,ea^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{fx^7}{8\,b}\sqrt{bx^2+a}-\frac{7\,afx^5}{48\,b^2}\sqrt{bx^2+a}$$
$$+\frac{35\,a^2fx^3}{192\,b^3}\sqrt{bx^2+a}-\frac{35\,a^3fx}{128\,b^4}\sqrt{bx^2+a}+\frac{35\,fa^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d*x^3/b*(b*x^2+a)^(1/2)-3/8*d*a/b^2*x*(b*x^2+a)^(1/2)+3/8*d*a
^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c*x/b*(b*x^2+a)^(1/2
)-1/2*c*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*e*x^5/b*(b*x^
2+a)^(1/2)-5/24*e*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*e*a^2/b^3*x*(b*x
^2+a)^(1/2)-5/16*e*a^3/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/8*
f*x^7*(b*x^2+a)^(1/2)/b-7/48*f*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*f
*a^2/b^3*x^3*(b*x^2+a)^(1/2)-35/128*f*a^3/b^4*x*(b*x^2+a)^(1/2)+3
5/128*f*a^4/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.37357, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,b^3fx^7+8\left(8\,b^3e-7\,ab^2f\right)x^5+2\left(48\,b^3d-40\,ab^2e+35\,a^2bf\right)x^3+3\left(64\,b^3c-48\,ab^2d+40\,a^2be-35\,a^3f\right)x\right)\sqrt{bx^2}}{768\,b^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^3*f*x^7 + 8*(8*b^3*e - 7*a*b^2*f)*x^5 + 2*(48*b^3
*d - 40*a*b^2*e + 35*a^2*b*f)*x^3 + 3*(64*b^3*c - 48*a*b^2*d + 40
*a^2*b*e - 35*a^3*f)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(64*a*b^3*c -
 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*log(-2*sqrt(b*x^2 + a)*b*x
 - (2*b*x^2 + a)*sqrt(b)))/b^(9/2), 1/384*((48*b^3*f*x^7 + 8*(8*b
```

```
^3*e - 7*a*b^2*f)*x^5 + 2*(48*b^3*d - 40*a*b^2*e + 35*a^2*b*f)*x^
3 + 3*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x)*sqrt(b*x
^2 + a)*sqrt(-b) - 3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35
*a^4*f)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^4)]
```

Sympy [A] time = 34.2524, size = 444, normalized size = 2.29

$$-\frac{35a^{\frac{7}{2}}fx}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}ex}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}fx^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}dx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}ex^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}fx^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}dx^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}ex^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}fx^7}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^4f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3e\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+\frac{dx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{ex^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**(7/2)*f*x/(128*b**4*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*e*x/(
16*b**3*sqrt(1 + b*x**2/a)) - 35*a**(5/2)*f*x**3/(384*b**3*sqrt(1
 + b*x**2/a)) - 3*a**(3/2)*d*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a*
*(3/2)*e*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + 7*a**(3/2)*f*x**5/(1
92*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*x*sqrt(1 + b*x**2/a)/(2*b
) - sqrt(a)*d*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt(a)*e*x**5/(24*
b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**7/(48*b*sqrt(1 + b*x**2/a))
+ 35*a**4*f*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*e*as
inh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*d*asinh(sqrt(b)*x/s
qrt(a))/(8*b**(5/2)) - a*c*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2))
+ d*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + e*x**7/(6*sqrt(a)*sqrt(
1 + b*x**2/a)) + f*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.226074, size = 236, normalized size = 1.22

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,fx^2}{b}-\frac{7\,ab^5f-8\,b^6e}{b^7}\right)x^2+\frac{48\,b^6d+35\,a^2b^4f-40\,ab^5e}{b^7}\right)x^2+\frac{3\left(64\,b^6c-48\,ab^5d-35\,a^3b^3f+40\,a^2b^4e\right)}{b^7}\right)\sqrt{}+\frac{\left(64\,ab^3c-48\,a^2b^2d-35\,a^4f+40\,a^3be\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)*x^2/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b - (7*a*b^5*f - 8*b^6*e)/b^7)*x^2 + (48*b^6
*d + 35*a^2*b^4*f - 40*a*b^5*e)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5
*d - 35*a^3*b^3*f + 40*a^2*b^4*e)/b^7)*sqrt(b*x^2 + a)*x + 1/128*
(64*a*b^3*c - 48*a^2*b^2*d - 35*a^4*f + 40*a^3*b*e)*ln(abs(-sqrt(
b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=145

$$\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*
b*e - 5*a*f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^
2])/(6*b) + ((16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rubi [A] time = 0.271066, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*
b*e - 5*a*f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^
2])/(6*b) + ((16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rubi in Sympy [A] time = 47.3924, size = 138, normalized size = 0.95

$$\frac{fx^5\sqrt{a+bx^2}}{6b}-\frac{x^3\sqrt{a+bx^2}\,(5af-6be)}{24b^2}+\frac{x\sqrt{a+bx^2}\,\left(a\,(5af-6be)+8b^2d\right)}{16b^3}-\frac{\left(5a^3f-6a^2be+8ab^2d-16b^3c\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] f*x**5*sqrt(a + b*x**2)/(6*b) - x**3*sqrt(a + b*x**2)*(5*a*f - 6*
b*e)/(24*b**2) + x*sqrt(a + b*x**2)*(a*(5*a*f - 6*b*e) + 8*b**2*d
)/(16*b**3) - (5*a**3*f - 6*a**2*b*e + 8*a*b**2*d - 16*b**3*c)*at
anh(sqrt(b)*x/sqrt(a + b*x**2))/(16*b**(7/2))
```

Mathematica [A] time = 0.147923, size = 121, normalized size = 0.83

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^2f-2ab\left(9e+5fx^2\right)+4b^2\left(6d+3ex^2+2fx^4\right)\right)+3\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(-5a^3f+6a^2be-8ab^2d+\right.}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*f - 2*a*b*(9*e + 5*f*x^2) + 4*
b^2*(6*d + 3*e*x^2 + 2*f*x^4)) + 3*(16*b^3*c - 8*a*b^2*d + 6*a^2*
b*e - 5*a^3*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Maple [A] time = 0.01, size = 203, normalized size = 1.4

$$c\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{dx}{2\,b}\sqrt{bx^2+a}-\frac{ad}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{ex^3}{4\,b}\sqrt{bx^2+a}-\frac{3\,aex}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2e}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{fx^5}{6\,b}\sqrt{bx^2+a}-\frac{5\,afx^3}{24\,b^2}\sqrt{bx^2+a}+\frac{5\,a^2fx}{16\,b^3}\sqrt{bx^2+a}-\frac{5\,a^3f}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] c*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/2*d*x/b*(b*x^2+a)^(1/2)
-1/2*d*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*e*x^3/b*(b*x^2
+a)^(1/2)-3/8*e*a/b^2*x*(b*x^2+a)^(1/2)+3/8*e*a^2/b^(5/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))+1/6*f*x^5*(b*x^2+a)^(1/2)/b-5/24*f*a/b^2*x
^3*(b*x^2+a)^(1/2)+5/16*f*a^2/b^3*x*(b*x^2+a)^(1/2)-5/16*f*a^3/b^
(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281155, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^2fx^5+2\left(6\,b^2e-5\,abf\right)x^3+3\left(8\,b^2d-6\,abe+5\,a^2f\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f\right)\log\left(2\right.}{96\,b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*f*x^5 + 2*(6*b^2*e - 5*a*b*f)*x^3 + 3*(8*b^2*d -
6*a*b*e + 5*a^2*f)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(16*b^3*c - 8*a
*b^2*d + 6*a^2*b*e - 5*a^3*f)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^
2 + a)*sqrt(b)))/b^(7/2), 1/48*((8*b^2*f*x^5 + 2*(6*b^2*e - 5*a*b
*f)*x^3 + 3*(8*b^2*d - 6*a*b*e + 5*a^2*f)*x)*sqrt(b*x^2 + a)*sqrt
(-b) + 3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 19.264, size = 362, normalized size = 2.5

$$\frac{5a^{\frac{5}{2}}fx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}ex}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}fx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}ex^3}{8b\sqrt{1+\frac{bx^2}{a}}}$$
$$-\frac{\sqrt{a}fx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$
$$+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)+\frac{ex^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*f*x/(16*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*e*x/(8*b
**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*f*x**3/(48*b**2*sqrt(1 + b*x
**2/a)) + sqrt(a)*d*x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*e*x**3/(
8*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**5/(24*b*sqrt(1 + b*x**2/a)
) - 5*a**3*f*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*e*as
inh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*d*asinh(sqrt(b)*x/sqrt(a)
)/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(
a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a
 > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b >
0) & (a < 0))) + e*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**7/(
6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.22088, size = 174, normalized size = 1.2

$$\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}-\frac{5\,ab^3f-6\,b^4e}{b^5}\right)x^2+\frac{3\left(8\,b^4d+5\,a^2b^2f-6\,ab^3e\right)}{b^5}\right)\sqrt{bx^2+a}x$$
$$-\frac{\left(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b - (5*a*b^3*f - 6*b^4*e)/b^5)*x^2 + 3*(8*b^4*d
+ 5*a^2*b^2*f - 6*a*b^3*e)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*
c - 8*a*b^2*d - 5*a^3*f + 6*a^2*b*e)*ln(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(7/2)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}} dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2]
)/(8*b^2) + (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e +
 3*a^2*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.289503, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2]
)/(8*b^2) + (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e +
 3*a^2*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi in Sympy [A] time = 52.7855, size = 104, normalized size = 0.89

$$\frac{fx^3\sqrt{a+bx^2}}{4b}-\frac{x\sqrt{a+bx^2}\,(3af-4be)}{8b^2}+\frac{\left(a\,(3af-4be)+8b^2d\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}}-\frac{c\sqrt{a+bx^2}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] f*x**3*sqrt(a + b*x**2)/(4*b) - x*sqrt(a + b*x**2)*(3*a*f - 4*b*e
)/(8*b**2) + (a*(3*a*f - 4*b*e) + 8*b**2*d)*atanh(sqrt(b)*x/sqrt(
a + b*x**2))/(8*b**(5/2)) - c*sqrt(a + b*x**2)/(a*x)
```

Mathematica [A] time = 0.23801, size = 100, normalized size = 0.85

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\sqrt{a+bx^2}\left(-\frac{x(3af-4be)}{8b^2}-\frac{c}{ax}+\frac{fx^3}{4b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] Sqrt[a + b*x^2]*(-(c/(a*x)) - ((-4*b*e + 3*a*f)*x)/(8*b^2) + (f*x
^3)/(4*b)) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*Log[b*x + Sqrt[b]*Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.014, size = 140, normalized size = 1.2

$$d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-\frac{c}{ax}\sqrt{bx^2+a}+\frac{ex}{2b}\sqrt{bx^2+a}-\frac{ae}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{fx^3}{4b}\sqrt{bx^2+a}-\frac{3afx}{8b^2}\sqrt{bx^2+a}+\frac{3a^2f}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] d*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-c*(b*x^2+a)^(1/2)/a/x+1/2
*e*x/b*(b*x^2+a)^(1/2)-1/2*e*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))+1/4*f*x^3*(b*x^2+a)^(1/2)/b-3/8*f*a/b^2*x*(b*x^2+a)^(1/2)+3/8
*f*a^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^2),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261276, size = 1, normalized size = 0.01

$$\left[\frac{\left(8ab^2d-4a^2be+3a^3f\right)x\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)+2\left(2abfx^4-8b^2c+\left(4abe-3a^2f\right)x^2\right)\sqrt{bx^2+a}\sqrt{b}}{16ab^{\frac{5}{2}}x},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^2),x, algorithm="fricas"
```

```
[Out] [1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*x*log(-2*sqrt(b*x^2 + a)
*b*x - (2*b*x^2 + a)*sqrt(b)) + 2*(2*a*b*f*x^4 - 8*b^2*c + (4*a*b
*e - 3*a^2*f)*x^2)*sqrt(b*x^2 + a)*sqrt(b))/(a*b^(5/2)*x), 1/8*((
8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (2*a*b*f*x^4 - 8*b^2*c + (4*a*b*e - 3*a^2*f)*x^2)*sqrt(b*x
^2 + a)*sqrt(-b))/(a*sqrt(-b)*b^2*x)]
```

Sympy [A] time = 10.7989, size = 250, normalized size = 2.14

$$-\frac{3a^{\frac{3}{2}}fx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}ex\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}fx^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{fx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*e*x*sqrt(1
+ b*x**2/a)/(2*b) - sqrt(a)*f*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a
**2*f*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*e*asinh(sqrt(b)*x
/sqrt(a))/(2*b**(3/2)) + d*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a
))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqr
t(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a
), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a + f*x**
5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.227348, size = 163, normalized size = 1.39

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,fx^2}{b}-\frac{3\,abf-4\,b^2e}{b^3}\right)x+\frac{2\,\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(8\,b^{\frac{5}{2}}d+3\,a^2\sqrt{b}f-4\,ab^{\frac{3}{2}}e\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b - (3*a*b*f - 4*b^2*e)/b^3)*x + 2*s
qrt(b)*c/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^(5/2)*
d + 3*a^2*sqrt(b)*f - 4*a*b^(3/2)*e)*ln((sqrt(b)*x - sqrt(b*x^2 +
 a))^2)/b^3
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a+bx^2}(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2]
)/(3*a^2*x) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.325037, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\sqrt{a+bx^2}(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2]
)/(3*a^2*x) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 82.5935, size = 95, normalized size = 0.86

$$\frac{fx\sqrt{a+bx^2}}{2b}-\frac{(af-2be)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}}-\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\sqrt{a+bx^2}\,(3ad-2bc)}{3a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] f*x*sqrt(a + b*x**2)/(2*b) - (a*f - 2*b*e)*atanh(sqrt(b)*x/sqrt(a
 + b*x**2))/(2*b**(3/2)) - c*sqrt(a + b*x**2)/(3*a*x**3) - sqrt(a
 + b*x**2)*(3*a*d - 2*b*c)/(3*a**2*x)
```

Mathematica [A] time = 0.183779, size = 93, normalized size = 0.85

$$\sqrt{a+bx^2}\left(\frac{2bc-3ad}{3a^2x}-\frac{c}{3ax^3}+\frac{fx}{2b}\right)+\frac{(2be-af)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (-c/(3*a*x^3) + (2*b*c - 3*a*d)/(3*a^2*x) + (f*x)/(2*b))*Sqrt[a +
 b*x^2] + ((2*b*e - a*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b
^(3/2))
```

Maple [A] time = 0.015, size = 117, normalized size = 1.1

$$e\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)\frac{1}{\sqrt{b}}+\frac{fx}{2\,b}\sqrt{bx^{2}+a}-\frac{af}{2}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{3}{2}}$$
$$-\frac{c}{3\,ax^{3}}\sqrt{bx^{2}+a}+\frac{2\,bc}{3\,xa^{2}}\sqrt{bx^{2}+a}-\frac{d}{ax}\sqrt{bx^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] e*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/2*f*x*(b*x^2+a)^(1/2)/b
-1/2*f*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/3*c*(b*x^2+a)^(1
/2)/a/x^3+2/3*c*b/a^2/x*(b*x^2+a)^(1/2)-d/a/x*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^4),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268033, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(2\,a^{2}be-a^{3}f\right)x^{3}\log\left(2\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)-2\left(3\,a^{2}fx^{4}-2\,abc+2\left(2\,b^{2}c-3\,abd\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{b}}{12\,a^{2}b^{\frac{3}{2}}x^{3}},\frac{3\,(2\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^4),x, algorithm="fricas"
```

```
[Out] [-1/12*(3*(2*a^2*b*e - a^3*f)*x^3*log(2*sqrt(b*x^2 + a)*b*x - (2*
b*x^2 + a)*sqrt(b)) - 2*(3*a^2*f*x^4 - 2*a*b*c + 2*(2*b^2*c - 3*a
*b*d)*x^2)*sqrt(b*x^2 + a)*sqrt(b))/(a^2*b^(3/2)*x^3), 1/6*(3*(2*
a^2*b*e - a^3*f)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*a^2*
f*x^4 - 2*a*b*c + 2*(2*b^2*c - 3*a*b*d)*x^2)*sqrt(b*x^2 + a)*sqrt
(-b))/(a^2*sqrt(-b)*b*x^3)]
```

Sympy [A] time = 6.06868, size = 197, normalized size = 1.79

$$\frac{\sqrt{a}fx\sqrt{1+\frac{bx^{2}}{a}}}{2b}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+e\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)$$
$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^{2}}+1}}{a}+\frac{2b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + b*x**2/a)/(2*b) - a*f*asinh(sqrt(b)*x/sqrt(a
))/(2*b**(3/2)) + e*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt
(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (
a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b >
 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

GIAC/XCAS [A] time = 0.230516, size = 238, normalized size = 2.16

$$\frac{\sqrt{bx^2+a}fx}{2\,b}+\frac{\left(a\sqrt{b}f-2\,b^{\frac{3}{2}}e\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^2}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4\sqrt{b}d+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d-2\,ab^{\frac{3}{2}}c+3\,a^2\sqrt{b}d\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + 1/4*(a*sqrt(b)*f - 2*b^(3/2)*e)*ln((s
qrt(b)*x - sqrt(b*x^2 + a))^2)/b^2 + 2/3*(3*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*sqrt(b)*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*
c - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d - 2*a*b^(3/2)*c
 + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{a+bx^2}(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{15a^3x}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2]
)/(15*a^2*x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2]
)/(15*a^3*x) + (f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.359818, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{\sqrt{a+bx^2}(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{15a^3x}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```



```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2]
)/(15*a^2*x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2]
)/(15*a^3*x) + (f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 79.0095, size = 117, normalized size = 0.99

$$\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{c\sqrt{a+bx^2}}{5ax^5}-\frac{e\sqrt{a+bx^2}}{ax}-\frac{\sqrt{a+bx^2}\,(5ad-4bc)}{15a^2x^3}+\frac{2b\sqrt{a+bx^2}\,(5ad-4bc)}{15a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] f*atanh(sqrt(b)*x/sqrt(a + b*x**2))/sqrt(b) - c*sqrt(a + b*x**2)/
(5*a*x**5) - e*sqrt(a + b*x**2)/(a*x) - sqrt(a + b*x**2)*(5*a*d -
 4*b*c)/(15*a**2*x**3) + 2*b*sqrt(a + b*x**2)*(5*a*d - 4*b*c)/(15
*a**3*x)
```

Mathematica [A] time = 0.215144, size = 98, normalized size = 0.83

$$\frac{f\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}\left(a^2\left(3c+5dx^2+15ex^4\right)-2abx^2\left(2c+5dx^2\right)+8b^2cx^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*
(3*c + 5*d*x^2 + 15*e*x^4)))/(15*a^3*x^5) + (f*Log[b*x + Sqrt[b]*
Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.015, size = 136, normalized size = 1.2

$$f\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-\frac{c}{5\,ax^5}\sqrt{bx^2+a}+\frac{4\,bc}{15\,x^3a^2}\sqrt{bx^2+a}-\frac{8\,b^2c}{15\,a^3x}\sqrt{bx^2+a}-\frac{d}{3\,ax^3}\sqrt{bx^2+a}+\frac{2\,bd}{3\,xa^2}\sqrt{bx^2+a}-\frac{e}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] f*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x
^5+4/15*c*b/a^2/x^3*(b*x^2+a)^(1/2)-8/15*c*b^2/a^3/x*(b*x^2+a)^(1
/2)-1/3*d/a/x^3*(b*x^2+a)^(1/2)+2/3*d*b/a^2/x*(b*x^2+a)^(1/2)-e/a
/x*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^6),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277702, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^3fx^5\log\left(-2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)-2\left(\left(8\,b^2c-10\,abd+15\,a^2e\right)x^4+3\,a^2c-\left(4\,abc-5\,a^2d\right)x^2\right)\sqrt{bx^2+a}\sqrt{b}}{30\,a^3\sqrt{b}x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^6),x, algorithm="fricas"
```

```
[Out] [1/30*(15*a^3*f*x^5*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sq
rt(b)) - 2*((8*b^2*c - 10*a*b*d + 15*a^2*e)*x^4 + 3*a^2*c - (4*a*
b*c - 5*a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(b))/(a^3*sqrt(b)*x^5), 1
/15*(15*a^3*f*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - ((8*b^2*c
- 10*a*b*d + 15*a^2*e)*x^4 + 3*a^2*c - (4*a*b*c - 5*a^2*d)*x^2)*s
qrt(b*x^2 + a)*sqrt(-b))/(a^3*sqrt(-b)*x^5)]
```

Sympy [A] time = 4.53568, size = 456, normalized size = 3.86

$$-\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12ab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{8b^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+f\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3
*b**6*x**8) - 3*a**2*b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a*
*5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*a*b**(
15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b*
*5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*x**2)
 + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 + f*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) &
(b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0
)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
 - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqrt(b)*e*sqrt(a/(
b*x**2) + 1)/a + 2*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

GIAC/XCAS [A] time = 0.229113, size = 437, normalized size = 3.7

$$-\frac{f\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8\sqrt{b}e+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{3}{2}}d-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a\sqrt{b}e+80\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{5}{2}}c-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] -1/2*f*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*b^(3/2)*d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(
b)*e + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 90*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(5
/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d - 60*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10
*a^3*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))
^2 - a)^5
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{a+bx^2}(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)}{105a^4x}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2]
)/(35*a^2*x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2
])/(105*a^3*x^3) + ((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3
*f)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rubi [A] time = 0.335471, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{\sqrt{a+bx^2}(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)}{105a^4x}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2]
)/(35*a^2*x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2
])/(105*a^3*x^3) + ((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3
*f)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rubi in Sympy [A] time = 94.4329, size = 163, normalized size = 1.16

$$-\frac{f\sqrt{a+bx^2}}{2bx^3}-\frac{c\sqrt{a+bx^2}}{7ax^7}-\frac{\sqrt{a+bx^2}\,(7ad-6bc)}{35a^2x^5}+\frac{\sqrt{a+bx^2}\left(105a^3f-70a^2be+56ab^2d-48b^3c\right)}{210a^3bx^3}-\frac{\sqrt{a+bx^2}\left(105a^3f-70a^2be+56ab^2d-48b^3c\right)}{105a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -f*sqrt(a + b*x**2)/(2*b*x**3) - c*sqrt(a + b*x**2)/(7*a*x**7) -
sqrt(a + b*x**2)*(7*a*d - 6*b*c)/(35*a**2*x**5) + sqrt(a + b*x**2
)*(105*a**3*f - 70*a**2*b*e + 56*a*b**2*d - 48*b**3*c)/(210*a**3*
b*x**3) - sqrt(a + b*x**2)*(105*a**3*f - 70*a**2*b*e + 56*a*b**2*
d - 48*b**3*c)/(105*a**4*x)
```

Mathematica [A] time = 0.139509, size = 103, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-a^3\left(15c+21dx^2+35x^4\left(e+3fx^2\right)\right)+2a^2bx^2\left(9c+14dx^2+35ex^4\right)-8ab^2x^4\left(3c+7dx^2\right)+48b^3cx^6\right)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*
a^2*b*x^2*(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35
*x^4*(e + 3*f*x^2))))/(105*a^4*x^7)
```

Maple [A] time = 0.009, size = 111, normalized size = 0.8

$$-\frac{105\,a^3fx^6-70\,a^2bex^6+56\,ab^2dx^6-48\,b^3cx^6+35\,a^3ex^4-28\,a^2bdx^4+24\,ab^2cx^4+21\,a^3dx^2-18\,a^2bcx^2+15\,ca^3}{105\,x^7a^4}\sqrt{bx^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(105*a^3*f*x^6-70*a^2*b*e*x^6+56*a*b^2*d*x
^6-48*b^3*c*x^6+35*a^3*e*x^4-28*a^2*b*d*x^4+24*a*b^2*c*x^4+21*a^3
*d*x^2-18*a^2*b*c*x^2+15*a^3*c)/x^7/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^8),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.416038, size = 135, normalized size = 0.96

$$\frac{\left(\left(48\,b^3c-56\,ab^2d+70\,a^2be-105\,a^3f\right)x^6-\left(24\,ab^2c-28\,a^2bd+35\,a^3e\right)x^4-15\,a^3c+3\left(6\,a^2bc-7\,a^3d\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^8),x, algorithm="fricas"
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24
*a*b^2*c - 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c -
 7*a^3*d)*x^2)*sqrt(b*x^2 + a)/(a^4*x^7)
```

Sympy [A] time = 7.21836, size = 891, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
9*a**5*b**(21/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 +
 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12
) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x*
*6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x
**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4
```

```
 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x*
*6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8
+ 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*
d*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**
6 + 15*a**3*b**6*x**8) + 30*a**2*b**(27/2)*c*x**8*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x
**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*sqrt(a/(b*x
**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*
x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12
*x**12) - 12*a*b**(15/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**
4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 16*b**(31/2)*c*
x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x*
*8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*d*
x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6
+ 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2)
- sqrt(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2)
 + 1)/(3*a**2)
```

GIAC/XCAS [A] time = 0.230394, size = 748, normalized size = 5.34

$$\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}f-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}f+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e+560\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^8),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f - 630*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a*sqrt(b)*f + 210*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*b^(3/2)*e + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(
5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*sqrt(b)*f - 910
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(3/2)*e + 1680*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*a*b^(5/2)*d - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sqrt(b)
*f + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*e - 1008*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*a^4*sqrt(b)*f - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3
/2)*e + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c - 392*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d - 630*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*a^5*sqrt(b)*f + 490*(sqrt(b)*x - sqrt(b*x^2 + a
))^2*a^4*b^(3/2)*e - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d + 105*a^
6*sqrt(b)*f - 70*a^5*b^(3/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^7
```

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx^2}(8bc-9ad)}{63a^2x^7}-\frac{\sqrt{a+bx^2}\left(21a^2e-18abd+16b^2c\right)}{105a^3x^5}-\frac{2b\sqrt{a+bx^2}\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2]
)/(63*a^2*x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2
])/(105*a^3*x^5) + ((64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3
*f)*Sqrt[a + b*x^2])/(315*a^4*x^3) - (2*b*(64*b^3*c - 72*a*b^2*d
+ 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^5*x)
```

Rubi [A] time = 0.495286, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{a+bx^2}(8bc-9ad)}{63a^2x^7}-\frac{\sqrt{a+bx^2}\left(21a^2e-18abd+16b^2c\right)}{105a^3x^5}-\frac{2b\sqrt{a+bx^2}\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2]
)/(63*a^2*x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2
])/(105*a^3*x^5) + ((64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3
*f)*Sqrt[a + b*x^2])/(315*a^4*x^3) - (2*b*(64*b^3*c - 72*a*b^2*d
+ 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^5*x)
```

Rubi in Sympy [A] time = 101.347, size = 216, normalized size = 1.14

$$-\frac{f\sqrt{a+bx^2}}{4bx^5}-\frac{c\sqrt{a+bx^2}}{9ax^9}-\frac{\sqrt{a+bx^2}\,(9ad-8bc)}{63a^2x^7}+\frac{\sqrt{a+bx^2}\left(105a^3f-84a^2be+72ab^2d-64b^3c\right)}{420a^3bx^5}-\frac{\sqrt{a+bx^2}\left(105a^3f-84a^2be+72ab^2d-64b^3c\right)}{315a^4x^3}+\frac{2b\sqrt{a+bx^2}\left(105a^3f-84a^2be+72ab^2d-64b^3c\right)}{315a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -f*sqrt(a + b*x**2)/(4*b*x**5) - c*sqrt(a + b*x**2)/(9*a*x**9) -
sqrt(a + b*x**2)*(9*a*d - 8*b*c)/(63*a**2*x**7) + sqrt(a + b*x**2
)*(105*a**3*f - 84*a**2*b*e + 72*a*b**2*d - 64*b**3*c)/(420*a**3*
b*x**5) - sqrt(a + b*x**2)*(105*a**3*f - 84*a**2*b*e + 72*a*b**2*
d - 64*b**3*c)/(315*a**4*x**3) + 2*b*sqrt(a + b*x**2)*(105*a**3*f
 - 84*a**2*b*e + 72*a*b**2*d - 64*b**3*c)/(315*a**5*x)
```

Mathematica [A] time = 0.150821, size = 134, normalized size = 0.71

$$-\frac{\sqrt{a+bx^2}\left(a^4\left(35c+45dx^2+63ex^4+105fx^6\right)-2a^3bx^2\left(20c+27dx^2+42ex^4+105fx^6\right)+24a^2b^2x^4\left(2c+3dx^2+7ex^4\right)\right.}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) +
 24*a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 2
7*d*x^2 + 42*e*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4
 + 105*f*x^6)))/(315*a^5*x^9)
```

Maple [A] time = 0.01, size = 157, normalized size = 0.8

$$-\frac{-210\,a^3bfx^8+168\,a^2b^2ex^8-144\,ab^3dx^8+128\,b^4cx^8+105\,a^4fx^6-84\,a^3bex^6+72\,a^2b^2dx^6-64\,ab^3cx^6+63\,a^4ex^4-54}{315\,x^9a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(1/2)*(-210*a^3*b*f*x^8+168*a^2*b^2*e*x^8-144*a*
b^3*d*x^8+128*b^4*c*x^8+105*a^4*f*x^6-84*a^3*b*e*x^6+72*a^2*b^2*d
*x^6-64*a*b^3*c*x^6+63*a^4*e*x^4-54*a^3*b*d*x^4+48*a^2*b^2*c*x^4+
45*a^4*d*x^2-40*a^3*b*c*x^2+35*a^4*c)/x^9/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^10),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.716944, size = 190, normalized size = 1.01

$$-\frac{\left(2\left(64\,b^4c-72\,ab^3d+84\,a^2b^2e-105\,a^3bf\right)x^8-\left(64\,ab^3c-72\,a^2b^2d+84\,a^3be-105\,a^4f\right)x^6+35\,a^4c+3\left(16\,a^2b^2c-18\right.}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^10),x, algorithm="fricas
```

```
[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^
8 - (64*a*b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35
*a^4*c + 3*(16*a^2*b^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*
b*c - 9*a^4*d)*x^2)*sqrt(b*x^2 + a)/(a^5*x^9)
```

Sympy [A] time = 11.0686, size = 1642, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 +
1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x
**14 + 315*a**5*b**20*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(
b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*
a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16)
- 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b
**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b
*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b
**11*x**10 + 35*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt
(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1
890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**
16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*
x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12
*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 126
0*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x
**4*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8
 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*
e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqrt(a/(b*x**2) +
1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18
*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b
**(25/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**
6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a*
*3*b**(11/2)*e*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*
a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 560*a**2*b**(45/2)*c*x**12*
sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10
 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20
*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*
b**12*x**12) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a
**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b*
*(47/2)*c*x**14*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*
a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
+ 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x*
*10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2
) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8
) - 128*b**(49/2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x*
*8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) + 16*b**(31/2)*d*x**12*sqrt(a/(
b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*
b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8*sqrt(a/(b
*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f
*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

GIAC/XCAS [A] time = 0.228773, size = 900, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6 + e*x^4 + d*x^2 + c)/(sqrt(b*x^2 + a)*x^10),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f - 1995*(sqr
t(b)*x - sqrt(b*x^2 + a))^12*a*b^(3/2)*f + 840*(sqrt(b)*x - sqrt(
b*x^2 + a))^12*b^(5/2)*e + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
b^(7/2)*d + 5355*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*f -
 3780*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*e + 8064*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*a*b^(7/2)*d - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(3/2)*f + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*e - 5
376*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x
```

```
 - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*d + 6825*(sqrt(b)*x - sqrt(b*x^
2 + a))^6*a^4*b^(3/2)*f - 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^
3*b^(5/2)*e + 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c
- 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)*d - 3465*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f + 3024*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*a^4*b^(5/2)*e - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)
*d + 945*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f - 756*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*e + 64*a^4*b^(9/2)*c - 72
*a^5*b^(7/2)*d - 105*a^7*b^(3/2)*f + 84*a^6*b^(5/2)*e)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^9
```

3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=381

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16ab^7}+\frac{x^3\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{24a^2b^6}-\frac{x^5\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{30a^2b^5\sqrt{a+bx^2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)}{16b^{15/2}}+\frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2
)) - ((2*A*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*
b^3*(a + b*x^2)^(5/2)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C +
143*a^2*D))*x^7)/(210*a^2*b^4*(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3
*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143
*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*A*b^3 - 3*a*(24
*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7) + (
(16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b
*x^2])/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429
*a^3*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rubi [A] time = 1.62045, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16ab^7}+\frac{x^3\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{24a^2b^6}-\frac{x^5\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{30a^2b^5\sqrt{a+bx^2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)}{16b^{15/2}}+\frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```


```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2
)) - ((2*A*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*
b^3*(a + b*x^2)^(5/2)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C +
143*a^2*D))*x^7)/(210*a^2*b^4*(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3
*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143
*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*A*b^3 - 3*a*(24
*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7) + (
(16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b
*x^2])/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429
*a^3*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.524869, size = 252, normalized size = 0.66

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16b^{15/2}}+\frac{x\left(45045a^6D-2310a^5b\left(9C-65Dx^2\right)+42a^4b^2\left(180B-1650Cx^2+4147Dx^4\right)-12a^3b^3\left(140A-2100Bx^2+6699Cx^4-6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B
 - 1650*C*x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 66
99*C*x^4 - 6292*D*x^6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*
C*x^4 + 455*D*x^6) + a^2*b^4*x^2*(-5600*A + 29232*B*x^2 - 34848*C
*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A + 35*(6*B*x^2 + 3*C*x^4 +
2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((16*A*b^3 - 3*a*(24*b
^2*B - 66*a*b*C + 143*a^2*D))*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])
/(16*b^(15/2))
```

Maple [A] time = 0.402, size = 517, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 9/2*B*a/b^5*x/(b*x^2+a)^(1/2)-11/8*C*a/b^2*x^9/(b*x^2+a)^(7/2)-99
/56*C*a^2/b^3*x^7/(b*x^2+a)^(7/2)-13/24*D*a/b^2*x^11/(b*x^2+a)^(7
/2)+143/48*D*a^2/b^3*x^9/(b*x^2+a)^(7/2)+429/112*D*a^3/b^4*x^7/(b
*x^2+a)^(7/2)+9/14*B*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*B*a/b^3*x^5/(
b*x^2+a)^(5/2)+3/2*B*a/b^4*x^3/(b*x^2+a)^(3/2)+429/80*D*a^3/b^5*x
^5/(b*x^2+a)^(5/2)+143/16*D*a^3/b^6*x^3/(b*x^2+a)^(3/2)+429/16*D*
a^3/b^7*x/(b*x^2+a)^(1/2)-99/40*C*a^2/b^4*x^5/(b*x^2+a)^(5/2)-33/
8*C*a^2/b^5*x^3/(b*x^2+a)^(3/2)-99/8*C*a^2/b^6*x/(b*x^2+a)^(1/2)+
A/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/7*A*x^7/b/(b*x^2+a)^(7/
2)-1/5*A/b^2*x^5/(b*x^2+a)^(5/2)-1/3*A/b^3*x^3/(b*x^2+a)^(3/2)-A/
b^4*x/(b*x^2+a)^(1/2)+1/4*C*x^11/b/(b*x^2+a)^(7/2)+99/8*C*a^2/b^(
13/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*B*x^9/b/(b*x^2+a)^(7/2)-9
/2*B*a/b^(11/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*D*x^13/b/(b*x^2
+a)^(7/2)-429/16*D*a^3/b^(15/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^8/(b*x^2 + a)^(9/2),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 1.11271, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^8/(b*x^2 + a)^(9/2),x, algorithm="fricas"
```

```
[Out] [1/3360*(2*(280*D*b^6*x^13 - 70*(13*D*a*b^5 - 6*C*b^6)*x^11 + 35*
(143*D*a^2*b^4 - 66*C*a*b^5 + 24*B*b^6)*x^9 + 176*(429*D*a^3*b^3
- 198*C*a^2*b^4 + 72*B*a*b^5 - 16*A*b^6)*x^7 + 406*(429*D*a^4*b^2
 - 198*C*a^3*b^3 + 72*B*a^2*b^4 - 16*A*a*b^5)*x^5 + 350*(429*D*a^
5*b - 198*C*a^4*b^2 + 72*B*a^3*b^3 - 16*A*a^2*b^4)*x^3 + 105*(429
*D*a^6 - 198*C*a^5*b + 72*B*a^4*b^2 - 16*A*a^3*b^3)*x)*sqrt(b*x^2
 + a)*sqrt(b) - 105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6
- 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a
^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a
*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16*
A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 -
16*A*a^3*b^4)*x^2)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqr
t(b)))/((b^11*x^8 + 4*a*b^10*x^6 + 6*a^2*b^9*x^4 + 4*a^3*b^8*x^2
+ a^4*b^7)*sqrt(b)), 1/1680*((280*D*b^6*x^13 - 70*(13*D*a*b^5 - 6
*C*b^6)*x^11 + 35*(143*D*a^2*b^4 - 66*C*a*b^5 + 24*B*b^6)*x^9 + 1
76*(429*D*a^3*b^3 - 198*C*a^2*b^4 + 72*B*a*b^5 - 16*A*b^6)*x^7 +
406*(429*D*a^4*b^2 - 198*C*a^3*b^3 + 72*B*a^2*b^4 - 16*A*a*b^5)*x
^5 + 350*(429*D*a^5*b - 198*C*a^4*b^2 + 72*B*a^3*b^3 - 16*A*a^2*b
^4)*x^3 + 105*(429*D*a^6 - 198*C*a^5*b + 72*B*a^4*b^2 - 16*A*a^3*
b^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 105*((429*D*a^3*b^4 - 198*C*a^
2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72
*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72
*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 +
 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^
2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)))/((b^11*x^8 + 4*a*b^10*x^6 + 6*a^2*b^9*x^4 + 4*a^3*b^8*x^
2 + a^4*b^7)*sqrt(-b))]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A]   time = 0.231411, size = 462, normalized size = 1.21

$$\frac{\left(\left(\left(\left(35\left(2\left(\frac{4\,Dx^2}{b}-\frac{13\,Da^4b^{11}-6\,Ca^3b^{12}}{a^3b^{13}}\right)x^2+\frac{143\,Da^5b^{10}-66\,Ca^4b^{11}+24\,Ba^3b^{12}}{a^3b^{13}}\right)x^2+\frac{176\left(429\,Da^6b^9-198\,Ca^5b^{10}+72\,Ba^4b^{11}-16\,Aa^3b^{12}\right)}{a^3b^{13}}\right)x^2+\cdots}{16\cdots}$$

$$+\frac{\left(429\,Da^3-198\,Ca^2b+72\,Bab^2-16\,Ab^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^8/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3
*b^13))*x^2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a
^3*b^13))*x^2 + 176*(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^
11 - 16*A*a^3*b^12)/(a^3*b^13))*x^2 + 406*(429*D*a^7*b^8 - 198*C*
a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)/(a^3*b^13))*x^2 + 350*(4
29*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A*a^5*b^10)/(a^3
*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*b^8 -
 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3
- 198*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*ln(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(15/2)
```

3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=279

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(99a^2D-36abC+8b^2B)}{8b^{13/2}}-\frac{x\sqrt{a+bx^2}\,(99a^2D-36abC+8b^2B)}{8ab^6}+\frac{x^3(99a^2D-36abC+8b^2B)}{12ab^5\sqrt{a+bx^2}}+\frac{x^5(99a^2D-36abC+8b^2B)}{60ab^4(a+bx^2)^{3/2}}+\frac{x^7(3a^2D-2abC+b^2B)}{5ab^3(a+bx^2)^{5/2}}+\frac{Dx^7}{4b^3(a+bx^2)^{3/2}}$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2
)) + ((b^2*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)
) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(
3/2)) + (D*x^7)/(4*b^3*(a + b*x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C
+ 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^2]) - ((8*b^2*B - 36*a*b*
C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b^2*B - 36*a*b*C
 + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/2))
```

Rubi [A] time = 0.989022, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(99a^2D-36abC+8b^2B)}{8b^{13/2}}-\frac{x\sqrt{a+bx^2}\,(99a^2D-36abC+8b^2B)}{8ab^6}+\frac{x^3(99a^2D-36abC+8b^2B)}{12ab^5\sqrt{a+bx^2}}+\frac{x^5(99a^2D-36abC+8b^2B)}{60ab^4(a+bx^2)^{3/2}}+\frac{x^7(3a^2D-2abC+b^2B)}{5ab^3(a+bx^2)^{5/2}}+\frac{Dx^7}{4b^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2
)) + ((b^2*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)
) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(
3/2)) + (D*x^7)/(4*b^3*(a + b*x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C
+ 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^2]) - ((8*b^2*B - 36*a*b*
C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b^2*B - 36*a*b*C
 + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.384186, size = 208, normalized size = 0.75

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\left(99a^2D-36abC+8b^2B\right)}{8b^{13/2}} + \frac{x\left(-10395a^6D+630a^5b\left(6C-55Dx^2\right)-42a^4b^2\left(20B-300Cx^2+957Dx^4\right)-8a^3b^3x^2\left(350B-1827Cx^2+2178Dx^4\right)+a^2\right.}{840ab^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-10395*a^6*D + 120*A*b^6*x^6 + 630*a^5*b*(6*C - 55*D*x^2) + a
^2*b^4*x^4*(-3248*B + 6336*C*x^2 - 1155*D*x^4) - 42*a^4*b^2*(20*B
 - 300*C*x^2 + 957*D*x^4) - 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2
178*D*x^4) + 2*a*b^5*x^6*(-704*B + 105*(2*C*x^2 + D*x^4))))/(840*
a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*Log[b
*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(13/2))
```

Maple [A] time = 0.016, size = 460, normalized size = 1.7

$$\begin{aligned}
&-\frac{Ax^5}{2b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{5\,aAx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15\,a^2Ax}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,aAx}{56\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}\\
&+\frac{Ax}{14\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Ax}{7\,ab^3}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Bx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}\\
&-\frac{Bx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^4}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{Cx^9}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}\\
&+\frac{9\,aCx^7}{14\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{9\,aCx^5}{10\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{3\,aCx^3}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}\\
&+\frac{9\,Cxa}{2\,b^5}\frac{1}{\sqrt{bx^2+a}}-\frac{9\,aC}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{11}{2}}+\frac{Dx^{11}}{4\,b}\left(bx^2+a\right)^{-\frac{7}{2}}\\
&-\frac{11\,aDx^9}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{99\,a^2Dx^7}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{99\,a^2Dx^5}{40\,b^4}\left(bx^2+a\right)^{-\frac{5}{2}}\\
&-\frac{33\,Dx^3a^2}{8\,b^5}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{99\,Dxa^2}{8\,b^6}\frac{1}{\sqrt{bx^2+a}}+\frac{99\,a^2D}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{13}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2*A*x^5/b/(b*x^2+a)^(7/2)-5/8*A*a/b^2*x^3/(b*x^2+a)^(7/2)-15/5
6*A*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*A*a/b^3*x/(b*x^2+a)^(5/2)+1/14
*A/b^3*x/(b*x^2+a)^(3/2)+1/7*A/a/b^3*x/(b*x^2+a)^(1/2)-1/7*B*x^7/
b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^3/(b*
x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))+1/2*C*x^9/b/(b*x^2+a)^(7/2)+9/14*C*a/b^2*x^7/(b*x^2+a
)^(7/2)+9/10*C*a/b^3*x^5/(b*x^2+a)^(5/2)+3/2*C*a/b^4*x^3/(b*x^2+a
)^(3/2)+9/2*C*a/b^5*x/(b*x^2+a)^(1/2)-9/2*C*a/b^(11/2)*ln(x*b^(1/
2)+(b*x^2+a)^(1/2))+1/4*D*x^11/b/(b*x^2+a)^(7/2)-11/8*D*a/b^2*x^9
/(b*x^2+a)^(7/2)-99/56*D*a^2/b^3*x^7/(b*x^2+a)^(7/2)-99/40*D*a^2/
b^4*x^5/(b*x^2+a)^(5/2)-33/8*D*a^2/b^5*x^3/(b*x^2+a)^(3/2)-99/8*D
*a^2/b^6*x/(b*x^2+a)^(1/2)+99/8*D*a^2/b^(13/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.722354, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="fricas"
```

```
[Out] [1/1680*(2*(210*D*a*b^5*x^11 - 105*(11*D*a^2*b^4 - 4*C*a*b^5)*x^9
 - 8*(2178*D*a^3*b^3 - 792*C*a^2*b^4 + 176*B*a*b^5 - 15*A*b^6)*x^
7 - 406*(99*D*a^4*b^2 - 36*C*a^3*b^3 + 8*B*a^2*b^4)*x^5 - 350*(99
*D*a^5*b - 36*C*a^4*b^2 + 8*B*a^3*b^3)*x^3 - 105*(99*D*a^6 - 36*C
*a^5*b + 8*B*a^4*b^2)*x)*sqrt(b*x^2 + a)*sqrt(b) + 105*((99*D*a^3
*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 36*C*a^6*b + 8*
B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6 + 6
*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b
- 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*log(-2*sqrt(b*x^2 + a)*b*x - (
2*b*x^2 + a)*sqrt(b)))/((a*b^10*x^8 + 4*a^2*b^9*x^6 + 6*a^3*b^8*x
^4 + 4*a^4*b^7*x^2 + a^5*b^6)*sqrt(b)), 1/840*((210*D*a*b^5*x^11
- 105*(11*D*a^2*b^4 - 4*C*a*b^5)*x^9 - 8*(2178*D*a^3*b^3 - 792*C*
a^2*b^4 + 176*B*a*b^5 - 15*A*b^6)*x^7 - 406*(99*D*a^4*b^2 - 36*C*
a^3*b^3 + 8*B*a^2*b^4)*x^5 - 350*(99*D*a^5*b - 36*C*a^4*b^2 + 8*B
*a^3*b^3)*x^3 - 105*(99*D*a^6 - 36*C*a^5*b + 8*B*a^4*b^2)*x)*sqrt
(b*x^2 + a)*sqrt(-b) + 105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*
b^6)*x^8 + 99*D*a^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3
- 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^
3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3
)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^10*x^8 + 4*a^2*b
^9*x^6 + 6*a^3*b^8*x^4 + 4*a^4*b^7*x^2 + a^5*b^6)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228934, size = 358, normalized size = 1.28

$$\frac{\left(\left(\left(\left(105 \left(\frac{2 Dx^2}{b} - \frac{11 Da^4b^9 - 4 Ca^3b^{10}}{a^3b^{11}}\right)x^2 - \frac{8 \left(2178 Da^5b^8 - 792 Ca^4b^9 + 176 Ba^3b^{10} - 15 Aa^2b^{11}\right)}{a^3b^{11}}\right)x^2 - \frac{406 \left(99 Da^6b^7 - 36 Ca^5b^8 + 8 Ba^4b^9\right)}{a^3b^{11}}\right)x^2 - 3 \cdots}{840 \left(bx^2 + a\right)^{\frac{7}{2}}}$$
$$- \frac{\left(99 Da^2 - 36 Cab + 8 Bb^2\right) \ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{8 b^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^6/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^1
1))*x^2 - 8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15
*A*a^2*b^11)/(a^3*b^11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 +
 8*B*a^4*b^9)/(a^3*b^11))*x^2 - 350*(99*D*a^7*b^6 - 36*C*a^6*b^7
+ 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99*D*a^8*b^5 - 36*C*a^7*b^6
```

```
 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) - 1/8*(99*D*a^2 -
 36*C*a*b + 8*B*b^2)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(13/
2)
```

3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=210

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{Dx\sqrt{a+bx^2}}{2b^5}$$

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2
)) + ((2*A*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*
b^3*(a + b*x^2)^(5/2)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(
3/2)) - ((4*b*C - 15*a*D)*x)/(3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[
a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(11/2))
```

Rubi [A] time = 0.980274, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{Dx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2
)) + ((2*A*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*
b^3*(a + b*x^2)^(5/2)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(
3/2)) - ((4*b*C - 15*a*D)*x)/(3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[
a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(11/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.367485, size = 174, normalized size = 0.83

$$\frac{x\left(945a^6D-210a^5b\left(C-15Dx^2\right)+14a^4b^2x^2\left(261Dx^2-50C\right)+4a^3b^3x^4\left(396Dx^2-203C\right)+a^2b^4x^6\left(105Dx^2-352C\right)+6\right.}{210a^2b^5\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*
a^5*b*(C - 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*
b^2*x^2*(-50*C + 261*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2))
)/(210*a^2*b^5*(a + b*x^2)^(7/2)) + ((2*b*C - 9*a*D)*Log[b*x + Sq
rt[b]*Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Maple [B] time = 0.02, size = 405, normalized size = 1.9

$$-\frac{Ax^3}{4\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{3\,aAx}{28\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,Ax}{140\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Ax}{35\,ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$
$$+\frac{2\,Ax}{35\,a^2b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx^5}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{5\,aBx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15\,Bxa^2}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$+\frac{3\,Bxa}{56\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Bx}{14\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Bx}{7\,ab^3}\frac{1}{\sqrt{bx^2+a}}-\frac{Cx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$-\frac{Cx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Cx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Cx}{b^4}\frac{1}{\sqrt{bx^2+a}}+C\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$
$$+\frac{Dx^9}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{9\,aDx^7}{14\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{9\,aDx^5}{10\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}$$
$$+\frac{3\,Dx^3a}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{9\,Dxa}{2\,b^5}\frac{1}{\sqrt{bx^2+a}}-\frac{9\,aD}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/4*A*x^3/b/(b*x^2+a)^(7/2)-3/28*A*a/b^2*x/(b*x^2+a)^(7/2)+3/140
*A/b^2*x/(b*x^2+a)^(5/2)+1/35*A/a/b^2*x/(b*x^2+a)^(3/2)+2/35*A/a^
2/b^2*x/(b*x^2+a)^(1/2)-1/2*B*x^5/b/(b*x^2+a)^(7/2)-5/8*B*a/b^2*x
^3/(b*x^2+a)^(7/2)-15/56*B*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*B*a/b^3
*x/(b*x^2+a)^(5/2)+1/14*B/b^3*x/(b*x^2+a)^(3/2)+1/7*B/a/b^3*x/(b*
x^2+a)^(1/2)-1/7*C*x^7/b/(b*x^2+a)^(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^
(5/2)-1/3*C/b^3*x^3/(b*x^2+a)^(3/2)-C/b^4*x/(b*x^2+a)^(1/2)+C/b^(
9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*D*x^9/b/(b*x^2+a)^(7/2)+9/
14*D*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*D*a/b^3*x^5/(b*x^2+a)^(5/2)+3
/2*D*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*D*a/b^5*x/(b*x^2+a)^(1/2)-9/2*
D*a/b^(11/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.517611, size = 1, normalized size = 0.

$$\left[\frac{2\left(105\,Da^2b^4x^9+2\left(792\,Da^3b^3-176\,Ca^2b^4+15\,Bab^5+6\,Ab^6\right)x^7+14\left(261\,Da^4b^2-58\,Ca^3b^3+3\,Aab^5\right)x^5+350\left(9\,Da^5b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="fricas"
```

```
[Out] [1/420*(2*(105*D*a^2*b^4*x^9 + 2*(792*D*a^3*b^3 - 176*C*a^2*b^4 +
 15*B*a*b^5 + 6*A*b^6)*x^7 + 14*(261*D*a^4*b^2 - 58*C*a^3*b^3 + 3
*A*a*b^5)*x^5 + 350*(9*D*a^5*b - 2*C*a^4*b^2)*x^3 + 105*(9*D*a^6
- 2*C*a^5*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 105*((9*D*a^3*b^4 - 2*C
*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D*a^4*b^3 - 2*C*a^3*b^
4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*a^6*b - 2*C*a
^5*b^2)*x^2)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))
/((a^2*b^9*x^8 + 4*a^3*b^8*x^6 + 6*a^4*b^7*x^4 + 4*a^5*b^6*x^2 +
a^6*b^5)*sqrt(b)), 1/210*((105*D*a^2*b^4*x^9 + 2*(792*D*a^3*b^3 -
 176*C*a^2*b^4 + 15*B*a*b^5 + 6*A*b^6)*x^7 + 14*(261*D*a^4*b^2 -
58*C*a^3*b^3 + 3*A*a*b^5)*x^5 + 350*(9*D*a^5*b - 2*C*a^4*b^2)*x^3
 + 105*(9*D*a^6 - 2*C*a^5*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 105*((
9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D*a^4
*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(
9*D*a^6*b - 2*C*a^5*b^2)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))
/((a^2*b^9*x^8 + 4*a^3*b^8*x^6 + 6*a^4*b^7*x^4 + 4*a^5*b^6*x^2 +
a^6*b^5)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229591, size = 274, normalized size = 1.3

$$\frac{\left(\left(\left(\left(\frac{105\,Dx^2}{b}+\frac{2\,(792\,Da^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10})}{a^3b^9}\right)x^2+\frac{14\,(261\,Da^5b^6-58\,Ca^4b^7+3\,Aa^2b^9)}{a^3b^9}\right)x^2+\frac{350\,(9\,Da^6b^5-2\,Ca^5b^6)}{a^3b^9}\right)x^2+\frac{105\,(9\,D}{}}{210\,(bx^2+a)^{\frac{7}{2}}}$$

$$+\frac{(9\,Da-2\,Cb)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a
^2*b^9 + 6*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^
4*b^7 + 3*A*a^2*b^9)/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*
b^6)/(a^3*b^9))*x^2 + 105*(9*D*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*
x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b)*ln(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(11/2)
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x^3\left(Ab^3-10a^3D\right)}{3ab^3\left(a+bx^2\right)^{7/2}}-\frac{a^3Dx}{b^4\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(-176a^3D+15a^2bC+6ab^2B+8Ab^3\right)}{105a^3b\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(-58a^3D+3ab^2B+4Ab^3\right)}{15a^2b^2\left(a+bx^2\right)^{7/2}}+\frac{D\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -((a^3*D*x)/(b^4*(a + b*x^2)^(7/2))) + ((A*b^3 - 10*a^3*D)*x^3)/(
3*a*b^3*(a + b*x^2)^(7/2)) + ((4*A*b^3 + 3*a*b^2*B - 58*a^3*D)*x^
5)/(15*a^2*b^2*(a + b*x^2)^(7/2)) + ((8*A*b^3 + 6*a*b^2*B + 15*a^
2*b*C - 176*a^3*D)*x^7)/(105*a^3*b*(a + b*x^2)^(7/2)) + (D*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.760792, antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{x^3\left(a\left(17a^2D-10abC+3b^2B\right)+4Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{105a^3b^3\left(a+bx^2\right)^{3/2}}+\frac{D\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{Dx}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2
)) + ((4*A*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*
b^3*(a + b*x^2)^(5/2)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a
^2*D))*x^3)/(105*a^3*b^3*(a + b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a +
 b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.382749, size = 147, normalized size = 0.82

$$\frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5-176a^3b^3Dx^7+a^2b^4x^3\left(35A+21Bx^2+15Cx^4\right)+2ab^5x^5\left(14A+3Bx^2\right)+8Ab^6x^7}{105a^3b^4\left(a+bx^2\right)^{7/2}}+\frac{D\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7
 - 176*a^3*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3
*(35*A + 21*B*x^2 + 15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) +
(D*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [B] time = 0.016, size = 363, normalized size = 2.

$$-\frac{Ax}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{Ax}{35\,ab}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{4\,Ax}{105\,a^2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{8\,Ax}{105\,a^3b}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx^3}{4\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{3\,Bxa}{28\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,Bx}{140\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Bx}{35\,ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,Bx}{35\,a^2b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{Cx^5}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{5\,aCx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15\,Cxa^2}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,Cxa}{56\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Cx}{14\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Cx}{7\,ab^3}\frac{1}{\sqrt{bx^2+a}}-\frac{Dx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Dx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Dx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Dx}{b^4}\frac{1}{\sqrt{bx^2+a}}+D\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*A/b*x/(b*x^2+a)^(7/2)+1/35*A/a/b*x/(b*x^2+a)^(5/2)+4/105*A/a
^2/b*x/(b*x^2+a)^(3/2)+8/105*A/a^3/b*x/(b*x^2+a)^(1/2)-1/4*B*x^3/
b/(b*x^2+a)^(7/2)-3/28*B*a/b^2*x/(b*x^2+a)^(7/2)+3/140*B/b^2*x/(b
*x^2+a)^(5/2)+1/35*B/a/b^2*x/(b*x^2+a)^(3/2)+2/35*B*x/a^2/b^2/(b*
x^2+a)^(1/2)-1/2*C*x^5/b/(b*x^2+a)^(7/2)-5/8*C*a/b^2*x^3/(b*x^2+a
)^(7/2)-15/56*C*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*C*a/b^3*x/(b*x^2+a
)^(5/2)+1/14*C/b^3*x/(b*x^2+a)^(3/2)+1/7*C/a/b^3*x/(b*x^2+a)^(1/2
)-1/7*D*x^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5/(b*x^2+a)^(5/2)-1/3*D
/b^3*x^3/(b*x^2+a)^(3/2)-D*x/b^4/(b*x^2+a)^(1/2)+D/b^(9/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="maxima"
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.461723, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(105\,Da^6x+\left(176\,Da^3b^3-15\,Ca^2b^4-6\,Bab^5-8\,Ab^6\right)x^7+7\left(58\,Da^4b^2-3\,Ba^2b^4-4\,Aab^5\right)x^5+35\left(10\,Da^5b-Aa^2b^4\right)\cdots}{210\left(a^3b^8x^8+4\,a^4b^7x^6+6\,a^5\cdots\right.}\right.$$

$$-\frac{\left(105\,Da^6x+\left(176\,Da^3b^3-15\,Ca^2b^4-6\,Bab^5-8\,Ab^6\right)x^7+7\left(58\,Da^4b^2-3\,Ba^2b^4-4\,Aab^5\right)x^5+35\left(10\,Da^5b-Aa^2b^4\right)x^3\cdots}{105\left(a^3b^8x^8+4\,a^4b^7x^6+6\,a^5b^6x^4+4\,a^6b^5\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="fricas"
```

```
[Out] [-1/210*(2*(105*D*a^6*x + (176*D*a^3*b^3 - 15*C*a^2*b^4 - 6*B*a*b
^5 - 8*A*b^6)*x^7 + 7*(58*D*a^4*b^2 - 3*B*a^2*b^4 - 4*A*a*b^5)*x^
5 + 35*(10*D*a^5*b - A*a^2*b^4)*x^3)*sqrt(b*x^2 + a)*sqrt(b) - 10
5*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b*
x^2 + D*a^7)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))
/((a^3*b^8*x^8 + 4*a^4*b^7*x^6 + 6*a^5*b^6*x^4 + 4*a^6*b^5*x^2 +
a^7*b^4)*sqrt(b)), -1/105*((105*D*a^6*x + (176*D*a^3*b^3 - 15*C*a
^2*b^4 - 6*B*a*b^5 - 8*A*b^6)*x^7 + 7*(58*D*a^4*b^2 - 3*B*a^2*b^4
 - 4*A*a*b^5)*x^5 + 35*(10*D*a^5*b - A*a^2*b^4)*x^3)*sqrt(b*x^2 +
 a)*sqrt(-b) - 105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2
*x^4 + 4*D*a^6*b*x^2 + D*a^7)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))
/((a^3*b^8*x^8 + 4*a^4*b^7*x^6 + 6*a^5*b^6*x^4 + 4*a^6*b^5*x^2 +
a^7*b^4)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228807, size = 216, normalized size = 1.21

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9\right)x^2}{a^3b^7}+\frac{7\left(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8\right)}{a^3b^7}\right)+\frac{35\left(10\,Da^5b^4-Aa^2b^7\right)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{D\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9
)*x^2/(a^3*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3
*b^7)) + 35*(10*D*a^5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3
/b^4)*x/(b*x^2 + a)^(7/2) - D*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(9/2)
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=134

$$\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x
^2)^(7/2)) + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*
x^2)^(7/2)) + ((48*A*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)
/(105*a^4*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.404128, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x
^2)^(7/2)) + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*
x^2)^(7/2)) + ((48*A*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)
/(105*a^4*(a + b*x^2)^(7/2))
```

Rubi in Sympy [A] time = 82.8628, size = 206, normalized size = 1.54

$$\frac{x\left(Ab^3-Bab^2+Ca^2b-Da^3\right)}{7ab^3\left(a+bx^2\right)^{\frac{7}{2}}}+\frac{x\left(6Ab^3+Bab^2-8Ca^2b+15Da^3\right)}{35a^2b^3\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{x\left(24Ab^3+4Bab^2+3Ca^2b-10Da^3+35Da^2bx^2\right)}{105a^3b^3\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{2x\left(24Ab^3+4Bab^2+3Ca^2b-10Da^3\right)}{105a^4b^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] x*(A*b**3 - B*a*b**2 + C*a**2*b - D*a**3)/(7*a*b**3*(a + b*x**2)*
*(7/2)) + x*(6*A*b**3 + B*a*b**2 - 8*C*a**2*b + 15*D*a**3)/(35*a*
*2*b**3*(a + b*x**2)**(5/2)) + x*(24*A*b**3 + 4*B*a*b**2 + 3*C*a*
*2*b - 10*D*a**3 + 35*D*a**2*b*x**2)/(105*a**3*b**3*(a + b*x**2)*
*(3/2)) + 2*x*(24*A*b**3 + 4*B*a*b**2 + 3*C*a**2*b - 10*D*a**3)/(
105*a**4*b**3*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.13307, size = 98, normalized size = 0.73

$$\frac{a^3\left(105Ax+35Bx^3+21Cx^5+15Dx^7\right)+2a^2bx^3\left(105A+14Bx^2+3Cx^4\right)+8ab^2x^5\left(21A+Bx^2\right)+48Ab^3x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A +
 14*B*x^2 + 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*
```

```
x^7))/(105*a^4*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.008, size = 109, normalized size = 0.8

$$\frac{x\left(48 Ab^{3}x^{6}+8 Bab^{2}x^{6}+6 a^{2}bCx^{6}+15 Da^{3}x^{6}+168 aAb^{2}x^{4}+28 Ba^{2}bx^{4}+21 a^{3}Cx^{4}+210 a^{2}Abx^{2}+35 a^{3}Bx^{2}+105 Aa^{3}\right)}{105 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x*(48*A*b^3*x^6+8*B*a*b^2*x^6+6*C*a^2*b*x^6+15*D*a^3*x^6+16
8*A*a*b^2*x^4+28*B*a^2*b*x^4+21*C*a^3*x^4+210*A*a^2*b*x^2+35*B*a^
3*x^2+105*A*a^3)/(b*x^2+a)^(7/2)/a^4
```

Maxima [A] time = 1.37042, size = 452, normalized size = 3.37

$$-\frac{Dx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{5\,Dax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Cx^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^{2}+a}a^{4}}+\frac{8\,Ax}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,Ax}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{Ax}{7\,(bx^{2}+a)^{\frac{7}{2}}a}+\frac{Dx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Dx}{7\,\sqrt{bx^{2}+a}ab^{3}}+\frac{3\,Dax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Da^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{3\,Cx}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Cx}{35\,\sqrt{bx^{2}+a}a^{2}b^{2}}+\frac{Cx}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Cax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Bx}{7\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^{2}+a}a^{3}b}+\frac{4\,Bx}{105\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b}+\frac{Bx}{35\,(bx^{2}+a)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)
*b^2) - 1/4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 +
 a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 +
a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2
+ a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((
b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3
/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*
b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 +
a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b
*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x
/((b*x^2 + a)^(5/2)*a*b)
```

Fricas [A] time = 0.358683, size = 190, normalized size = 1.42

$$\frac{\left(\left(15\,Da^{3}+6\,Ca^{2}b+8\,Bab^{2}+48\,Ab^{3}\right)x^{7}+7\left(3\,Ca^{3}+4\,Ba^{2}b+24\,Aab^{2}\right)x^{5}+105\,Aa^{3}x+35\left(Ba^{3}+6\,Aa^{2}b\right)x^{3}\right)\sqrt{bx^{2}+a}}{105\,(a^{4}b^{4}x^{8}+4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C
*a^3 + 4*B*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*
A*a^2*b)*x^3)*sqrt(b*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^
6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22427, size = 177, normalized size = 1.32

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ba^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*
x^2/(a^4*b^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b
^3)) + 35*(B*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(
b*x^2 + a)^(7/2)
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)
^(7/2)) - ((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(
7/2)) - ((4*b*(48*A*b^2 - a*(6*b*B + a*C)) - 3*a^3*D)*x^5)/(15*a^
4*(a + b*x^2)^(7/2)) - (2*b*(4*b*(48*A*b^2 - a*(6*b*B + a*C)) - 3
*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.498562, antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)
^(7/2)) - ((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(
7/2)) - ((192*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4
*(a + b*x^2)^(7/2)) - (2*b*(192*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a
^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rubi in Sympy [A] time = 155.612, size = 230, normalized size = 1.24

$$-\frac{Dx}{4b^2\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{x\left(\frac{Ab^3}{x^2}-\frac{Bab^2}{x^2}+\frac{Ca^2b}{x^2}-\frac{Da^3}{x^2}\right)}{7ab^3\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{Bb^2-Cab+Da^2}{ab^3x\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{x\left(24Bb^2-28Cab+27Da^2\right)}{20a^2b^2\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{x\left(24Bb^2-28Cab+27Da^2\right)}{15a^3b^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{2x\left(24Bb^2-28Cab+27Da^2\right)}{15a^4b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -D*x/(4*b**2*(a + b*x**2)**(5/2)) + x*(A*b**3/x**2 - B*a*b**2/x**
2 + C*a**2*b/x**2 - D*a**3/x**2)/(7*a*b**3*(a + b*x**2)**(7/2)) -
 (B*b**2 - C*a*b + D*a**2)/(a*b**3*x*(a + b*x**2)**(5/2)) - x*(24
*B*b**2 - 28*C*a*b + 27*D*a**2)/(20*a**2*b**2*(a + b*x**2)**(5/2)
) - x*(24*B*b**2 - 28*C*a*b + 27*D*a**2)/(15*a**3*b**2*(a + b*x**
2)**(3/2)) - 2*x*(24*B*b**2 - 28*C*a*b + 27*D*a**2)/(15*a**4*b**2
*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.180635, size = 133, normalized size = 0.72

$$\frac{-7a^4\left(15A-15Bx^2-5Cx^4-3Dx^6\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-210A+21Bx^2+Cx^4\right)+48}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-
210*A + 21*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*
D*x^6) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(
105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.01, size = 157, normalized size = 0.9

$$-\frac{384 Ab^4x^8 - 48 Bab^3x^8 - 8 Ca^2b^2x^8 - 6 Da^3bx^8 + 1344 Aab^3x^6 - 168 Ba^2b^2x^6 - 28 Ca^3bx^6 - 21 Da^4x^6 + 1680 Aa^2b^2x^4 -}{105 xa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^
8+1344*A*a*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+
1680*A*a^2*b^2*x^4-210*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-1
05*B*a^4*x^2+105*A*a^4)/x/(b*x^2+a)^(7/2)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.418137, size = 246, normalized size = 1.33

$$\frac{\left(2\left(3 Da^3b + 4 Ca^2b^2 + 24 Bab^3 - 192 Ab^4\right)x^8 + 7\left(3 Da^4 + 4 Ca^3b + 24 Ba^2b^2 - 192 Aab^3\right)x^6 - 105 Aa^4 + 35\left(Ca^4 + 6 Ba^3b\right.}{105\left(a^5b^4x^9 + 4 a^6b^3x^7 + 6 a^7b^2x^5 + 4 a^8bx^3 + a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="frica
```

```
[Out] 1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 +
 7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A
*a^4 + 35*(C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8
*A*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a
^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.224518, size = 285, normalized size = 1.54

$$\frac{\left(\left(x^2\left(\frac{\left(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right)x^2+\frac{105}{}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

$$+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm="giac"
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A
*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*
a^11*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^1
2*b^4 - 30*A*a^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^
12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=242

$$\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{3a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b
*x^2)^(7/2)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x
^2)^(7/2)) + ((160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(
3*a^4*(a + b*x^2)^(7/2)) + (4*b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*
C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/2)) + (8*b^2*(160*A*b^3 -
 a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.670315, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{3a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b
*x^2)^(7/2)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x
^2)^(7/2)) + ((160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(
3*a^4*(a + b*x^2)^(7/2)) + (4*b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*
C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/2)) + (8*b^2*(160*A*b^3 -
 a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a + b*x^2)^(7/2))
```

Rubi in Sympy [A] time = 163.22, size = 270, normalized size = 1.12

$$-\frac{D}{6b^2x\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{x\left(\frac{Ab^3}{x^4}-\frac{Bab^2}{x^4}+\frac{Ca^2b}{x^4}-\frac{Da^3}{x^4}\right)}{7ab^3\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{Bb^2-Cab+Da^2}{3ab^3x^3\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{16Bb^2-22Cab+23Da^2}{6a^2b^2x\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{x\left(16Bb^2-22Cab+23Da^2\right)}{5a^3b\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{4x\left(16Bb^2-22Cab+23Da^2\right)}{15a^4b\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{8x\left(16Bb^2-22Cab+23Da^2\right)}{15a^5b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] -D/(6*b**2*x*(a + b*x**2)**(5/2)) + x*(A*b**3/x**4 - B*a*b**2/x**
4 + C*a**2*b/x**4 - D*a**3/x**4)/(7*a*b**3*(a + b*x**2)**(7/2)) -
 (B*b**2 - C*a*b + D*a**2)/(3*a*b**3*x**3*(a + b*x**2)**(5/2)) +
(16*B*b**2 - 22*C*a*b + 23*D*a**2)/(6*a**2*b**2*x*(a + b*x**2)**(
5/2)) + x*(16*B*b**2 - 22*C*a*b + 23*D*a**2)/(5*a**3*b*(a + b*x**
2)**(5/2)) + 4*x*(16*B*b**2 - 22*C*a*b + 23*D*a**2)/(15*a**4*b*(a
 + b*x**2)**(3/2)) + 8*x*(16*B*b**2 - 22*C*a*b + 23*D*a**2)/(15*a
```

```
**5*b*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.211512, size = 165, normalized size = 0.68

$$\frac{-35a^5\left(A+3Bx^2-3Cx^4-Dx^6\right)+14a^4bx^2\left(25A-60Bx^2+15Cx^4+2Dx^6\right)+8a^3b^2x^4\left(350A-210Bx^2+21Cx^4+Dx^6\right)+}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^
6*(350*A - 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 -
D*x^6) + 8*a^3*b^2*x^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 1
4*a^4*b*x^2*(25*A - 60*B*x^2 + 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*
(a + b*x^2)^(7/2))
```

Maple [A] time = 0.012, size = 205, normalized size = 0.9

$$-\frac{-1280\,Ab^5x^{10}+384\,Bab^4x^{10}-48\,Ca^2b^3x^{10}-8\,Da^3b^2x^{10}-4480\,Aab^4x^8+1344\,Ba^2b^3x^8-168\,Ca^3b^2x^8-28\,Da^4bx^8-5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-1280*A*b^5*x^10+384*B*a*b^4*x^10-48*C*a^2*b^3*x^10-8*D*a
^3*b^2*x^10-4480*A*a*b^4*x^8+1344*B*a^2*b^3*x^8-168*C*a^3*b^2*x^8
-28*D*a^4*b*x^8-5600*A*a^2*b^3*x^6+1680*B*a^3*b^2*x^6-210*C*a^4*b
*x^6-35*D*a^5*x^6-2800*A*a^3*b^2*x^4+840*B*a^4*b*x^4-105*C*a^5*x^
4-350*A*a^4*b*x^2+105*B*a^5*x^2+35*A*a^5)/x^3/(b*x^2+a)^(7/2)/a^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^4),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.680248, size = 304, normalized size = 1.26

$$\frac{\left(8\left(Da^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5\right)x^{10}+28\left(Da^4b+6\,Ca^3b^2-48\,Ba^2b^3+160\,Aab^4\right)x^8+35\left(Da^5+6\,Ca^4b-48\,Ba^3\right.\right.}{105\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^4),x, algorithm="frica
```

```
[Out] 1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10
+ 28*(D*a^4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 3
5*(D*a^5 + 6*C*a^4*b - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a
^5 + 35*(3*C*a^5 - 24*B*a^4*b + 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 -
```

```
10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 +
6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22656, size = 471, normalized size = 1.95

$$\frac{\left(\left(x^2\left(\frac{\left(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8\right)x^2}{a^{18}b^3}+\frac{7\left(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6+365\,Aa^{13}b^7\right)}{a^{18}b^3}\right)+\frac{35\left(Da^{17}b^3+6\,Ca^{16}b^4-30\,Ba^{15}b^5+80\,Aa^{1}\right.}{a^{18}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^4),x, algorithm="giac"
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790
*A*a^12*b^8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 1
32*B*a^14*b^6 + 365*A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*
C*a^16*b^4 - 30*B*a^15*b^5 + 80*A*a^14*b^6)/(a^18*b^3))*x^2 + 105
*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*b^5)/(a^18*b^3))*x/(b*x^2
 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)
- 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{35a^5\left(a+bx^2\right)^{5/2}}-\frac{x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{21a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a
+ b*x^2)^(7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b
*x^2)^(7/2)) - ((192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x
)/(21*a^4*(a + b*x^2)^(7/2)) - (2*(192*A*b^3 - a*(80*b^2*B - 24*a
*b*C + 3*a^2*D))*x)/(35*a^5*(a + b*x^2)^(5/2)) - (8*(192*A*b^3 -
a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a + b*x^2)^(3/2))
 - (16*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^
7*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.843296, antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{35a^5\left(a+bx^2\right)^{5/2}}-\frac{x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{21a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a
+ b*x^2)^(7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b
*x^2)^(7/2)) - ((192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)
/(21*a^4*(a + b*x^2)^(7/2)) - (2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a
*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^(5/2)) - (8*(192*A*b^3 - a*
(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a + b*x^2)^(3/2)) -
 (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(105*a^7*S
qrt[a + b*x^2])
```

Rubi in Sympy [A] time = 171.121, size = 309, normalized size = 1.1

$$-\frac{D}{8b^2x^3\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{x\left(\frac{Ab^3}{x^6}-\frac{Bab^2}{x^6}+\frac{Ca^2b}{x^6}-\frac{Da^3}{x^6}\right)}{7ab^3\left(a+bx^2\right)^{\frac{7}{2}}}-\frac{Bb^2-Cab+Da^2}{5ab^3x^5\left(a+bx^2\right)^{\frac{5}{2}}}+\frac{16Bb^2-24Cab+27Da^2}{24a^2b^2x^3\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{16Bb^2-24Cab+27Da^2}{3a^3bx\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{2x\left(16Bb^2-24Cab+27Da^2\right)}{5a^4\left(a+bx^2\right)^{\frac{5}{2}}}-\frac{8x\left(16Bb^2-24Cab+27Da^2\right)}{15a^5\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{16x\left(16Bb^2-24Cab+27Da^2\right)}{15a^6\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] -D/(8*b**2*x**3*(a + b*x**2)**(5/2)) + x*(A*b**3/x**6 - B*a*b**2/
x**6 + C*a**2*b/x**6 - D*a**3/x**6)/(7*a*b**3*(a + b*x**2)**(7/2)
) - (B*b**2 - C*a*b + D*a**2)/(5*a*b**3*x**5*(a + b*x**2)**(5/2))
 + (16*B*b**2 - 24*C*a*b + 27*D*a**2)/(24*a**2*b**2*x**3*(a + b*x
**2)**(5/2)) - (16*B*b**2 - 24*C*a*b + 27*D*a**2)/(3*a**3*b*x*(a
+ b*x**2)**(5/2)) - 2*x*(16*B*b**2 - 24*C*a*b + 27*D*a**2)/(5*a**
4*(a + b*x**2)**(5/2)) - 8*x*(16*B*b**2 - 24*C*a*b + 27*D*a**2)/(
15*a**5*(a + b*x**2)**(3/2)) - 16*x*(16*B*b**2 - 24*C*a*b + 27*D*
a**2)/(15*a**6*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.293513, size = 202, normalized size = 0.72

$$\frac{-7a^6\left(3A+5x^2\left(B+3Cx^2-3Dx^4\right)\right)+14a^5bx^2\left(6A+25Bx^2-60Cx^4+15Dx^6\right)+56a^4b^2x^4\left(-15A+50Bx^2-30Cx^4+3Dx\right.}{105a^7x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^
4*x^8*(105*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350
*B*x^2 - 84*C*x^4 + 3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 -
 30*C*x^4 + 3*D*x^6) + 14*a^5*b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 +
15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*x^2 - 3*D*x^4)))/(105*a^7
*x^5*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.012, size = 253, normalized size = 0.9

$$-\frac{3072\,Ab^6x^{12}-1280\,Bab^5x^{12}+384\,Ca^2b^4x^{12}-48\,Da^3b^3x^{12}+10752\,Aab^5x^{10}-4480\,Ba^2b^4x^{10}+1344\,Ca^3b^3x^{10}-168\,Da^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(3072*A*b^6*x^12-1280*B*a*b^5*x^12+384*C*a^2*b^4*x^12-48*D
*a^3*b^3*x^12+10752*A*a*b^5*x^10-4480*B*a^2*b^4*x^10+1344*C*a^3*b
^3*x^10-168*D*a^4*b^2*x^10+13440*A*a^2*b^4*x^8-5600*B*a^3*b^3*x^8
+1680*C*a^4*b^2*x^8-210*D*a^5*b*x^8+6720*A*a^3*b^3*x^6-2800*B*a^4
*b^2*x^6+840*C*a^5*b*x^6-105*D*a^6*x^6+840*A*a^4*b^2*x^4-350*B*a^
5*b*x^4+105*C*a^6*x^4-84*A*a^5*b*x^2+35*B*a^6*x^2+21*A*a^6)/x^5/(
b*x^2+a)^(7/2)/a^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^6),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.12548, size = 365, normalized size = 1.3

$$\frac{\left(16\left(3\,Da^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6\right)x^{12}+56\left(3\,Da^4b^2-24\,Ca^3b^3+80\,Ba^2b^4-192\,Aab^5\right)x^{10}+70\left(3\,Da^5b-24\,C\right.\right.}{105\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^6),x, algorithm="frica
```

```
[Out] 1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x
^12 + 56*(3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5
)*x^10 + 70*(3*D*a^5*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*
b^4)*x^8 - 21*A*a^6 + 35*(3*D*a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 1
92*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a^5*b + 24*A*a^4*b^2)*x^4
- 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a^7*b^4*x^13 + 4
*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.233374, size = 799, normalized size = 2.84

$$\frac{\left(\left(x^2\left(\frac{\left(48\,Da^{18}b^6-279\,Ca^{17}b^7+790\,Ba^{16}b^8-1686\,Aa^{15}b^9\right)x^2}{a^{22}b^3}+\frac{7\left(24\,Da^{19}b^5-132\,Ca^{18}b^6+365\,Ba^{17}b^7-768\,Aa^{16}b^8\right)}{a^{22}b^3}\right)+\frac{35\left(6\,Da^{20}b^4-30\,Ca^{19}b^5+80\,Ba^{18}b^{6}\cdots\right.}{a^{22}b^3}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

$$+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca^2\sqrt{b}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6C\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^6),x, algorithm="giac"
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1
686*A*a^15*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^
6 + 365*B*a^17*b^7 - 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b
^4 - 30*C*a^19*b^5 + 80*B*a^18*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*
x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 10*B*a^19*b^5 - 20*A*a^18*
b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(sqrt(b)*x - sqrt
(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*A*a*b^(5/2) + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqr
t(b) - 430*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 1260*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 60*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=334

$$-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3(a+bx^2)^{7/2}}+\frac{2Ab-aB}{5a^2x^5(a+bx^2)^{7/2}}+\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7(a+bx^2)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{35a^6(a+bx^2)^{5/2}}+\frac{8bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{21a^5(a+bx^2)^{7/2}}+\frac{48Ab^3-a\left(3a^2D-10abC+24b^2B\right)}{3a^4x(a+bx^2)^{7/2}}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*
x^2)^(7/2)) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*
x^2)^(7/2)) + (48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a
^4*x*(a + b*x^2)^(7/2)) + (8*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C
 + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2)) + (16*b*(48*A*b^3 - a*
(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b*x^2)^(5/2)) +
(64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105*a^7*
(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C +
3*a^2*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.992973, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3(a+bx^2)^{7/2}}+\frac{2Ab-aB}{5a^2x^5(a+bx^2)^{7/2}}+\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7(a+bx^2)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{35a^6(a+bx^2)^{5/2}}+\frac{8bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{21a^5(a+bx^2)^{7/2}}+\frac{48Ab^3-a\left(3a^2D-10abC+24b^2B\right)}{3a^4x(a+bx^2)^{7/2}}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```



```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*
x^2)^(7/2)) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*
x^2)^(7/2)) + (48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a
^4*x*(a + b*x^2)^(7/2)) + (8*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C
 + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2)) + (16*b*(48*A*b^3 - a*
(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b*x^2)^(5/2)) +
(64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105*a^7*
(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C +
3*a^2*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 179.705, size = 354, normalized size = 1.06

$$-\frac{D}{10b^2x^5(a+bx^2)^{\frac{5}{2}}}+\frac{x\left(\frac{Ab^3}{x^8}-\frac{Bab^2}{x^8}+\frac{Ca^2b}{x^8}-\frac{Da^3}{x^8}\right)}{7ab^3(a+bx^2)^{\frac{7}{2}}}-\frac{Bb^2-Cab+Da^2}{7ab^3x^7(a+bx^2)^{\frac{5}{2}}}+\frac{24Bb^2-38Cab+45Da^2}{70a^2b^2x^5(a+bx^2)^{\frac{5}{2}}}-\frac{24Bb^2-38Cab+45Da^2}{21a^3bx^3(a+bx^2)^{\frac{5}{2}}}+\frac{8\left(24Bb^2-38Cab+45Da^2\right)}{21a^4x(a+bx^2)^{\frac{5}{2}}}+\frac{16bx\left(24Bb^2-38Cab+45Da^2\right)}{35a^5(a+bx^2)^{\frac{5}{2}}}+\frac{64bx\left(24Bb^2-38Cab+45Da^2\right)}{105a^6(a+bx^2)^{\frac{3}{2}}}+\frac{128bx\left(24Bb^2-38Cab+45Da^2\right)}{105a^7\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] -D/(10*b**2*x**5*(a + b*x**2)**(5/2)) + x*(A*b**3/x**8 - B*a*b**2
/x**8 + C*a**2*b/x**8 - D*a**3/x**8)/(7*a*b**3*(a + b*x**2)**(7/2
)) - (B*b**2 - C*a*b + D*a**2)/(7*a*b**3*x**7*(a + b*x**2)**(5/2)
) + (24*B*b**2 - 38*C*a*b + 45*D*a**2)/(70*a**2*b**2*x**5*(a + b*
x**2)**(5/2)) - (24*B*b**2 - 38*C*a*b + 45*D*a**2)/(21*a**3*b*x**
3*(a + b*x**2)**(5/2)) + 8*(24*B*b**2 - 38*C*a*b + 45*D*a**2)/(21
*a**4*x*(a + b*x**2)**(5/2)) + 16*b*x*(24*B*b**2 - 38*C*a*b + 45*
D*a**2)/(35*a**5*(a + b*x**2)**(5/2)) + 64*b*x*(24*B*b**2 - 38*C*
a*b + 45*D*a**2)/(105*a**6*(a + b*x**2)**(3/2)) + 128*b*x*(24*B*b
**2 - 38*C*a*b + 45*D*a**2)/(105*a**7*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.428251, size = 234, normalized size = 0.7

$$\frac{-a^7\left(15A+21Bx^2+35x^4\left(C+3Dx^2\right)\right)+14a^6bx^2\left(3A+6Bx^2+25Cx^4-60Dx^6\right)-56a^5b^2x^4\left(3A+15Bx^2-50Cx^4+30Dx\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x
^10*(105*A - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 +
25*C*x^4 - 60*D*x^6) + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^
4 - 12*D*x^6) + 128*a^3*b^4*x^8*(105*A - 105*B*x^2 + 35*C*x^4 - 3
*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2 - 50*C*x^4 + 30*D*x^6) -
 a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(105*a^8*x^7*(a +
b*x^2)^(7/2))
```

Maple [A] time = 0.012, size = 301, normalized size = 0.9

$$-\frac{-6144\,Ab^7x^{14}+3072\,Bab^6x^{14}-1280\,Ca^2b^5x^{14}+384\,Da^3b^4x^{14}-21504\,Aab^6x^{12}+10752\,Ba^2b^5x^{12}-4480\,Ca^3b^4x^{12}+134}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6144*A*b^7*x^14+3072*B*a*b^6*x^14-1280*C*a^2*b^5*x^14+38
4*D*a^3*b^4*x^14-21504*A*a*b^6*x^12+10752*B*a^2*b^5*x^12-4480*C*a
^3*b^4*x^12+1344*D*a^4*b^3*x^12-26880*A*a^2*b^5*x^10+13440*B*a^3*
b^4*x^10-5600*C*a^4*b^3*x^10+1680*D*a^5*b^2*x^10-13440*A*a^3*b^4*
x^8+6720*B*a^4*b^3*x^8-2800*C*a^5*b^2*x^8+840*D*a^6*b*x^8-1680*A*
a^4*b^3*x^6+840*B*a^5*b^2*x^6-350*C*a^6*b*x^6+105*D*a^7*x^6+168*A
*a^5*b^2*x^4-84*B*a^6*b*x^4+35*C*a^7*x^4-42*A*a^6*b*x^2+21*B*a^7*
x^2+15*A*a^7)/x^7/(b*x^2+a)^(7/2)/a^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^8),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96954, size = 420, normalized size = 1.26

$$-\frac{\left(128\left(3\,Da^{3}b^{4}-10\,Ca^{2}b^{5}+24\,Bab^{6}-48\,Ab^{7}\right)x^{14}+448\left(3\,Da^{4}b^{3}-10\,Ca^{3}b^{4}+24\,Ba^{2}b^{5}-48\,Aab^{6}\right)x^{12}+560\left(3\,Da^{5}b^{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^8),x, algorithm="frica
```

```
[Out] -1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*
x^14 + 448*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^
6)*x^12 + 560*(3*D*a^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a
^2*b^5)*x^10 + 280*(3*D*a^6*b - 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*
A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*a^7 - 10*C*a^6*b + 24*B*a^5*b
^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^6*b + 24*A*a^5*b^2)*
x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8*b^4*x^15 +
 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.255198, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^8),x, algorithm="giac"
```

```
[Out] Done
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{256b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{5/2}}-\frac{16b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{63a^6\left(a+bx^2\right)^{7/2}}-\frac{2b\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{9a^5x\left(a+bx^2\right)^{7/2}}+\frac{128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)}{45a^4x^3\left(a+bx^2\right)^{7/2}}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a
+ b*x^2)^(7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a +
 b*x^2)^(7/2)) + (128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D)
)/(45*a^4*x^3*(a + b*x^2)^(7/2)) - (2*b*(128*A*b^3 - 3*a*(24*b^2*
B - 12*a*b*C + 5*a^2*D)))/(9*a^5*x*(a + b*x^2)^(7/2)) - (16*b^2*(
128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))*x)/(63*a^6*(a +
b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5
*a^2*D))*x)/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 3
*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))*x)/(315*a^8*(a + b*x^2)^(3/2)
) - (256*b^2*(128*A*b^3 - 3*a*(24*b^2*B - 12*a*b*C + 5*a^2*D))*x)
/(315*a^9*Sqrt[a + b*x^2])
```

Rubi [A] time = 1.13509, antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{256b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{105a^7\left(a+bx^2\right)^{5/2}}-\frac{16b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{63a^6\left(a+bx^2\right)^{7/2}}-\frac{2b\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{9a^5x\left(a+bx^2\right)^{7/2}}+\frac{-15a^3D-36ab(2bB-aC)+128Ab^3}{45a^4x^3\left(a+bx^2\right)^{7/2}}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a
+ b*x^2)^(7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a +
 b*x^2)^(7/2)) + (128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(4
5*a^4*x^3*(a + b*x^2)^(7/2)) - (2*b*(128*A*b^3 - 36*a*b*(2*b*B -
a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)^(7/2)) - (16*b^2*(128*A*b^
3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*(a + b*x^2)^(7/2)
) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(105
*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a
*C) - 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*
b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2
])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.475293, size = 270, normalized size = 0.69

$$\frac{-a^8\left(35A+45Bx^2+63Cx^4+105Dx^6\right)+2a^7bx^2\left(40A+21\left(3Bx^2+6Cx^4+25Dx^6\right)\right)-56a^6b^2x^4\left(4A+9Bx^2+45Cx^4-150\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2
*b^6*x^12*(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*
B*x^2 + 45*C*x^4 - 150*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2
- 9*C*x^4 + 3*D*x^6) + 256*a^3*b^5*x^10*(-280*A + 315*B*x^2 - 126
*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^2 + 63*C*x^4 + 105*D*x^6)
 + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 150*D*x^6) + 2*a
^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^9*x^9
*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.013, size = 349, normalized size = 0.9

$$-\frac{32768\,Ab^8x^{16}-18432\,Bab^7x^{16}+9216\,Ca^2b^6x^{16}-3840\,Da^3b^5x^{16}+114688\,Aab^7x^{14}-64512\,Ba^2b^6x^{14}+32256\,Ca^3b^5x^{14}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/315*(32768*A*b^8*x^16-18432*B*a*b^7*x^16+9216*C*a^2*b^6*x^16-3
840*D*a^3*b^5*x^16+114688*A*a*b^7*x^14-64512*B*a^2*b^6*x^14+32256
*C*a^3*b^5*x^14-13440*D*a^4*b^4*x^14+143360*A*a^2*b^6*x^12-80640*
B*a^3*b^5*x^12+40320*C*a^4*b^4*x^12-16800*D*a^5*b^3*x^12+71680*A*
a^3*b^5*x^10-40320*B*a^4*b^4*x^10+20160*C*a^5*b^3*x^10-8400*D*a^6
*b^2*x^10+8960*A*a^4*b^4*x^8-5040*B*a^5*b^3*x^8+2520*C*a^6*b^2*x^
8-1050*D*a^7*b*x^8-896*A*a^5*b^3*x^6+504*B*a^6*b^2*x^6-252*C*a^7*
b*x^6+105*D*a^8*x^6+224*A*a^6*b^2*x^4-126*B*a^7*b*x^4+63*C*a^8*x^
4-80*A*a^7*b*x^2+45*B*a^8*x^2+35*A*a^8)/x^9/(b*x^2+a)^(7/2)/a^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^10),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.1156, size = 478, normalized size = 1.22

$$\frac{\left(256\left(15\,Da^3b^5-36\,Ca^2b^6+72\,Bab^7-128\,Ab^8\right)x^{16}+896\left(15\,Da^4b^4-36\,Ca^3b^5+72\,Ba^2b^6-128\,Aab^7\right)x^{14}+1120\left(15\,Da\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^10),x, algorithm="fric
```

```
[Out] 1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)
*x^16 + 896*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a
*b^7)*x^14 + 1120*(15*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 1
28*A*a^2*b^6)*x^12 + 560*(15*D*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*
b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8 + 70*(15*D*a^7*b - 36*C*a^6*
b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(15*D*a^8 - 36*C*a^7*
b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 18*B*a^7*b +
 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 + a
)/(a^9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^
11 + a^13*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259431, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^10),x, algorithm="giac
```

```
[Out] Done
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (
a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/
(3*b^6) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)
^(5/2))/(5*b^6) + ((b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2)
)/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(9/2))/(9*b^6) + (f*(a + b
*x^2)^(11/2))/(11*b^6)
```

Rubi [A] time = 0.463463, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (
a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/
(3*b^6) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)
^(5/2))/(5*b^6) + ((b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2)
)/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(9/2))/(9*b^6) + (f*(a + b
*x^2)^(11/2))/(11*b^6)
```

Rubi in Sympy [A] time = 86.4853, size = 206, normalized size = 0.96

$$-\frac{a^2\sqrt{a+bx^2}\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^6}+\frac{a\left(a+bx^2\right)^{\frac{3}{2}}\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{3b^6}+\frac{f\left(a+bx^2\right)^{\frac{11}{2}}}{11b^6}-\frac{\left(a+bx^2\right)^{\frac{9}{2}}(5af-be)}{9b^6}+\frac{\left(a+bx^2\right)^{\frac{7}{2}}\left(10a^2f-4abe+b^2d\right)}{7b^6}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}\left(10a^3f-6a^2be+3ab^2d-b^3c\right)}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] -a**2*sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b*
*6 + a*(a + b*x**2)**(3/2)*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d -
2*b**3*c)/(3*b**6) + f*(a + b*x**2)**(11/2)/(11*b**6) - (a + b*x*
*2)**(9/2)*(5*a*f - b*e)/(9*b**6) + (a + b*x**2)**(7/2)*(10*a**2*
f - 4*a*b*e + b**2*d)/(7*b**6) - (a + b*x**2)**(5/2)*(10*a**3*f -
```

```
6*a**2*b*e + 3*a*b**2*d - b**3*c)/(5*b**6)
```

Mathematica [A] time = 0.222092, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-2\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a
^3*b^2*(99*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2
 + 66*e*x^4 + 50*f*x^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*
x^4 + 175*f*x^6) + b^5*x^4*(693*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f
*x^6))))/(3465*b^6)
```

Maple [A] time = 0.01, size = 193, normalized size = 0.9

$$-\frac{-315\,fx^{10}b^5+350\,ab^4fx^8-385\,b^5ex^8-400\,a^2b^3fx^6+440\,ab^4ex^6-495\,b^5dx^6+480\,a^3b^2fx^4-528\,a^2b^3ex^4+594\,ab^4d}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*
e*x^8-400*a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2
*f*x^4-528*a^2*b^3*e*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*
f*x^2+704*a^3*b^2*e*x^2-792*a^2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^
5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2*b^3*c)/b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11 + e*x^9 + d*x^7 + c*x^5)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262701, size = 239, normalized size = 1.12

$$\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4be-1280\right.}{3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11 + e*x^9 + d*x^7 + c*x^5)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b
^5*d - 88*a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3
*b^2*d + 1408*a^4*b*e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d +
 176*a^2*b^3*e - 160*a^3*b^2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^
```

```
3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*sqrt(b*x^2 + a)/b^6
```

Sympy [A] time = 6.76848, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} + \frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3} \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*s
qrt(a + b*x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(69
3*b**5) - 16*a**3*d*sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*s
qrt(a + b*x**2)/(315*b**4) - 32*a**3*f*x**4*sqrt(a + b*x**2)/(231
*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b**3) + 8*a**2*d*x**2*sqrt
(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b*x**2)/(105*b**
3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2*sqrt
(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) -
8*a*e*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x*
*2)/(99*b**2) + c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b
*x**2)/(7*b) + e*x**8*sqrt(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b
*x**2)/(11*b), Ne(b, 0)), ((c*x**6/6 + d*x**8/8 + e*x**10/10 + f*
x**12/12)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.230981, size = 387, normalized size = 1.81

$$\frac{693\left(bx^2+a\right)^{\frac{5}{2}}b^3c - 2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c + 3465\sqrt{bx^2+a}a^2b^3c + 495\left(bx^2+a\right)^{\frac{7}{2}}b^2d - 2079\left(bx^2+a\right)^{\frac{5}{2}}ab^2d + 3465\left(bx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^11 + e*x^9 + d*x^7 + c*x^5)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/3465*(693*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^
3*c + 3465*sqrt(b*x^2 + a)*a^2*b^3*c + 495*(b*x^2 + a)^(7/2)*b^2*
d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*b
^2*d - 3465*sqrt(b*x^2 + a)*a^3*b^2*d + 315*(b*x^2 + a)^(11/2)*f
- 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f - 693
0*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f - 3465*s
qrt(b*x^2 + a)*a^5*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 +
a)^(7/2)*a*b*e + 4158*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a
)^(3/2)*a^3*b*e + 3465*sqrt(b*x^2 + a)*a^4*b*e)/b^6
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d)}{5b^5}+\frac{(a+bx^2)^{3/2}(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{7/2}(be-4af)}{7b^5}+\frac{f(a+bx^2)^{9/2}}{9b^5}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) +
((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*
b^5) + ((b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) +
((b*e - 4*a*f)*(a + b*x^2)^(7/2))/(7*b^5) + (f*(a + b*x^2)^(9/2))
/(9*b^5)
```

Rubi [A] time = 0.365978, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d)}{5b^5}+\frac{(a+bx^2)^{3/2}(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{7/2}(be-4af)}{7b^5}+\frac{f(a+bx^2)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) +
((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*
b^5) + ((b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) +
((b*e - 4*a*f)*(a + b*x^2)^(7/2))/(7*b^5) + (f*(a + b*x^2)^(9/2))
/(9*b^5)
```

Rubi in Sympy [A] time = 73.8337, size = 156, normalized size = 0.93

$$\frac{a\sqrt{a+bx^2}(a^3f-a^2be+ab^2d-b^3c)}{b^5}+\frac{f(a+bx^2)^{\frac{9}{2}}}{9b^5}-\frac{(a+bx^2)^{\frac{7}{2}}(4af-be)}{7b^5}+\frac{(a+bx^2)^{\frac{5}{2}}(6a^2f-3abe+b^2d)}{5b^5}-\frac{(a+bx^2)^{\frac{3}{2}}(4a^3f-3a^2be+2ab^2d-b^3c)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] a*sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b**5 +
 f*(a + b*x**2)**(9/2)/(9*b**5) - (a + b*x**2)**(7/2)*(4*a*f - b*
e)/(7*b**5) + (a + b*x**2)**(5/2)*(6*a**2*f - 3*a*b*e + b**2*d)/(
5*b**5) - (a + b*x**2)**(3/2)*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d
 - b**3*c)/(3*b**5)
```

Mathematica [A] time = 0.154335, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(105c\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b
^2*(7*d + 3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x
^4 + 20*f*x^6) + b^4*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)
))/(315*b^5)
```

Maple [A] time = 0.008, size = 145, normalized size = 0.9

$$\frac{35\, f x^8 b^4 - 40\, ab^3 f x^6 + 45\, b^4 e x^6 + 48\, a^2 b^2 f x^4 - 54\, ab^3 e x^4 + 63\, b^4 d x^4 - 64\, a^3 b f x^2 + 72\, a^2 b^2 e x^2 - 84\, ab^3 d x^2 + 105\, b^4 c x^2 +}{315\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+4
8*a^2*b^2*f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2
*b^2*e*x^2-84*a*b^3*d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168
*a^2*b^2*d-210*a*b^3*c)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^7 + d*x^5 + c*x^3)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262873, size = 181, normalized size = 1.08

$$\frac{\left(35\, b^4 f x^8 + 5\left(9\, b^4 e - 8\, ab^3 f\right) x^6 - 210\, ab^3 c + 168\, a^2 b^2 d - 144\, a^3 be + 128\, a^4 f + 3\left(21\, b^4 d - 18\, ab^3 e + 16\, a^2 b^2 f\right) x^4 + (105}{315\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^9 + e*x^7 + d*x^5 + c*x^3)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c +
 168*a^2*b^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3
*e + 16*a^2*b^2*f)*x^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e -
 64*a^3*b*f)*x^2)*sqrt(b*x^2 + a)/b^5
```

Sympy [A] time = 4.01114, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4 f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3 e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3 f x^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2 d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2 e x^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2 f x^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - \frac{6ae \cdots}{} \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqr
t(a + b*x**2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b*
*4) + 8*a**2*d*sqrt(a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a
+ b*x**2)/(35*b**3) + 16*a**2*f*x**4*sqrt(a + b*x**2)/(105*b**3)
- 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*d*x**2*sqrt(a + b*x**2)/(
15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*f*x**6*sqr
t(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*x**4*
sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*s
qrt(a + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/
8 + f*x**10/10)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.224144, size = 296, normalized size = 1.77

$$\frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c-315\sqrt{bx^2+a}ab^3c+63\left(bx^2+a\right)^{\frac{5}{2}}b^2d-210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+315\sqrt{bx^2+a}a^2b^2d+35\left(bx^2+a\right)^{\frac{9}{2}}f-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9 + e*x^7 + d*x^5 + c*x^3)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*b^3*c - 315*sqrt(b*x^2 + a)*a*b^3*c
+ 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)*a*b^2*d + 31
5*sqrt(b*x^2 + a)*a^2*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2
 + a)^(7/2)*a*f + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(
3/2)*a^3*f + 315*sqrt(b*x^2 + a)*a^4*f + 45*(b*x^2 + a)^(7/2)*b*e
 - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e -
315*sqrt(b*x^2 + a)*a^3*b*e)/b^5
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2
*d - 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*
f)*(a + b*x^2)^(5/2))/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.284795, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2
*d - 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*
f)*(a + b*x^2)^(5/2))/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi in Sympy [A] time = 58.647, size = 110, normalized size = 0.91

$$\frac{f\left(a+bx^2\right)^{\frac{7}{2}}}{7b^4}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}(3af-be)}{5b^4}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{\sqrt{a+bx^2}\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(7/2)/(7*b**4) - (a + b*x**2)**(5/2)*(3*a*f - b*e
)/(5*b**4) + (a + b*x**2)**(3/2)*(3*a**2*f - 2*a*b*e + b**2*d)/(3
*b**4) - sqrt(a + b*x**2)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
/b**4
```

Mathematica [A] time = 0.122034, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b\left(7e+3fx^2\right)-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(
35*d + 14*e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 1
```

```
5*f*x^6)))/(105*b^4)
```

Maple [A] time = 0.006, size = 99, normalized size = 0.8

$$-\frac{-15\,f x^6 b^3+18\,a b^2 f x^4-21\,b^3 e x^4-24\,a^2 b f x^2+28\,a b^2 e x^2-35\,b^3 d x^2+48\,a^3 f-56\,a^2 b e+70\,a b^2 d-105\,b^3 c}{105\,b^4}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4
-24*a^2*b*f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+7
0*a*b^2*d-105*b^3*c)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7 + e*x^5 + d*x^3 + c*x)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274592, size = 127, normalized size = 1.05

$$\frac{\left(15\,b^3 f x^6+3\left(7\,b^3 e-6\,a b^2 f\right)x^4+105\,b^3 c-70\,a b^2 d+56\,a^2 b e-48\,a^3 f+\left(35\,b^3 d-28\,a b^2 e+24\,a^2 b f\right)x^2\right)\sqrt{b x^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7 + e*x^5 + d*x^3 + c*x)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 7
0*a*b^2*d + 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a
^2*b*f)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 2.49304, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3 f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2 e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2 f x^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+ \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3)
- 2*a*d*sqrt(a + b*x**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(
15*b**2) - 6*a*f*x**4*sqrt(a + b*x**2)/(35*b**2) + c*sqrt(a + b*x
**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) + e*x**4*sqrt(a + b*x**2)/
(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((c*x**2/2 + d*
```

```
x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.219525, size = 207, normalized size = 1.71

$$\frac{105\,\sqrt{bx^2+a}b^3c + 35\,\left(bx^2+a\right)^{\frac{3}{2}}b^2d - 105\,\sqrt{bx^2+a}ab^2d + 15\,\left(bx^2+a\right)^{\frac{7}{2}}f - 63\,\left(bx^2+a\right)^{\frac{5}{2}}af + 105\,\left(bx^2+a\right)^{\frac{3}{2}}a^2f - 105}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7 + e*x^5 + d*x^3 + c*x)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(b*x^2 + a)*b^3*c + 35*(b*x^2 + a)^(3/2)*b^2*d - 1
05*sqrt(b*x^2 + a)*a*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 +
 a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2)*a^2*f - 105*sqrt(b*x^2 + a)
*a^3*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e +
105*sqrt(b*x^2 + a)*a^2*b*e)/b^4
```

3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=261

$$\frac{x^3\left(a\left(162a^3F-71a^2bD+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{7ab^4\left(a+bx^2\right)^{7/2}}+\frac{(2bD-9aF)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(bD-4aF)}{b^5\sqrt{a+bx^2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}$$

```
[Out] ((A*b^4 - a*(b^3*B - a*b^2*C + a^2*b*D - a^3*F))*x^3)/(7*a*b^4*(a
 + b*x^2)^(7/2)) + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b
*D - 24*a^3*F))*x^3)/(35*a^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 +
 a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*D + 162*a^3*F))*x^3)/(105*a^3
*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b^5*Sqrt[a + b*x^2])
 + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rubi [A] time = 1.54255, antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$

$$\frac{x^3\left(a\left(162a^3F-71a^2bD+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{(2bD-9aF)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(bD-4aF)}{b^5\sqrt{a+bx^2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x
^2)^(7/2)) + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 2
4*a^3*F))*x^3)/(35*a^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*
b^3*B + 15*a*b^2*C - 71*a^2*b*D + 162*a^3*F))*x^3)/(105*a^3*b^4*(
a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b^5*Sqrt[a + b*x^2]) + (F*
x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.437863, size = 201, normalized size = 0.77

$$\frac{x\left(945a^7F-210a^6b\left(D-15Fx^2\right)+14a^5b^2x^2\left(261Fx^2-50D\right)+4a^4b^3x^4\left(396Fx^2-203D\right)+a^3b^4x^6\left(105Fx^2-352D\right)+2a\right.}{210a^3b^5\left(a+bx^2\right)^{7/2}}$$
$$+\frac{(2bD-9aF)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210
*a^6*b*(D - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5
*b^2*x^2*(-50*D + 261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2)
 + 2*a^2*b^5*x^2*(35*A + 21*B*x^2 + 15*C*x^4)))/(210*a^3*b^5*(a +
 b*x^2)^(7/2)) + ((2*b*D - 9*a*F)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^
2]])/(2*b^(11/2))
```

Maple [B] time = 0.016, size = 478, normalized size = 1.8

$$-\frac{Dx}{b^4}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,Cxa}{56\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Cx}{7\,ab^3}\frac{1}{\sqrt{bx^2+a}}+\frac{9\,Fax}{2\,b^5}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,aCx^3}{8\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$+\frac{Ax}{35\,ab}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Ax}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Bx^3}{4\,b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3\,Bx}{140\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}$$
$$-\frac{Cx^5}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{Cx}{14\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Dx^7}{7\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{Dx^5}{5\,b^2}\left(bx^2+a\right)^{-\frac{5}{2}}$$
$$-\frac{Dx^3}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{8\,Ax}{105\,a^3b}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,Bxa}{28\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{Bx}{35\,ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$
$$+\frac{3\,Fax^3}{2\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Fx^9}{2\,b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{9\,Fa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{11}{2}}$$
$$+\frac{2\,Bx}{35\,a^2b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,Cxa^2}{56\,b^3}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{4\,Ax}{105\,a^2b}\left(bx^2+a\right)^{-\frac{3}{2}}$$
$$+D\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{9\,Fax^5}{10\,b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{9\,Fax^7}{14\,b^2}\left(bx^2+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -D*x/b^4/(b*x^2+a)^(1/2)+3/56*C*a/b^3*x/(b*x^2+a)^(5/2)+1/7*C/a/b
^3*x/(b*x^2+a)^(1/2)+9/2*F*a/b^5*x/(b*x^2+a)^(1/2)-5/8*C*a/b^2*x^
3/(b*x^2+a)^(7/2)+1/35*A/a/b*x/(b*x^2+a)^(5/2)-1/7*A/b*x/(b*x^2+a
)^(7/2)-1/4*B*x^3/b/(b*x^2+a)^(7/2)+3/140*B/b^2*x/(b*x^2+a)^(5/2)
-1/2*C*x^5/b/(b*x^2+a)^(7/2)+1/14*C/b^3*x/(b*x^2+a)^(3/2)-1/7*D*x
^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5/(b*x^2+a)^(5/2)-1/3*D/b^3*x^3/
(b*x^2+a)^(3/2)+8/105*A/a^3/b*x/(b*x^2+a)^(1/2)-3/28*B*a/b^2*x/(b
*x^2+a)^(7/2)+1/35*B/a/b^2*x/(b*x^2+a)^(3/2)+3/2*F*a/b^4*x^3/(b*x
^2+a)^(3/2)+1/2*F*x^9/b/(b*x^2+a)^(7/2)-9/2*F*a/b^(11/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)-15/56*C*a^
2/b^3*x/(b*x^2+a)^(7/2)+4/105*A/a^2/b*x/(b*x^2+a)^(3/2)+D/b^(9/2)
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+9/10*F*a/b^3*x^5/(b*x^2+a)^(5/2)+9
/14*F*a/b^2*x^7/(b*x^2+a)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm='
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.604369, size = 1, normalized size = 0.

$$\left[\frac{2\left(105\,Fa^3b^4x^9 + 2\left(792\,Fa^4b^3 - 176\,Da^3b^4 + 15\,Ca^2b^5 + 6\,Bab^6 + 8\,Ab^7\right)x^7 + 14\left(261\,Fa^5b^2 - 58\,Da^4b^3 + 3\,Ba^2b^5 + 4\,Aab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm='
```

```
[Out] [1/420*(2*(105*F*a^3*b^4*x^9 + 2*(792*F*a^4*b^3 - 176*D*a^3*b^4 +
 15*C*a^2*b^5 + 6*B*a*b^6 + 8*A*b^7)*x^7 + 14*(261*F*a^5*b^2 - 58
*D*a^4*b^3 + 3*B*a^2*b^5 + 4*A*a*b^6)*x^5 + 70*(45*F*a^6*b - 10*D
*a^5*b^2 + A*a^2*b^5)*x^3 + 105*(9*F*a^7 - 2*D*a^6*b)*x)*sqrt(b*x
^2 + a)*sqrt(b) - 105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a
^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2
- 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*log(-2*sqrt
(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a^3*b^9*x^8 + 4*a^4*b
^8*x^6 + 6*a^5*b^7*x^4 + 4*a^6*b^6*x^2 + a^7*b^5)*sqrt(b)), 1/210
*((105*F*a^3*b^4*x^9 + 2*(792*F*a^4*b^3 - 176*D*a^3*b^4 + 15*C*a^
2*b^5 + 6*B*a*b^6 + 8*A*b^7)*x^7 + 14*(261*F*a^5*b^2 - 58*D*a^4*b
^3 + 3*B*a^2*b^5 + 4*A*a*b^6)*x^5 + 70*(45*F*a^6*b - 10*D*a^5*b^2
 + A*a^2*b^5)*x^3 + 105*(9*F*a^7 - 2*D*a^6*b)*x)*sqrt(b*x^2 + a)*
sqrt(-b) - 105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)
*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a
^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/((a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4 +
 4*a^6*b^6*x^2 + a^7*b^5)*sqrt(-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229006, size = 302, normalized size = 1.16

$$\frac{\left(\left(\left(\left(\frac{105\,Fx^2}{b} + \frac{2\left(792\,Fa^4b^7 - 176\,Da^3b^8 + 15\,Ca^2b^9 + 6\,Bab^{10} + 8\,Ab^{11}\right)}{a^3b^9}\right)x^2 + \frac{14\left(261\,Fa^5b^6 - 58\,Da^4b^7 + 3\,Ba^2b^9 + 4\,Aab^{10}\right)}{a^3b^9}\right)x^2 + \frac{70\left(45\,Fa^6b^5 - 10\,Da^5b^6 +\right.}{a^3b^9}\right.\right.}{210\left(bx^2 + a\right)^{\frac{7}{2}}}$$

$$+\frac{(9\,Fa - 2\,Db)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)*x^2/(b*x^2 + a)^(9/2),x, algorithm='
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a
^2*b^9 + 6*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^
6 - 58*D*a^4*b^7 + 3*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*
(45*F*a^6*b^5 - 10*D*a^5*b^6 + A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9
*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9
*F*a - 2*D*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}} \, dx$

Optimal. Leaf size=214

$$\frac{x(Ab^4-a^4F)}{ab^4(a+bx^2)^{7/2}}+\frac{x^5\left(a\left(-58a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{15a^3b^2(a+bx^2)^{7/2}}+\frac{x^3\left(-10a^4F+ab^3B+6Ab^4\right)}{3a^2b^3(a+bx^2)^{7/2}}+\frac{x^7\left(a\left(-176a^3F+15a^2bD+6ab^2C+8b^3B\right)+48Ab^4\right)}{105a^4b(a+bx^2)^{7/2}}+\frac{F\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] ((A*b^4 - a^4*F)*x)/(a*b^4*(a + b*x^2)^(7/2)) + ((6*A*b^4 + a*b^3
*B - 10*a^4*F)*x^3)/(3*a^2*b^3*(a + b*x^2)^(7/2)) + ((24*A*b^4 +
a*(4*b^3*B + 3*a*b^2*C - 58*a^3*F))*x^5)/(15*a^3*b^2*(a + b*x^2)^
(7/2)) + ((48*A*b^4 + a*(8*b^3*B + 6*a*b^2*C + 15*a^2*b*D - 176*a
^3*F))*x^7)/(105*a^4*b*(a + b*x^2)^(7/2)) + (F*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi [A] time = 0.947392, antiderivative size = 252, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x\left(a\left(122a^3F-45a^2bD+3ab^2C+4b^3B\right)+24Ab^4\right)}{105a^3b^4(a+bx^2)^{3/2}}+\frac{x\left(a\left(-22a^3F+15a^2bD-8ab^2C+b^3B\right)+6Ab^4\right)}{35a^2b^4(a+bx^2)^{5/2}}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7(a+bx^2)^{7/2}}+\frac{x\left(a\left(-176a^3F+15a^2bD+6ab^2C+8b^3B\right)+48Ab^4\right)}{105a^4b^4\sqrt{a+bx^2}}+\frac{F\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2
)^(7/2)) + ((6*A*b^4 + a*(b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3
*F))*x)/(35*a^2*b^4*(a + b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B
+ 3*a*b^2*C - 45*a^2*b*D + 122*a^3*F))*x)/(105*a^3*b^4*(a + b*x^2
)^(3/2)) + ((48*A*b^4 + a*(8*b^3*B + 6*a*b^2*C + 15*a^2*b*D - 176
*a^3*F))*x)/(105*a^4*b^4*Sqrt[a + b*x^2]) + (F*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.424071, size = 176, normalized size = 0.82

$$\frac{x\left(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4-176a^4b^3Fx^6+a^3b^4\left(105A+35Bx^2+21Cx^4+15Dx^6\right)+2a^2b^5x^2\left(105A+14Bx^2\right.\right.}{105a^4b^4(a+bx^2)^{7/2}}+\frac{F\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x
^6 - 176*a^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x
^2*(105*A + 14*B*x^2 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*
C*x^4 + 15*D*x^6)))/(105*a^4*b^4*(a + b*x^2)^(7/2)) + (F*Log[b*x
+ Sqrt[b]*Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [B] time = 0.01, size = 427, normalized size = 2.

$$\frac{Ax}{7a}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{6Ax}{35a^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{8Ax}{35a^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{16Ax}{35a^4}\frac{1}{\sqrt{bx^2+a}}$$
$$-\frac{Bx}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{Bx}{35ab}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{4Bx}{105a^2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{8Bx}{105a^3b}\frac{1}{\sqrt{bx^2+a}}$$
$$-\frac{Cx^3}{4b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{3Cxa}{28b^2}\left(bx^2+a\right)^{-\frac{7}{2}}+\frac{3Cx}{140b^2}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Cx}{35ab^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$
$$+\frac{2Cx}{35a^2b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{Dx^5}{2b}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{5Dx^3a}{8b^2}\left(bx^2+a\right)^{-\frac{7}{2}}-\frac{15Dxa^2}{56b^3}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$+\frac{3Dxa}{56b^3}\left(bx^2+a\right)^{-\frac{5}{2}}+\frac{Dx}{14b^3}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Dx}{7ab^3}\frac{1}{\sqrt{bx^2+a}}-\frac{Fx^7}{7b}\left(bx^2+a\right)^{-\frac{7}{2}}$$
$$-\frac{Fx^5}{5b^2}\left(bx^2+a\right)^{-\frac{5}{2}}-\frac{Fx^3}{3b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Fx}{b^4}\frac{1}{\sqrt{bx^2+a}}+F\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*A*x/a/(b*x^2+a)^(7/2)+6/35*A/a^2*x/(b*x^2+a)^(5/2)+8/35*A/a^3
*x/(b*x^2+a)^(3/2)+16/35*A/a^4*x/(b*x^2+a)^(1/2)-1/7*B/b*x/(b*x^2
+a)^(7/2)+1/35*B/a/b*x/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^
(3/2)+8/105*B*x/a^3/b/(b*x^2+a)^(1/2)-1/4*C*x^3/b/(b*x^2+a)^(7/2)
-3/28*C*a/b^2*x/(b*x^2+a)^(7/2)+3/140*C/b^2*x/(b*x^2+a)^(5/2)+1/3
5*C/a/b^2*x/(b*x^2+a)^(3/2)+2/35*C/a^2/b^2*x/(b*x^2+a)^(1/2)-1/2*
D*x^5/b/(b*x^2+a)^(7/2)-5/8*D*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*D*a
^2/b^3*x/(b*x^2+a)^(7/2)+3/56*D*a/b^3*x/(b*x^2+a)^(5/2)+1/14*D/b^
3*x/(b*x^2+a)^(3/2)+1/7*D/a/b^3*x/(b*x^2+a)^(1/2)-1/7*F*x^7/b/(b*
x^2+a)^(7/2)-1/5*F/b^2*x^5/(b*x^2+a)^(5/2)-1/3*F/b^3*x^3/(b*x^2+a
)^(3/2)-F/b^4*x/(b*x^2+a)^(1/2)+F/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.604811, size = 1, normalized size = 0.

$$\left[-\frac{2\left(\left(176\,Fa^4b^3-15\,Da^3b^4-6\,Ca^2b^5-8\,Bab^6-48\,Ab^7\right)x^7+7\left(58\,Fa^5b^2-3\,Ca^3b^4-4\,Ba^2b^5-24\,Aab^6\right)x^5+35\left(10\,Fa^6\right.\right.}{210\left(a^4b\right.}\right.$$

$$-\frac{\left(\left(176\,Fa^4b^3-15\,Da^3b^4-6\,Ca^2b^5-8\,Bab^6-48\,Ab^7\right)x^7+7\left(58\,Fa^5b^2-3\,Ca^3b^4-4\,Ba^2b^5-24\,Aab^6\right)x^5+35\left(10\,Fa^6b-\right.\right.}{105\left(a^4b^8x^8+4\,a^5b^7x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="fric
```

```
[Out] [-1/210*(2*((176*F*a^4*b^3 - 15*D*a^3*b^4 - 6*C*a^2*b^5 - 8*B*a*b
^6 - 48*A*b^7)*x^7 + 7*(58*F*a^5*b^2 - 3*C*a^3*b^4 - 4*B*a^2*b^5
- 24*A*a*b^6)*x^5 + 35*(10*F*a^6*b - B*a^3*b^4 - 6*A*a^2*b^5)*x^3
 + 105*(F*a^7 - A*a^3*b^4)*x)*sqrt(b*x^2 + a)*sqrt(b) - 105*(F*a^
4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b*x^2 + F
*a^8)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a^4*
b^8*x^8 + 4*a^5*b^7*x^6 + 6*a^6*b^6*x^4 + 4*a^7*b^5*x^2 + a^8*b^4
)*sqrt(b)), -1/105*(((176*F*a^4*b^3 - 15*D*a^3*b^4 - 6*C*a^2*b^5
- 8*B*a*b^6 - 48*A*b^7)*x^7 + 7*(58*F*a^5*b^2 - 3*C*a^3*b^4 - 4*B
*a^2*b^5 - 24*A*a*b^6)*x^5 + 35*(10*F*a^6*b - B*a^3*b^4 - 6*A*a^2
*b^5)*x^3 + 105*(F*a^7 - A*a^3*b^4)*x)*sqrt(b*x^2 + a)*sqrt(-b) -
 105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7
*b*x^2 + F*a^8)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a^4*b^8*x^8
 + 4*a^5*b^7*x^6 + 6*a^6*b^6*x^4 + 4*a^7*b^5*x^2 + a^8*b^4)*sqrt(
-b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228094, size = 275, normalized size = 1.29

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Fa^4b^6-15\,Da^3b^7-6\,Ca^2b^8-8\,Bab^9-48\,Ab^{10}\right)x^2}{a^4b^7}+\frac{7\left(58\,Fa^5b^5-3\,Ca^3b^7-4\,Ba^2b^8-24\,Aab^9\right)}{a^4b^7}\right)+\frac{35\left(10\,Fa^6b^4-Ba^3b^7-6\,Aa^2b^8\right)}{a^4b^7}\right)x^2+\frac{105}{}\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

$$-\frac{F\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/2),x, algorithm="giac
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a
*b^9 - 48*A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 -
 4*B*a^2*b^8 - 24*A*a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*
b^7 - 6*A*a^2*b^8)/(a^4*b^7))*x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(
a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*ln(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(9/2)
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)
^(7/2)) - ((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(
7/2)) - ((192*A*b^3 - a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*
a^4*(a + b*x^2)^(7/2)) - ((384*A*b^4 - a*(48*b^3*B + 8*a*b^2*C +
6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rubi [A] time = 0.65614, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)
^(7/2)) - ((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(
7/2)) - ((192*A*b^3 - a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*
a^4*(a + b*x^2)^(7/2)) - ((384*A*b^4 - a*(48*b^3*B + 8*a*b^2*C +
6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.226936, size = 138, normalized size = 0.72

$$\frac{a^4\left(-105A+105Bx^2+35Cx^4+21Dx^6+15Fx^8\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-210A+21Bx^2 -\right.}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-
210*A + 21*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*
C*x^4 + 3*D*x^6) + a^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6
```

```
+ 15*F*x^8))/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.011, size = 166, normalized size = 0.9

$$-\frac{384\,Ab^4x^8 - 48\,Bab^3x^8 - 8\,Ca^2b^2x^8 - 6\,Da^3bx^8 - 15\,Fa^4x^8 + 1344\,Aab^3x^6 - 168\,Ba^2b^2x^6 - 28\,Ca^3bx^6 - 21\,Da^4x^6 + 168}{105\,xa^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^
8-15*F*a^4*x^8+1344*A*a*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-
21*D*a^4*x^6+1680*A*a^2*b^2*x^4-210*B*a^3*b*x^4-35*C*a^4*x^4+840*
A*a^3*b*x^2-105*B*a^4*x^2+105*A*a^4)/x/(b*x^2+a)^(7/2)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.513402, size = 252, normalized size = 1.31

$$\frac{\left(\left(15\,Fa^4 + 6\,Da^3b + 8\,Ca^2b^2 + 48\,Bab^3 - 384\,Ab^4\right)x^8 + 7\left(3\,Da^4 + 4\,Ca^3b + 24\,Ba^2b^2 - 192\,Aab^3\right)x^6 - 105\,Aa^4 + 35\left(Ca^4 + \right.\right.}{105\left(a^5b^4x^9 + 4\,a^6b^3x^7 + 6\,a^7b^2x^5 + 4\,a^8bx^3 + a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm
```

```
[Out] 1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b
^4)*x^8 + 7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^
6 - 105*A*a^4 + 35*(C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(
B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*
x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228539, size = 297, normalized size = 1.54

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

$$+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8 + D*x^6 + C*x^4 + B*x^2 + A)/((b*x^2 + a)^(9/2)*x^2),x, algorithm
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*
a^10*b^6 - 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*
a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^
13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a
^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2) + 2*A*sqr
t(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```